UNCERTAINTY AND THE THEORY OF TAX INCIDENCE IN A STOCK MARKET ECONOMY

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1. INTRODUCTION

Commencing with Harberger's (1962) classic paper, a number of studies have analyzed the incidence of taxation in the context of a deterministic, two-sector, two-factor general equilibrium model. Recently, R. N. Batra (1975) and R. A. Ratti and P. Shome (1977a, 1977b) have reexamined the robustness of these deterministic results for the case in which production uncertainty is incorporated into the model. By using "entrepreneurial" models in which the firm is assumed to maximize the expected utility of profits, they find that the incidence of taxes depends on the preferences and probability assessments of the entrepreneur, and in general, the deterministic results no longer obtain.

Most firms, however, are not owned by a single individual, and Batra and Ratti and Shome do not indicate how appropriate their results are for other ownership forms. In particular, their models do not utilize any form of risk-sharing arrangements such as those available through the securities markets. In the presence of a stock market, it will be shown that the standard deterministic results continue to hold for the firm in their economy if the firm has publicly-traded securities and acts in the best interests of its shareholders. With this shareholders' interests criterion and the Batra-Ratti-Shome model, the securities market is sufficient to separate the production decisions of the firm from the portfolio-consumption decisions of shareholders. In a related analysis, Baron and Forsythe (1979) focus on the role of the securities market in establishing unanimity among shareholders about the value-maximization criterion for firms. Here, the emphasis is on the impact of taxes on production and factor rewards. Because of separation, the equilibrium in the securities, output, and factor markets has the same qualitative properties as in a deterministic model, and the standard propositions regarding the incidence of taxation continue to hold. For expositional purposes only, the analysis will be limited to the study of the effect of the corporate income tax, but in the final section the results for other forms of taxation, such as those considered by Mieszkowski (1967), will be shown to also extend to the stochastic economy considered here.

2. THE MODEL AND EQUILIBRIUM

A. Firms

Following the model specified by Harberger, a two-factor model is considered in which $X_1$ is produced in the corporate sector and $X_2$ is the noncorporate sector. Production takes place under conditions of perfect competition, full employment, inelastic factor supplies, and irreversible factor intensities. The quantities of capital and labor employed in the $j$th sector are denoted by $K_j$ and $L_j$, respectively, and $\bar{K}$ and $\bar{L}$ are the total fixed supplies of each factor.
The output of the corporate sector is subject to uncertainty with a production function of the form

\[ X_1 = \alpha F_1(K_1, L_1), \]

where \( \alpha \) is a random variable, \( \alpha \geq 0 \), representing exogenous and uncertain influences affecting output. The output of the noncorporate sector is assumed to be deterministic and given by

\[ X_2 = F_2(K_2, L_2). \]

Although each sector is assumed to be composed of many firms, only a representative firm in each sector will be analyzed in order to simplify the notation. The production functions \( F_j, j = 1,2 \), are assumed to be linear homogeneous and concave so that

\[ F_j(K_j, L_j) = L_j f_j(k_j), \quad j = 1,2, \]

where \( k_j \) is the capital-labor ratio in sector \( j \), \( f_j' > 0 \), and \( f_j'' < 0 \).

It is assumed that firms make their input decisions at the beginning of the period, prior to the realization \( \alpha \), by contracting for labor at the competitive wage rate \( w \) and financing their capital purchases by selling bonds, \( B_j = K_j, \quad j = 1,2 \), which yield a deterministic gross rate of return \( r_j (r_j \geq 1) \) determined in a securities market. At the completion of trades in the securities and factor markets, the contracted levels of inputs are employed and \( \alpha \) is realized, as is output. The market clearing price in the corporate sector output market depends on \( \alpha \) and hence is uncertain at the time input decisions are made.

