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THE EFFECT OF UNCERTAINTY IN REGULATORY DELAY ON THE RATE OF INNOVATION*

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1. Introduction

When a regulated firm considers the undertaking of an effort to innovate, it often faces incentives quite different from those confronting an unregulated firm. At least three types of uncertainty arise. First, will an innovative effort result in an implementable technology, and if so, when? Second, will the implementation of the technology be delayed by a regulatory authority, and if so, for how long? And finally, when the regulator permits the use of an innovation, what level of benefits will the firm ultimately receive?

The first of these questions is not unique to regulation. The rather extensive literature devoted to this issue has been summarized by Kamien and Schwartz. In the past decade a number of theoretical efforts have shown how the structure of industrial markets, the rate of technological progress in those markets, and the incentives for a firm to innovate are affected by uncertainty about the time required for an effort to innovate to reach technological fruition.[1]

Virtually all of the more theoretical studies on research and development have focused on unregulated markets, the most notable exception being a study by Klevorick.[2] Klevorick examined the effects of stochastic regulatory review on the innovative effort undertaken by a regulated firm, and characterized the optimal policy of a firm facing the prospect of regulatory lag. He emphasized the effects of changes in the length of regulatory lag, and in the probability that a firm will undergo regulatory review. Our work differs from his in that we focus on the uncertainty of the length of the delay between

the time of innovation and the time at which a firm receives permission to adopt the innovation.

A number of other studies have addressed innovation in regulated industries, but these have primarily been of descriptive or empirical nature. The empirical investigations have largely attempted to measure the rate of technological progress in regulated industries. For example, Kendrick has estimated the average annual rate of total factor productivity (output per unit of capital and labor, combined) for transportation industries to have been about 3.2 percent between 1899 and 1953. For railroads alone the corresponding estimate was 2.5 percent. These figures compare with an estimate for the aggregated private domestic economy of about 1.7 percent over the same time period.[3] Mansfield has estimated that output per man-hour in the railroad industry rose at an annual rate of 2.5 percent between 1890 and 1925, and at 3.0 percent between 1925 and 1953.[4]

These figures alone do not provide a basis for determining whether the transportation industry has performed well or poorly in innovation. Friedlaender emphasizes this by pointing out that "there is no operational standard against which performance can be judged."[5] The figures do not reflect the extent to which potential opportunities for innovation in the industry were exploited, a point that makes the comparison of rates of innovation across industries even more difficult.[6]

Instead of addressing the aggregate rate of innovation for an entire industry, we direct our analysis to the firm itself. We specify a model of a regulated firm that faces all three types of uncertainty described at the outset of this paper. We focus on the incentives created by regulatory delay, and in particular on the uncertainty introduced by the regulatory process. It is this focus that differentiates this work from most other studies of research and development. Material from selected case studies in surface freight transport will be used to shape and illustrate the analysis.

We first examine some widely held views about innovation and regulation, and show why efforts to innovate are diminished for higher discount rates, higher costs of participating in the regulatory process, and longer regulatory delays. We will also show why uncertainty about the level of benefits ultimately to be received may not affect the level of R&D effort for at least some firms. And finally, we will show why a regulated firm may actually prefer to face uncertainty about the length of regulatory delay, and choose to engage in more R&D under uncertainty than it would if the regulatory delay were known with certainty in advance.

2. Examples of Regulatory Delay

Before formulating a model of innovation with regulatory delay, it would be useful to consider a few examples that illustrate the nature of the dilemma that transport firms face when deciding whether to engage in R&D. If regulators delay the implementation of innovation, then there can be no doubt that incentives to innovate are dampened. Briefly, we examine two specific innovations, the Big John Hopper railroad car for shipping grain and the unit train.

