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MONEY IN CONSUMPTION-LOAN TYPE MODELS: AN ADDENDUM

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MONEY IN CONSUMPTION-LOAN TYPE MODELS: AN ADDENDUM*

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I. The Cass-Okuno-Zilcha Conjecture

In [1] we presented several examples of consumption-loan type models exhibiting the nonexistence of any competitive equilibrium which is Pareto optimal. One sort of example -- involving nonmonotonicity in tastes -- seemed very special (see [1, pp. 62-66]). Evidently, it depended on having just the right combination of some consumption satiation together with some boundary endowments. We conjectured that such speciality wasn't essential to the intuitive proposition being advanced, that somewhat less than total satiation might unfavorably restrict potential intertemporal market transfers -- even given the institution of money, in its role as a store of value, as a common means for facilitating trade between the present and the future. Rather, we proposed as a more likely reason for such speciality the particular analytic methodology being utilized, the limitation to considering only models for which competitive equilibria could be essentially characterized as the nonnegative solutions to a first order difference equation (and thus completely represented in terms of a two-dimensional diagram).

It turns out that while our intuition was correct, our reasoning wasn't. In fact, it is now apparent that the only operative constraint was simply our lack of sufficient ingenuity.

In the next section I sketch a robust nonoptimality example; nonexistence of Pareto optimal equilibrium persists in the presence of small perturbations in both tastes and endowments. The peculiar nature of the satiation phenomena introduced in this example entails other interesting anomalies as well. Two of these additional results are briefly discussed in the final section.

II. A Robust Nonoptimality Example

Consider the basic model described in [1, pp. 44-49], and suppose once again that there are two consumers in every generation but the oldest, $G_0 = \{0\}$ and $G_t = \{2t-1,2t\}$ for $t \geqq 1$, and that odd-numbered consumers, $h = 2t-1$ for $t \geqq 1$, are of α-type, even-numbered, $h = 2t$ for $t \geqq 1$, of β-type. Their respective tastes and endowments are assumed to be as specified in Figure 1a. The critical general features

[insert Fig. 1]

of this specification are that (i) both α- and β-type consumers can become satiated in second period consumption -- though at least one type can never become satiated in first period consumption -- while (ii) the α- but not the β-type consumer is relatively overburdened with second period income. By exploiting the more specific features introduced in order to simplify exposition -- especially, the particular linear structure of both type consumers' offer curves at all except very low real rates of return -- Figure 1b makes plain that, while there are a plethora of both barter and monetary equilibria in this example,