Ratti and Shome (1977a) recognize that the price is uncertain but, in order to avoid dealing with price uncertainty, they assume a small country for which product prices are given by world markets. Batra does not make the small country assumption, yet assumes that the output price is not random. In noting this, Ratti and Shome (1977b) suggest that when assuming a large country, the price which equates expected demand to expected supply should be used. As will be demonstrated, these assumptions are unnecessary, since an uncertain price does not affect the standard tax incidence results for the model considered here.\(^3\)

When the output market clears, factors are paid their wages, and the after-tax earnings are then distributed to shareholders in proportion to their holdings. Finally, it is assumed that all commitments to factor inputs are met and that there is no risk of default on the bond obligations.\(^4\) Letting \( p(\alpha) \) denote the price of output in the first sector expressed in terms of the price in the second sector, the after-tax earnings \( \Pi_1(\alpha) \) of the corporate sector may be expressed as

\[
\Pi_1(\alpha) = (1 - t)(p(\alpha)\alpha F_1(K_1, L_1) - r_1B_1 - wL_1)
\]

\[ = (1 - t)(p(\alpha)\alpha L_1 f_1(k_1) - L_1(r_1k_1 + \omega)), \]
where $t$ is the corporate income tax rate. The tax system is assumed to be such that the corporate tax involves full loss offset. The earnings of the noncorporate sector are given by

$$ H_2 = F_2(K_2, L_2) - r_2B_2 - wL_2 - L_2 f_2(k_2) - L_2 (r_2 k_2 + w). $$

Adopting the view of Harberger (p. 215) that the corporate income tax is one "which strikes the earnings of capital in the corporate sector, but not in the noncorporate sector," the return on the debt as well as the equity of a firm in the first sector is subject to the tax. In this case, the appropriate equilibrium condition in the bond market is

$$(1 - t)r_1 = r_2. $$

Thus, the after-tax return to the equity of the corporate sector can be rewritten as

$$ H_1(a) = (1 - t)[p(a)\alpha L_1 f_1(k_1) - wL_1 - r_1 k_1 L_1]. $$

It should be clear from this formulation that the Harberger assumption requires that interest payments are not deductible as usually is assumed in the finance literature. If interest were deductible, the after-tax return to equity in sector one would be

$$ H_1(a) = (1 - t)[p(a)\alpha L_1 f_1(k_1) - wL_1 - r_1 k_1 L_1], $$

since equilibrium in the bond market would require

$$ r_1 = r_2 = r. $$

To parallel the Harberger analysis, the specification given by (2) is used throughout the remainder of the paper. In the conclusion, however, it will be shown that if interest payments are deductible then the imposition of a corporate income tax is neutral, since it has no effect on the equilibrium in this model, and for realizations of $\alpha$ for which profits are positive (negative), the corporate income tax is exactly a lump-sum tax (subsidy).

**B. Consumers**

At the beginning of the period, consumers are assumed to make portfolio decisions and to allocate their labor and capital to firms, while at the end of the period they purchase commodities using their factor payments plus their share of the profits distributed by firms. At the end of the period consumer $i$'s consumption problem, conditional on $a$, is

$$ \text{maximize} \quad U^i(c_1^i, c_2^i) $$

$$ \text{subject to} \quad p(a)c_1^i + c_2^i \leq I^i(a^0), $$

where $U^i(c_1^i, c_2^i)$ is an ordinal, concave utility function for the two commodities and $I^i(a^0)$ is the income of consumer $i$ when $a^0$ is the realization of $\alpha$.5
Consumer i is assumed to be initially endowed with fixed amounts of labor $L^i$ and capital $K^i$ which may be hired by firms at prices $w$ and $r$, respectively. Each consumer $i$ is also endowed with a portfolio consisting of ownership shares, $y^i_1$, of the corporate sector firms. A consumer may sell his shares in the securities market at the market price $V_1$ and may purchase new shares $y^i_1$ or bonds $b^i$. Since the noncorporate sector does not include publicly-traded firms, it is assumed that consumers receive a fixed share, $y^i_2$, of their profits. Thus, income available for consumption is given by

$$I^i(\alpha) = y^i_1(V_1) + y^i_2V_2 + rb^i + wL^i.$$  

Each consumer is assumed to have a subjective probability assessment of $\alpha$ which may be represented by the absolutely continuous, distribution function $G^i(\alpha)$. At the beginning of the period each consumer solves the portfolio problem

$$\text{maximize} \quad E^i u^i(I^i(\alpha), p(\alpha))$$
$$\gamma^i_1, y^i_2, b$$

subject to

$$\gamma^i_1V_1 + b^i \leq \gamma^i_2V_1 + K^i,$$

where $u^i(I^i(\alpha), p(\alpha))$ obtained from (3) is consumer i's indirect utility function which is assumed to be strictly concave in $I^i(\alpha)$, and $E^i$ denotes the expectation operator.  