The Big John Hopper

During the late 1950s the shipments of grain to the Southeast grew at a rapid rate. Most of the increased shipments were made by barges or by motor carriers rather than by railroads.[7] The Southern Railway System designed a large aluminum Big John Hopper car to transport grain at a lower cost, and attempted to use the innovation to justify lower tariffs for its grain shipments. Southern proposed new tariffs for grain shipments that were, on average, sixty percent below the existing tariffs; without regulatory delay the tariffs would have become effective in August, 1961.[8]

Not surprisingly, the barge shippers objected to the new tariff.[9] After numerous delays, the ICC finally decided to disallow the lower tariffs on July 1, 1963, on the grounds that the proposed low tariffs would cause unfair and destructive competition in violation of the National Transportation Policy.[10] On appeal, a district court set aside the ICC decision in 1964, based on the lack of substantial evidence in the record to support the ICC decision.[11] The Supreme Court later vacated the decision of the district court and remanded the case to the ICC for reconsideration.[12] In August, 1965, the ICC finally approved the lower tariffs, four years after Southern announced its intention to introduce the Big John Hopper.[13]

The Unit Train

Railroad costs are also reduced when large amounts of a single commodity can be consolidated for movement on a single shipment. A train movement dedicated to hauling only one commodity, for example coal transported from a mine to a utility, will require less switching in transit, and lower costs overall.

Unit train movements were used by the federal government as early as World War I.[14] Yet they were not widely adopted by the railroads until much later. MacAvoy and Sloss have argued persuasively that ICC regulation delayed the adoption of the innovation because of rate restrictions placed on commodities.[15] In particular, the ICC restrictions would have required that a lower rate filed for unit train movements be applied to any shipper using similar services. Thus, "unless the savings on the innovation, in this case the unit trains, are sufficiently great to offset the revenue reductions on the traffic that currently moves at higher rates, the innovation will not be adopted."[16]

In the case of unit trains, the regulatory restriction on tariffs effectively reduced the expected benefit from the innovation. As we shall show more formally in section four this has the consequence of deterring the rate of expenditures on developing better unit train technology.

These examples are not presented for the purpose of examining the merits of the arguments involved. Rather, they illustrate the powerful effects of the regulatory process in determining the level of benefit ultimately accruing to an innovator, in introducing uncertainty about the level of benefits, and in causing regulatory delay of uncertain length to be a part of the environment within which a firm must make its decision whether to engage in R&D. A number of other examples of the effects of regulation on innovation could be

cited to illustrate similar points.[17] These considerations help to shape the model of innovation and regulation to which we now turn.

3. A Model of Innovation and Regulatory Delay

Consider a regulated firm that is deciding whether to undertake an effort to innovate, and that faces all three of the kinds of uncertainty described at the outset of section one of this paper. We shall assume that if the firm elects to undertake the project, it enters into a fixed cost R&D contract.[18] Let us denote the present value of the cost of the R&D effort as being x dollars, where the level of x is chosen by the firm.[19] This money will be spent whether or not the project results in an actual innovation, and independent of whether regulators ultimately allow the innovation to be implemented.

Given the decision to spend x dollars on R&D, there is uncertainty about the arrival time of a technologically feasible design. Although the firm does not know exactly when the innovation will occur, it does assign some probability that the innovation will occur before any given time (e.g., t_1) when the size of its R&D program is x dollars.[20] Moreover, the firm believes that by increasing x, it can increase the probability that the innovation will arrive by any specified time.[21]

Once the innovation is developed, the firm may have to seek regulatory approval in order to implement the innovation, particularly if the firm seeks changes in tariffs or operating authority as a result of the innovation. Of course, not all innovations will require regulatory approval prior to implementation, particularly those that do not present a problem with safety, pose a threat to other firms' profits, or require tariff adjustments for the innovating firm. These innovations do not concern us since regulatory delay is not an issue.

However, two further uncertainties do arise when the regulatory action is required. The first is the uncertainty about the length of the delay, which we denote by t_2. Thus, the total time from the initiation of the R&D effort until actual implementation will be $(t_1 + t_2)$, where implementation does not occur until the regulatory delay is completed.[22] The length of t_2 to some extent depends on the resistance that the innovator will encounter during the delay. Let us represent the extent of this resistance by R. For example, R might be thought of as the extent of rivalry from intermodal competitors, whose market shares might decline if the innovation is adopted at tariffs proposed by the innovator. As mentioned earlier, when the Southern decided to develop the Big John Hopper, it encountered much resistance from the barges and other shippers. Thus, qualitatively, R was large in that case.