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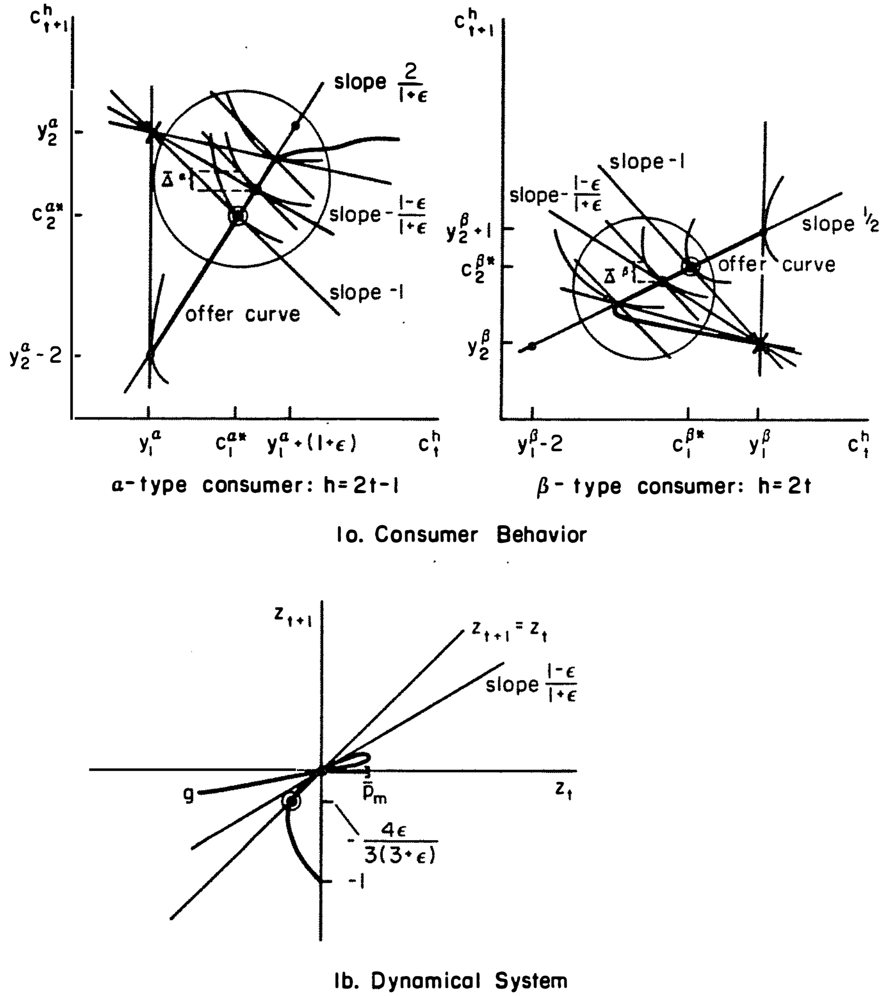


Figure 1. Nonoptimality due to nonmonotonicity: an example with potential satiation in second period consumption

nevertheless every competitive equilibrium must exhibit real rates of return which satisfy the uniform bound

$$\frac{\partial U^h(c^h)}{\partial c^h_t} \Big/ \frac{\partial U^h(c^h)}{\partial c^h_{t+1}} = p_t/p_{t+1} = z_{t+1}/z_t \leqq (1-\varepsilon)/(1+\varepsilon) < 1$$

for h = 2t-1, 2t and t ≧ 1.

Hence, it is easily seen (referring to the encircled portions of Figure 1a, and appealing to the specific characteristics of the geometric construction briefly remarked in the following paragraph) that every competitive allocation is dominated by a corresponding feasible allocation which is identical but for a sequence of sufficiently small one-for-one forward transfers between only α-type consumers

$$0 < -\Delta c^{2t-1}_t = \Delta c^{2t-1}_{t+1} = \Delta c^\alpha < \bar{\Delta}^\alpha \text{ for } t \geqq 1$$

(or, alternatively, only β-type consumers

$$0 < -\Delta c^{2t}_t = \Delta c^{2t}_{t+1} = \Delta c^\beta < \bar{\Delta}^\beta \text{ for } t \geqq 1).$$

That this example is legitimate, and, more importantly, that its welfare significance is invariant to sufficiently small pertubations in both tastes and endowments should become self-evident from close examination of Figure 2. This figure contains directions for

[insert Fig. 2]

constructing a well-behaved utility function consistent with my specification of the α-type consumer; a similar procedure can be employed to justify my specification of the β-type consumer. Since both constructs are very closely patterned after that already analyzed at great length in [1, pp. 71-78], I omit further elaboration here.