C. SECURITY AND FACTOR MARKET EQUILIBRIUM

It is assumed that firms act in the best interest of their shareholders and in this model it can be shown that shareholders unanimously prefer that the firm maximizes its market value. Since uncertainty enters linearly into the returns of firms in the corporate sector, it is easy to show that the random component of the return, $p(\alpha)a$, can be obtained by a linear combination of existing securities, i.e.,

$$p(\alpha)a = \beta_1 \Pi_1(\alpha) + \beta_2 \Pi_2,$$

where

$$\beta_1 = \frac{1}{(1-t)\mu_1(\nu_1)} \text{ and } \beta_2 = \frac{L_1[(1-t)w + rk_1]}{(1-t)\mu_1(\nu_1)}.$$

Given this spanning property, the "price," $E^{*}_T$, of the random component $p(\alpha)a$ of the return is the market certainty equivalent of the random variable $p(\alpha)a$ discounted to the beginning of the period. Due to the multiplicative nature of uncertainty, the market certainty equivalent may be determined directly from the market value of the corporate sector firm, the inputs, and the factor prices.

By assuming that the input decisions of one firm have a negligible effect on the availability of inputs of other firms and that all consumers perceive that the profit and market value of a given firm is independent of the decisions of any other firm, it may be shown that all shareholders prefer that firms in the corporate sector
maximize their market value given by

\[ V_1 = \frac{1}{r} \{ (1 - t)p^* f_1(k_1) - L_1 [(1 - t)w + rk_1] \}. \]

The preferred input levels for firms maximize the values \( V_1 \) and \( \Pi_2 \) and satisfy

\[ (1 - t)p^* f'_1(k_1) - r = 0 \]
\[ (1 - t)(p^* f_1(k_1) - w) - rk_1 = 0 \]
\[ f'_1(k_2) - r = 0 \]
\[ f'_2(k_2) - w - rk_2 = 0. \]

An equilibrium in factor markets requires that the returns to factors be the same in both sectors, so

\[ p^*(1 - t)f'_1(k_1) = f'_2(k_2) \]
and
\[ p^*(f_1(k_1) - k_1 f'_1(k_1)) = f_2(k_2) - k_2 f'_2(k_2). \]

At an equilibrium resources are fully employed, so

\[ \bar{K} = \sum_i \bar{Z}^i = L_1 k_1 + L_2 k_2 \]
(13)
\[ \bar{L} = \sum_i \bar{L}^i = L_1 + L_2 \]

It is assumed that an equilibrium exists and that positive amounts of both commodities are produced.

Some additional work is required to derive the output market clearing condition in this model. To accomplish this, it is useful to think of a firm as producing a bundle of "outputs" defined across states of the world. In this model there are two such outputs: the first provides the consumer with \( p(0^0) \) units of income if the realization of \( \alpha \) is \( 0^0 \), the second provides one unit of income independent of the realization of \( \alpha \). Substituting (6) into the budget constraint of the consumer's problem in (5) and rearranging terms, it can be seen that

\[ p^* [(1 - t)y_1^1 f_1(k_1)] + \frac{1}{r} \{ -y_1^1 [(1 - t)w + rk_1] + r^2 \}
\]
\[ \leq p^* [(1 - t)y_1^2 f_1(k_1)] + \frac{1}{r} \{ -y_1^2 [(1 - t)w + rk_1] + r^2 \}. \]