Given a level of resistance, R, there remains uncertainty about the length of the delay, t_2. Although the firm does not know exactly when the delay will end, it does assign some probability that the delay will be shorter than any given duration (e.g., t_2).[23] Moreover, the firm believes that a higher level of resistance, R, will reduce the probability that the regulatory delay will end by any specified time.[24]

Finally, there is uncertainty about the extent to which the firm will benefit from the innovation. For example, if the innovation

reduces the cost of providing a service subject to intermodal competition, the firm does not know in advance what will ultimately be the tariff in effect for its own service, for the services provided by its rivals, or the exact amount the cost reduction actually achieved. We assume that the level of dollar benefits (additional profits) accruing to the firm after t_2 will be a flow of b dollars. Although b is uncertain, we assume that the firm can assign some probability that the level of benefits will be less than any particular level (e.g., b_1), given the level of resistance, R.[25]

Although a change in R may affect the probability of receiving at least b_1 dollars, the direction of the effect is not obvious. On one hand, with strong intermodal competition an innovator may encounter stronger arguments against the tariffs and other conditions of service it proposes following an innovation. Thus, expected benefits might decline as rivalry increases.

On the other hand, a firm facing stronger intermodal competition might expect to capture large shares of the market from its competitors if an innovation is successful. Thus, expected benefits might increase with more rivalry.[26]

We assume that the firm selects the level of R&D expenditures, x, to maximize T, the present value of the expected profits resulting from the effort to innovate, where the discount rate for the firm is r.[27] There are three components that when added together, comprise T. First, as stated earlier, the present value of the cost of the R&D project is x.[28] Second, between t_1, when the innovation arrives, and $(t_1 + t_2)$, when the regulatory delay is concluded, the firm is involved in legal proceedings before the regulator, and perhaps the court system. We assume that the firm incurs a procedural cost flow of L dollars to maintain a "standard" level of effort in those proceedings. (We do not specify the level of effort as a decision variable for the firm; this possibility suggests an interesting extension of the present analysis.) Thus the present value of these procedural expenditures incurred between t_1 and $(t_1 + t_2)$ constitutes the second component of T.[29]

The third component of T is the present value of the benefit flow. We assume that the benefits, b, are received forever after time $(t_1 + t_2)$.[30] Finally, the firm determines T by adding all three components, and by taking the expectation of the sum over all of the possible values of the uncertain variables t_1, t_2, and b.[31] The firm then chooses that level of x that maximizes T.[32]

4. Effects of a Change in Discount Rate, Procedural Cost, and the Extent of Intermodal Competition on Innovation

Three observations can be stated immediately about the incentives to innovate for the regulated firm. These do not deal with uncertainty, a topic that we reserve for section five. We begin by asking how a change in the discount rate would affect the level of innovation.[33]

> Proposition 1. A higher discount rate reduces the amount of innovation undertaken by the firm. This effect is observed even in the absence of regulatory delay (i.e., even when $t_2 = 0$).[34]

The rationale for this statement is straightforward. The presence of a discount factor means that the firm weighs a dollar received (or spent) earlier in the stream of time more heavily than a dollar received (or spent) later on. Since the firm incurs R&D costs and procedural costs before it receives benefits, a higher discount rate makes innovation less attractive.

One might also expect that a higher procedural cost, L, leads to less innovation. Higher procedural costs make any prospective innovation less profitable if that innovation is subject to regulatory delays. Proposition 2 shows that this is true.

> Proposition 2. An increase in procedural cost, L, leads to a lower level of innovation by the firm.[35]

We next turn to the effect of increased rivalry (e.g., increased intermodal competition). Increased rivalry affects the incentives to innovate in two ways. First, more intense intermodal competition typically lengthens regulatory delay. Second, it may either increase or reduce the level of benefits, b, ultimately realized by the innovator, as noted earlier. Thus, the two effects combined make the effects of increased intermodal competition on the rate of innovation generally ambiguous.

> Proposition 3. Increased intermodal competition will decrease the level of innovative effort if the probability of receiving at least b_1 dollars of net benefits remains constant or falls as competition becomes more intense

(R rises) for all b_1. Otherwise, the level of innovative effort may either rise or fall as intermodal competition becomes more intense.[36]

In other words, increased intermodal competition can be expected to lengthen regulatory delay. If it also lowers expected benefits, then innovation looks less attractive to the firm. However, if it raises expected benefits, then the firm expects higher benefits at a more distant time, and the present value of the innovation (and thus the incentive to innovate) may either rise or fall in that case.