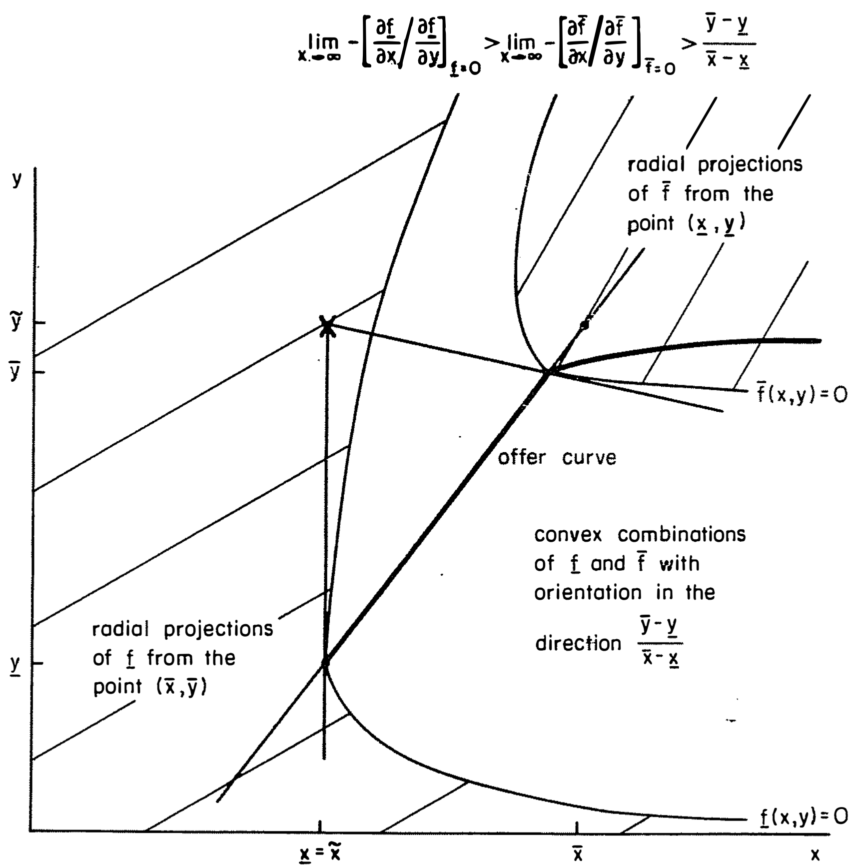


Figure 2. Construction of an offer curve exhibiting "perverse" behavior at extreme relative prices

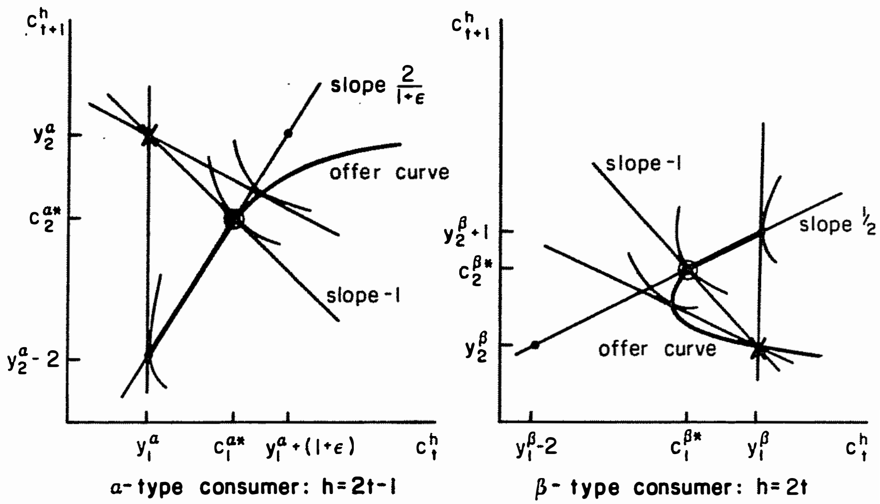
III. Additional Difficulties Associated with Satiation Phenomena

A. Nonexistence of Competitive Equilibrium

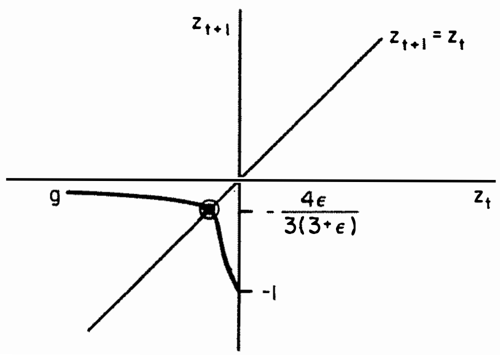
Figure 3 displays a technically minor modification of the preceding example which entails a substantively major conclusion: By merely shifting portions of both consumers' indifference maps -- so that the α-type (resp., β-type) consumer's offer curve is considerably less (resp., more) steeply sloped at negative real rates of return near zero -- competitive equilibrium ceases to exist.