Thus, consumer 1 is endowed with \( z_1^1 = (1 - t)y_1^1 f_1(k_1) \) units of output one and \( z_1^2 = \{-y_1^2 [(1 - t)w + rk_1] + r^2 \} \) units of output two and the consumer purchases \( z_1^1 = (1 - t)y_1^1 f_1(k_1) \) of output one and \( z_1^2 = \{-y_1^2 [(1 - t)w + rk_1] + r^2 \} \) units of output two. Thus, the reformulated budget constraint becomes

\[ \frac{1}{r} \left[ p^* z_1^1 + z_2^1 \right] \leq \frac{1}{r} \left[ p^* z_1^2 + z_2^2 \right]. \]
These conditions imply that the real rewards to capital and labor vary inversely with the corporate tax rate, since the inputs are substitutes. To determine the derivatives of the right sides of (16) and (17), substitute $L_2 = L - L_1$ from (14) and totally differentiate the equilibrium system of equations (13), (11), (12), and (16) with respect to the corporation income tax rate to obtain

\[
\begin{bmatrix}
k_1 - k_2 & L_1 & L_2 & 0 \\
0 & (1-t)p^* f''_1 & -f''_2 & (1-t)f'_1 \\
0 & p^* k_1 f''_1 & k_2 f''_2 & f'_1 k_1 f''_1 \\
0 & p^* f'_1 & p^* f''_1 & 0 & \sigma L_1 f_1
\end{bmatrix}
\begin{bmatrix}
dL_1 \\
dk_1 \\
dk_2 \\
dp^*
\end{bmatrix}
= \begin{bmatrix}
0 \\
p^* f_1' dt \\
0 \\
0
\end{bmatrix}
\]

where $dL_1 = -dL_2$, the derivative of (18) has been multiplied by $p^*$, and $\sigma$ is the price elasticity of demand given by

\[
\sigma = \frac{-dD(p^*)}{dp^*} = \frac{p^*}{D(p^*)} \frac{D'(p^*)}{L_1 f_1'}.
\]

The determinant $C$ of the coefficient matrix is

\[
C = p^* \left\{(f_1')^2 \left[ L_1 f_1'' (k_2 + \omega)(k_2 (1-t) + \omega) + p^*(1-t)L_2 f_1'' (k_1 + \omega)^2 \right] \\
- \sigma L_1 f_1' f_1'' (k_1 - k_2)(k_1 - (1-t)k_2) \right\}
\]

where

\[
\omega = \frac{w}{r} = (f_1 - k f'_1)/f_1
\]

is the wage-rental ratio. In the Harberger model, the system is evaluated at $t = 0$ which assures that $C < 0$. Alternatively, if $t > 0$, it is necessary to assume that $k_1 > (1-t)k_2$. This will certainly be satisfied if the corporate sector is capital intensive relative to the noncorporate sector. The solution to $dk_2/dt$ is given by

\[
dk_2/dt = \frac{1}{C} \left\{(p^* f_1')^2 L_1 \left[ k_1 f_1'' \sigma (\omega + k_1) (k_1 - k_2) - \omega f_1' (\omega + k_2) \right] \right\},
\]

and since $C < 0$, if the corporate sector is more capital intensive than the noncorporate sector, $dk_2/dt > 0$. With this result it can now be seen from (16) and (17) that a corporate income tax increases the wage paid to labor and decreases the return to capital, i.e.,

\[
(19) \quad dt/dt < 0 \text{ and } dw/dt > 0.
\]
Given this interpretation, Harberger's assumption will be employed; namely, that the government spends the proceeds of the tax to exactly counterbalance the reduction on private expenditures in the two outputs at the initial price $p^*$ and that redistributions of income among consumers leave the pattern of demand unchanged. That is, consumers purchase $(1 - t)L_1 f_1(k_1)$ units of the first output and the government receives $tL_1 f_1(k_1)$ units of that output. The government redistributes its receipts to the consumers so that the total amount available in the market is $L_1 f_1(k_1)$. When the tax revenue is spent in the same manner as consumers would at the existing prices, there is no direct tax effect and only relative commodity prices affect aggregate demand. Using each consumer's reformulated budget constraint in (15) it is seen that with full employment, the demand for product one determines the demand for product two. At the equilibrium certainty equivalent price $p^*$ the quantity demanded $D(p^*)$ of claims to output one is

$$D(p^*) = \sum_i z_1^i(p^*).$$

The supply of output one is

$$\sum_i z_1^i = L_1 f_1(k_1),$$

so in equilibrium

(16)

$$D(p^*) - tL_1 f_1(k_1) = 0.$$

With this market clearing condition, the equilibrium of this model may now be analyzed by examining the system of five equations (11) - (14) and (16), in five unknowns, $k_1$, $k_2$, $L_1$, $L_2$, and $p^*$.