5. Effects of Uncertainty on the Effort to Innovate

In this section we examine how uncertainty about the level of benefits and the extent of regulatory delay affect the firm. We ask whether a firm facing uncertainty about the length of regulatory delay and the ultimate flow of benefits would change its level of innovative effort if it knew in advance with certainty what the delay and benefit flow would be.

To start with we note that the phrase "with certainty" is imprecise. The choice of an appropriate meaning is not obvious. We employ the following notions in the work that follows.

Definition 1. "Certainty about the level of benefit flow" is defined to mean that the firm knows in advance that the flow of benefits will be equal to the mean of the probability distribution for b, $\bar{b}(R)$.[37]

Definition 2. "Certainty about the length of regulatory delay" is defined to mean that the firm knows in advance that the actual delay will be the mean of the probability distribution for t_2, $\bar{t}_2(R)$.[38]

Then the following two statements can be made immediately.[39]

Proposition 4. The firm undertakes the same level of innovative effort under either uncertainty or certainty about the level of benefit flow.

Proposition 5. The firm expects the same profit under uncertainty as under certainty about the level of benefit flow.

These statements are possible because the firm is assumed to be risk neutral.[40] Of course, if the firm knew in advance that the actual benefit flow would differ from the mean, $\bar{b}(R)$, then the firm would prefer that certain knowledge to a state of uncertainty. But absent such knowledge, the innovative effort of the firm is not affected by uncertainty about the level of benefit flow.

Finally, we turn to the effect of uncertainty about regulatory delay on the level of innovation. If anything, one might expect that the firm would prefer to operate with certainty rather than uncertainty, since timing is an important aspect of innovation, particularly if the firm incurs costs of coordination that arise if the actual time of regulatory delay differs from the expected length of the delay. Obviously, if these coordination costs are high enough, then the firm would indeed prefer certainty to uncertainty.

However, the point of the following two propositions is that a firm might not always prefer certainty to uncertainty. In particular, we show this for the case in which no such coordination costs are incurred. Using the notion of uncertainty of Definition 2, it turns out that, absent coordination costs, the firm would prefer uncertainty about the length of regulatory delay. Further, the firm would undertake a higher level of innovative effort under uncertainty.

<u>Proposition 6</u>. Without coordination costs, the firm innovates more under uncertainty than under certainty about the length of regulatory delay.[41]

<u>Proposition 7</u>. Without coordination costs, the firm prefers uncertainty to certainty about the length of the regulatory delay.[42]

Although these results may appear to be counterintuitive, they do make sense after some reflection. To show how, we construct the following example. Assume that an innovation has just been achieved, i.e., we have arrived at time t_1. The annual benefit flow that will be realized when t_2 is reached (after regulatory delay has occurred and the innovation can be implemented) is known to be \$1 million. The regulatory delay is uncertain, but the firm assumes that it will be one year, two years, or three years with an equal

probability of one-third in each case. There are no coordination costs.

For simplicity, assume that $L = 0$, i.e., the procedural costs are zero. Then Table 1 enables us to calculate the present value of the expected benefits of the innovation, discounted to time t_1. Note that the present value of the expected benefit of innovation is \$8.22 million, when the discount rate is assumed to be ten percent.

For the case of certainty, we assume that t_2 is known to equal the mean (or average) value of delay of two years. But, as Table 1 shows, the benefit corresponding to a delay of two years is \$8.18 million, which is less than the expected benefit of \$8.22 million for the case with uncertainty.

The reason for the preference for uncertainty is now apparent. The firm is willing to take a chance that the delay will be less than the mean since it discounts earlier earnings to a lesser extent than later earnings.

We reemphasize that a certain knowledge that the actual delay would differ from the mean may lead the firm to reverse its preference for uncertainty. However, absent such certain knowledge, and with small or no coordination costs, the firm may actually undertake an R&D project under uncertainty about the length of regulatory delay that it would not pursue if it knew in advance that the delay would be the mean of the distribution.