[insert Fig. 3]

On first blush this result is somewhat surprising. All consumers' tastes and endowments continue to enjoy the same regularity properties as before. Upon closer inspection, however, the source of the difficulty is easily discovered. The peculiar second-period nature of satiation has now combined with the intrinsic one-directional nature of time in just such a manner that there is simply not enough structural interrelationship between generations to admit trade -- even trade only among members of each generation -- through the market mechanism. Deficiency of this sort is well-know to create difficulties for existence of competitive equilibrium in the standard Arrow-Debreu model (see, in particular, the illuminating seminal analysis of McKenzie [3]). This version of the example suggests that -- in the intertemporal context -- these difficulties should not be peremptorily dismissed as mere curiosities. It also hints at a commonality between the circumstances giving rise for nonoptimality and those giving rise to nonexistence which should be thoroughly investigated.



3a. Consumer Behavior



3b. Dynamical System

Figure 3. Version of the example where there is no competitive equilibrium (barter or monetary)

B. Ineffectiveness of Social Security

Subject to well-known standard qualifications, the second basic theorem of welfare economics -- that every Pareto optimal allocation is a competitive equilibrium (given an appropriate initial distribution of endowments) -- obtains in consumption-loan type models (see, in particular, the even more comprehensive analysis of McFadden-Majumdar-Mitra [2]). In the present example (in either its original or modified form), for instance, any redistribution of endowments according to the scheme

$$y^{h\delta} = \begin{cases} y^0 - \frac{4\varepsilon}{3(3+\varepsilon)} & \text{for } h = 0 \\ c^{\alpha*} + (\delta,-\delta) & \text{for } h = 2t-1,\ t \geqq 1 \\ c^{\beta*} + (-\delta,\delta) & \text{otherwise} \end{cases}$$

permits attaining a barter equilibrium which is Pareto optimal (supported by prices $p_t = 1$ for $t \geqq 1$ and yielding the allocation

$$c^h = \begin{cases} y^0 - \frac{4\varepsilon}{3(3+\varepsilon)} & \text{for } h = 0 \\ c^{\alpha*} & \text{for } h = 2t-1,\ t \geqq 1 \\ c^{\beta*} & \text{otherwise.)} \end{cases}$$

The central message of this example, of course, is that the historical advent of money as a store of value may not be an adequate proxy for such a redistribution of endowments. Thus, there remains at least partly unanswered an extremely interesting question: What are the simplest extra-market institutional arrangements -- perhaps involving some direct redistribution of endowments -- which will permit attaining Pareto optimality in a wide class of intertemporal market environments?

I hazard the opinion -- based primarily on extensive analysis of consumption-loan type models embodying both variety of commodities and diversity of consumers -- that a minimal qualification for any such arrangement will be a large degree of flexibility in confronting heterogeneity across agents.

This view is suggestively supported (but by no means precisely or conclusively demonstrated), for instance, by considering the scope of a perpetual per capita transfer τ from young to old -- or a simple social security system -- in the present example. It is a fairly straightforward matter to establish that whether or not such uniform social transfers are potent enough to permit attaining Pareto optimality through additional market transfers is problematical, and ultimately depends on whether or not they conceivably admit barter equilibrium at some nonnegative real rate of return. Figure 4 presents a polar version of the example where a simple social security system is necessarily ineffective. The figure is more or less self-explanatory, once it is

[insert Fig. 4]

noted that any shift from first to second period income essentially results in some distorted rotation of the reflected generational offer curve (still defined relative to original endowments) around its intersection with the 45° line in the negative quadrant.