3. UNCERTAINTY AND TAX INCIDENCE

Batra and Ratti and Shome find that when firms maximize the expected utility of profits, the results of Harberger and Mieszczkowski fail to obtain. For example, Batra concludes that Harberger's principal result turns on the behavior of firm's relative and absolute risk aversion, since the factor returns in his model are dependent upon the utility functions and probability assessments of firms. With a securities market, however, the factor returns depend only on the certainty-equivalent market price and, as will be shown in this section, the results derived by Harberger remain unaffected. Due to the fact that the reduced form of the model is isomorphic to the certainty economy, this result is hardly surprising. In fact, the system of equations, (11) - (14) and (16) are identical to those considered by Harberger.

To illustrate that such is that case in this stochastic environment, it will be shown that if the corporate sector is capital intensive relative to the noncorporate sector, then, in proportion to its share of national income, capital will bear a greater burden of the corporate income tax than labor. Since $r = f_2'$ and $w = (f_2' - k_2 f_2)$ from (9) and (10), respectively, the effect on the real reward to factors is given by

$$\frac{dr}{dt} = f_2' \frac{dk_2}{dt},$$

and
Similarly, evaluating the effect of the tax on the certainty equivalent price gives

\[ \frac{dp^*}{dt} = \frac{1}{\kappa} \left\{ (p^*f_1^2) \left[ k_2L_1f_2''(w + k_2) + p^*k_1L_2f_1''(k_1 + \omega) \right] \right\} > 0, \]

if \( k_1 > (1 - t)k_2 \). Thus the corporate income tax unambiguously increases the relative commodity price.

If \( \theta_L \) denotes the share of labor and \( \theta_K \) the share of capital in national income \((y = wL + rK + T)\), then

\[ \theta_L = \frac{wL}{wL + rK + T} \]

and

\[ \theta_K = \frac{rK}{wL + rK + T}, \]

where \( T \) is tax revenue. Differentiating (20) and (21) with respect to the corporate income tax rate gives

\[ \frac{d\theta_L}{dt} = \left[ \frac{dw}{dt} L(rK + T) - wL \left( \frac{dr}{dt} K + \frac{dt}{dt} \right) \right] / y^2 \]

and

\[ \frac{d\theta_K}{dt} = \left[ \frac{dr}{dt} k(wL + T) - rK \left( \frac{dw}{dt} L + \frac{dt}{dt} \right) \right] / y^2. \]

In order to determine which factor bears the greater burden of the tax in proportion to its initial share, (22) and (23) may be used to obtain

\[ \frac{1}{\theta_L} \frac{d\theta_L}{dt} - \frac{1}{\theta_K} \frac{d\theta_K}{dt} = \frac{1}{w} \frac{dw}{dt} - \frac{1}{r} \frac{dr}{dt}. \]

Using (19), this quantity is seen to be positive if either 1) the system is evaluated at \( t = 0 \) and the corporate sector is more capital intensive than the noncorporate sector, or 2) the after-tax capital-labor ratio in the corporate sector exceeds the capital-labor ratio in the noncorporate sector. The Harberger result that capital bears a greater burden of the tax in proportion to its share of national income than does labor continues to hold under either of these two assumptions.