TABLE 1

Expected Benefit of Innovation: Example

Regulatory Delay (Years) (1)	Probability of Delay of Column 1 (2)	Present Value of Benefits at t_1 (3)[43]	Col (2) x Col (3) (4)
1	1/3	$9.05 million	$3.02 million
2	1/3	$8.18 million	$2.73 million
3	1/3	$7.41 million	$2.47 million
	Present value of expected benefit =		$8.22 million

6. Conclusion

We have analyzed a formal model of regulatory delay and drawn a number of conclusions. Several of these conclusions correspond to widely accepted notions about the effect of the regulatory process on innovation. Higher discount rates, higher costs of engaging the administrative process, and longer regulatory delay all serve to diminish the amount of R&D undertaken by the regulated firm. Increased intermodal competition has ambiguous effects on incentives for innovation. Uncertainty about the level of the benefit flow does not affect incentives for R&D for a risk neutral firm. And finally, uncertainty about the length of regulatory delay may actually increase incentives to innovate, particularly if coordination costs are small.

In an effort to focus on the effects of regulatory delay, we have made a number of simplifying structural assumptions that suggest directions for further research. One potential direction for research would be to expand the scope of the model to investigate broader questions about market equilibrium, in the same manner that Loury, and Lee and Wilde have extended the analysis of Kamien and Schwartz.[44] Such a model would explicitly include the threat of innovation by other firms competing with any given firm. It would also require a structure to replace the parametric resistance parameter R in this paper, as well as a more detailed representation of the decision mechanism used by regulators in determining the benefits allowed and the length of regulatory delay. While these issues are beyond the scope of this paper, the basic structure presented here should prove helpful in those further efforts.

FOOTNOTES

1. Kamien, M., and Schwartz, N., "Market Structure and Innovation: A Survey," 13 Jour. Econ. Lit., 1, pp. 1-37. Notable among these studies are the following. Kamien M. and Schwartz, N., "Timing of Innovations Under Rivalry," 40 Econometrica, 1, Jan. 1972, pp. 43-60. Kamien, M., and Schwartz, N., "On the Degree of Rivalry for Maximum Innovative Activity," 40 Quart. J. Econ., 1976, pp. 245-60. Loury, Glenn C., "Market Structure and Innovation, Quart. J. Econ., forthcoming, 1979. Scherer, F.M., "Firm Size, Market Structure, Opportunity, and the Output of Patented Inventions, 55 Amer. Econ. Rev., 5, pp. 1097-1125. Scherer, F.M., "Research and Development Resource Allocation Under Rivalry," 81 Quart. J. Econ., 3, pp. 359-94. Lee, T., and Wilde, L., "Market Structure and Innovation: A Reformulation," Quart. J. Econ., forthcoming (1979).

2. Klevorick, A., "The Behavior of the Firm Subject to Stochastic Regulatory Review," The Bell Journal of Economics and Management Science, Vol. 4, No. 1, pp. 57-88.

3. Kendrick, J., Productivity Trends in the United States, National Bureau of Economic Research, Princeton Univ. Press, 1961, pp. 136-137.

4. Mansfield, E., "Innovation and Technical Change in the Railroad Industry," Transportation Economics, National Bureau of Economic Research, Columbia Univ. Press, 1965, p. 171.

5. Friedlaender, A., The Dilemma of Freight Transport Regulation, The Brookings Institution, 1969, p. 88.

6. Ibid., pp. 88-89.

7. For a good discussion of the market conditions at the time of the Big John Hopper innovation, see Barber, R., "Technological Change in American Transportation: The Role of Government Action," 50 Virginia Law Review, 5, June 1964, p. 863.

8. Gellman, A., "Surface Freight Transportation," in Technological Change in Regulated Industries, ed. W. Capron, The Brookings Institution, 1971, p. 175.

9. "Petition of Protestants for Reconsideration of the Report and Order of Division 2, before the ICC, Investigation and Suspension Docket 7656 'Grain in Multiple Car Shipments -- River Crossing to the South,'" filed Feb. 25, 1963.

10. "Grain and Multiple car shipments -- River Crossings to the South," 321 ICC 582, July 1, 1963.