Clearly, this counterexample depends on the particular structure of the distortion displayed in Figure 4b, which in turn

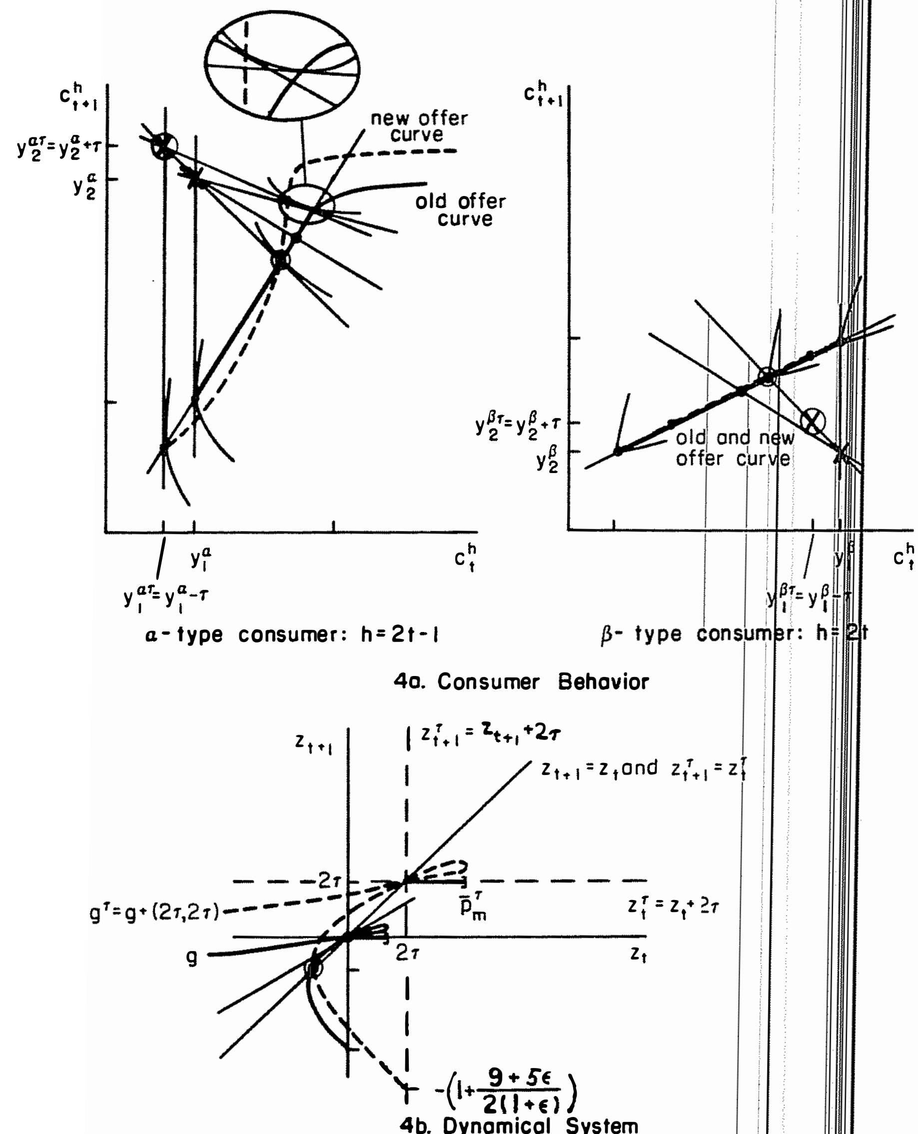


Figure 4. Version of the example where a simple social security system will not permit attaining Pareto optimality

depends on the particular character of the behavior displayed in Figure 4a -- especially the relative sensitivity (resp., absolute insensitivity) of the α-type (resp., β-type) consumer to shifts from first to second period income. The opposite polar version of the present example -- reversing these sensitivities -- yields a model in which a simple social security system is potentially effective.

References

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