11. The Cincinnati, New Orleans and Texas Pacific Railway Co. v. U.S., 29 F. Supp. 572 (1964).

12. 379 U.S. 642, Jan. 18, 1965.

13. 325 ICC 752, Aug. 30, 1965.

14. Friedlaender (n. 5, *supra*), p. 94.

15. MacAvoy, P., and Sloss, J., Regulation of Transport Innovation: The ICC and Unit Coal Trains to the East Coast, Random House, 1967, pp. 59-85.

16. Friedlaender (n. 5, *supra*), p. 94.

17. For example, the costing procedures of the ICC may have affected the timing of the introduction of railroad piggybacking (trailer on flat car) operations. For a good discussion of this, see Gellman (n. 8, *supra*), pp. 170-174.

18. The term fixed cost means that the level of the costs incurred does not vary with the time until the innovation is actually achieved. This is the form of R&D costs investigated by Loury (see n. 2, *supra*). Of course, other forms of R&D costs are possible. For example Lee and Wilde (see n. 2, *supra*) have investigated the case in which the firm incurs both some fixed cost, whose level does not vary with the time until the innovation is actually achieved, and a variable cost, which adds to the fixed cost an additional cost per time period until the innovation arrives.

19. The actual payment of the costs of the R&D project could be for x dollars in full at the outset of the project. Or, it may be incurred at some later time, t, in an amount of y dollars. If r is the nominal interest rate paid continuously, then the y dollars paid at time t has a present value of x dollars when $x = ye^{-rt}$, where e is approximately equal to 2.71828. See Chiang, A., Fundamental Methods of Mathematical Economics, second edition, McGraw-Hill, 1974, pp. 293-294 and 456-459.

20. Let $f(t_1,x)$ represent the probability density function for the arrival time, t_1, given an R&D expenditure level of x. Then $F(t_1,x)$ is the corresponding cumulative density function, which states the probability that the innovation will arrive by time t_1, given x.

21. This statement is equivalent to the condition that $F(t_1,x)$ increases as x increases. In other words, the partial derivative of $F(t_1,x)$ with respect to x is positive. For the balance of this paper we will denote the partial derivative of one variable

with respect to a second variable by subscribing the first with the second. Thus the partial derivative of $F(t_1,x)$ with respect to x is represented by $F_x(t_1,x)$.

22. In some cases the innovation may actually be introduced (to some extent) before the regulatory action is concluded. For example, in the Big John Hopper case, the Southern had placed some of the grain cars in service, and at some lower tariffs, during the delay. But the full benefits of the innovation were far from being realized during this time. See Gellman (n. 8 supra) p. 177. The qualitative results about the effects of regulatory delay on innovation, which we develop below, would hold even if some (but not all) benefits are realized before the delay is over.

23. Let $g(t_2,R)$ represent the probability density function for the length of regulatory delay, given R. Then $G(t_2,R)$ is the corresponding cumulative density function, which states the probability that the delay will be shorter than t_2, given R.

24. This statement is equivalent to the condition that $G(t_2,R)$ declines as R increases, or, following the notation of n. 21, supra, $G_R(t_2,R) < 0$.

25. Let h(b,R) represent the probability density function for the level of benefits, b, given R. Then H(b,R) is the corresponding cumulative density function, which states the probability that the level of benefit will be no more than b dollars given R.

26. This statement is equivalent to the condition that H(b,R) may rise or fall as R increases, or, following the notation of n. 21, supra, $H_R(b,R)$ has no determinate sign.

27. The notion of present value is discussed in n. 19, supra. By working with expected profits, we are assuming that the firm is risk neutral rather than risk averse or risk loving. (See Varian, H., Microeconomic Analysis, Norton, 1978, pp. 108-109.) Suppose the firm could accept a lottery which would give it w dollars with probability p, and y dollars with probability (1 - p). If the firm is indifferent between accepting the lottery and accepting [pw + (1 - p)y] dollars with certainty, then it is risk neutral. If it prefers the former, it is risk loving; if it prefers the latter, it is risk averse.

28. See n. 19, supra. Since it is a cost, its contribution to T is -x.

29. The expected present value of this component of the present value of profit is therefore

$$-\int_{t_1}^{t_1+t_2} L\, e^{-rt}dt \tag{1}$$

30. The expected present value of the benefit stream is

$$\int_{t_1+t_2}^{\infty} b\, e^{-rt}dt \tag{2}$$

31. Formally, the present value of the expected profits from the innovation can be written as

$$T(x,R,r,L) = -x + \int_0^{\infty}\int_0^{\infty} \Big\{(-L)\int_{t_1}^{t_1+t_2} e^{-rt}dt \tag{3}$$

$$+ \int_{t_1+t_2}^{\infty} \Big[\int_{-\infty}^{+\infty} bh(b,R)db\Big] e^{-rt}dt\Big\} f(t_1,x)\, g(t_2,R)dt_1dt_2$$

$$= -x + \frac{L+\bar{b}(R)}{r} \int_0^{\infty} e^{-rt_1}\, f(t_1,x)dt_1 \int_0^{\infty} e^{-rt_2}\, g(t_2,R)dt_2$$

$$- \frac{L}{R}\int_0^{\infty} e^{-rt_1}\, f(t_1,x)dt_1$$

$$= -x + (L+\bar{b}(R)) \int_0^{\infty} e^{-rt_2}\, F(t_1,x)dt_1 \int_0^{\infty} e^{-rt_2}\, g(t_2,R)dt_2$$

$$- L\int_0^{\infty} e^{-rt_1}\, F(t_1,x)dt_1$$

where

$$\bar{b}(R) = \int_{-\infty}^{\infty} bh(b,R)db \text{ , the mean of } h(b,R) \text{ given R.} \tag{4}$$

32. The first order necessary condition for an optimum at which $x > 0$ is:

$$T_x = -1 + (L+\bar{b}(R))\int_0^{\infty} e^{-rt_1}\, F_x(t_1,x)dt_1 \int_0^{\infty} e^{-rt_2}\, g(t_2,R)dt_2 \tag{5}$$

$$- L\int_0^{\infty} e^{-rt_1}\, F_x(t_1,x)dt_1 = 0.$$

Further, define $\phi_x(x)$ to be $\int_0^{\infty} e^{-rt_1}\, F_x(t_1,x)dt_1$. Then from Eq. 5, we have

$$\phi_x(x) = 1 \Big/ \Big\{\Big[L+\bar{b}(R)\Big]\int_0^{\infty} e^{-rt_2}\, g(t_2,R)dt_2 - L\Big\} > 0 \tag{6}$$

Finally, at a maximum of T, then $T_{xx} < 0$, which implies that $\phi_{xx}(x) < 0$. For a discussion of necessary and sufficient conditions for optimality, see Varian (n. 27, <u>supra</u>), pp. 262-267.

33. For a discussion of the discount rate, see n. 19 supra.

34. Proof: From equations (5) and (6), n. 32, it follows that

$$T_{xr} = -t_1\int_0^\infty e^{-rt_1} F_x(t_1,x)dt_1\left[(L+\bar{b}(R))\int_0^\infty e^{-rt_2} g(t_2,R)dt_2 - L\right]$$

$$-t_2\ (L+\bar{b}(R))\int_0^\infty e^{-rt_2} F_x(t_1,x)dt_1\int_0^\infty e^{-rt_2} g(t_2,R)dt_2 < 0$$

Thus $dx/dr = -T_{xr}/T_{xx} < 0$, even if $t_2 = 0$. For a discussion of comparative statics, used to prove this and subsequent propositions, see Varian (n. 27), pp. 267-269.

35. Proof: From Equation (5), n. 32, and the fact that

$\int_0^\infty e^{-rt_2} g(t_2,R)dt_2 < 1$ when there is any possibility of regulatory delay, it follows that

$$T_{xL} = \int_0^\infty e^{-rt_1} F_x(t_1,x)dt_1\left[\int_0^\infty e^{-rt_2} g(t_2,R)dt_2 - 1\right] < 0$$

Thus $dx/dL = -T_{xL}/T_{xx} < 0$.

36. We may rewrite equation (5), n. 32 using the fact that

$$\int_0^\infty e^{-rt_2} g(t_2,R)dt_2 = r\int_0^\infty e^{-rt_2} G(t_2,R)dt_2\ .$$

It follows that

$$T_{xR} = r\bar{b}_R(R)\int_0^\infty e^{-rt_1} F_x(t_1,x)dt_1\int_0^\infty e^{-rt_2} G(t_2,R)dt_2$$

$$+\ r(L+\bar{b}(R))\int_0^\infty e^{-rt_1} F_x(t_1,x)dt_1\int_0^\infty e^{-rt_2} G_R(t_2,R)dt_2 < 0\ .$$

Thus, $dx/dR = -T_{xR}/T_{xx} < 0$, when $\bar{b}_R(R) \leq 0$, and may be either positive or negative when $\bar{b}_R(R) > 0$.

37. Recall that $\bar{b}(R) = \int_{-\infty}^{+\infty} b\,h(b,R)db$, from n. 31, supra.

38. Thus, $\bar{t}_2(R) = \int_0^\infty t_2 g(t_2,R)dt_2$.

39. Proof of Proposition 4 and Proposition 5. Both follow directly from the observation that the level of x that satisfies equation (5), n. 32, remains unchanged when $\bar{b}(R)$ replaces $\int_{-\infty}^{\infty} b\,h(b,R)db$. Similarly, the value of T in equation (3), n. 31, is not affected.

40. For a discussion of risk neutrality, see n. 27, supra.

41. Proof: Let $\bar{t}_2(R)$ be as defined in n. 38, supra. By Jensen's Inequality, $\int_0^\infty e^{-rt_2} g(t_2,R)dt_2 > e^{-r\bar{t}_2(R)}$. Using equation (6),

n. 32, let $\phi_x^A(x)$ be the value of x when t_2 is known to be $\bar{t}_2(R)$. Let x_A solve

$$\phi_x^A(x) = 1/\{(L+\bar{b}(R))e^{-r\bar{t}_2(R)}-L\}$$

Further, let $\phi_x^B(x)$ be the value of $\phi_x(x)$ with uncertainty. Let x_B solve

$$\phi_x^B(x) = 1/\{(L+\bar{b}(R))\int_o^\infty e^{-rt_2}\, g(t_2,R)dt_2 - L\}$$

Then $\phi_x^A(x) > \phi_x^B(x)$ for any x. Since $\phi_{xx}(x) < 0$, from n. 32, supra, then $x_A < x_B$.

42. Proof: We first note that equation (5), n. 32, implies that T at x_B (which we denote by $T(x_B)$) is larger than T at x_A (which we denote by $T(x_A)$), i.e. $T(x_B) > T(x_A)$ for $x_A \neq x_B$. Further, for any (x,t_2) combination, define $v(x,R,r,L;t_2)$ to mean the value of the innovation given any t_2, where

$$v(x,R,r,L;t_2) = -x + [L+\bar{b}(R)](\int_o^\infty e^{-rt_1}\, F(t_1,x)dt_1)e^{-rt_2}$$

$$-L\int_o^\infty e^{-rt_1}\, F(x,t_1)dt_1$$

and that

$$v_{t_2t_2}(x,R,r,L;t_2) = r^2[L+\bar{b}(R)](\int_o^\infty e^{-rt_1}\, F(t_1,x)dt_1)e^{-rt_2} > 0.$$

By Jensen's Inequality, $T(x_A) > v(x_A,R,r,L;\bar{t}_2(R))$

Thus $T(x_B) > T(x_A) > v(x_A,R,r,L;\bar{t}_2(R))$.

Using this method of proof, by direct extension it follows that Propositions 6 and 7 hold for both risk neutral and risk preferring firms, and for some risk averse firms. However, for a strong enough aversion to risk, the preference for a firm would switch to certainty.

43. The present value of the benefit for each year is calculated for each entry in column (3) as follows, using the procedure of Chiang, n. 19, supra, pp. 456-459:

Delay (Years)	Present value of benefit, discounted to t_1
1	(\$1 million) $\int_1^\infty e^{-rt}dt$ = \$9.05 million
2	(\$1 million) $\int_2^\infty e^{-rt}dt$ = \$8.18 million
3	(\$1 million) $\int_3^\infty e^{-rt}dt$ = \$7.41 million

44. Citations to the contributions of Kamien and Schwartz, Loury, and Lee and Wilde are found in n. 1, *supra*.