As indicated above, the deductibility of interest payments will leave the decisions of firms unaffected by the tax structure. This result is immediate since with the deductibility assumption \( r_1 = r_2 = r \), and the right-hand-side of equation system (18) is identically zero. This is consistent with the earlier analyses of Stiglitz (1973) and King (1975) and, as Stiglitz points out "from an efficiency point of view, the whole corporate profits tax structure is just like a lump-sum tax on corporations.'
establishes a certainty equivalent price that firms can use in planning their inputs in a manner directly analogous to that in a deterministic model. The certainty equivalent price separates production decisions from a consumer's consumption-portfolio decisions, so it is hardly surprising that Harberger's tax incidence results continue to hold. Furthermore, the analysis of partial factor taxes also becomes straightforward in this model and, with the methodology developed here, Mieszkowski's results can be shown to extend to a stochastic world.

Studies of firm behavior under uncertainty that represent the objectives of firms in terms of the preferences and expectations of a decision maker, either an entrepreneur or a manager, will necessarily conclude that those preferences and expectations influence production decisions unless a market is present that prices out the uncertainty in the model. The tax incidence results of Batra and Ratti and Shome are thus applicable to firms owned and operated by a single entrepreneur but not to publicly-traded firms that are managed in the interests of their shareholders. An alternative justification for the expected utility maximization objective of a firm is that it is descriptive of managerial decision making when ownership is separated from the control of a firm. Even in the case of a manager who maximizes an arbitrary expected utility function, however, Baron and Forrythe have shown that separation obtains if the firm trades its own shares through treasury purchases. The conclusions of deterministic theory are then applicable.

To argue that uncertainty compromises the results of deterministic theory in a model in which the uncertainty enters in a linear manner thus requires rather special assumptions about the owner or manager of the firm. In a more general model, however, the necessary separation may not result. The correspondence between the uncertainty model considered here and the deterministic model results because the return vector (across states) of a corporate sector firm is spanned by the return vectors of the securities traded in the stock market. When the technology of the firm is such that this spanning property is not satisfied, shareholders will no longer, in general, be in agreement with respect to their preferences for the decisions of a firm, since there is no unambiguous objective for the firm to pursue. In this case there is little guidance as to how the firm should make its decisions and hence no framework in which the incidence of taxes can be investigated.
FOOTNOTES

1. These include Johnson (1956), Mieszkowski (1967), and Wells (1955).

2. An analysis of the conditions needed for this separation may be found in Baron (1979).

3. This result depends importantly on the form of the production function since it is linear in \( \alpha \).

4. The same results will obtain if there is default risk but no bankruptcy costs as demonstrated in Baron (1976).

5. The reformulation given in this section is based on that given in Helpman and Razin (1978) and used in Baron and Forsythe (1979). The optimal consumption is a function of \( p(\alpha) \) and \( \mathbf{i}^*(\alpha) \) and hence indirectly a function of \( \alpha \).

6. If the noncorporate sector is viewed as being composed of institutions such as mutual insurance companies or mutual savings associations, trading in ownership shares could be considered. Similarly, if farms are included in that sector, consumers could purchase or sell acreage or enter into sharecropping arrangements.

7. This analysis allows for noncorporate firms which may be wholly owned by a single individual. If individual \( i' \) owns such a firm, then \( \gamma_2^i = 1 \) and \( \gamma_2^i = 0 \) for \( i \neq i' \).

8. For a detailed derivation of this indirect utility approach, see Milne (1979).

9. The price \( \frac{p^*}{r} \) is given by

\[
\frac{p^*}{r} = \frac{\beta_1 Y_1 + \beta_2 p L_2}{r} = \left[ \frac{r(Y_1 + k_1 L_1) + (1 - t) w L_1}{(L_1 f'_1(k_1) - t)} \right] / \left( L_1 f'_1(k_1) - t \right)
\]

10. These conditions are derived by maximizing the consumer's expected utility at a securities market equilibrium. The derivation will not be presented here, since analogous conditions are derived in Baron and Forsythe (1979).

11. As Milne (1976, 1979) has shown the reduction of the asset economy to one in which

(i) consumers solve the portfolio problem in (5)

(ii) corporate firms maximize their market value in (6)

(iii) noncorporate firms maximize their profits in (8),

is isomorphic to a certainty economy in which the price is \( p^* \).

12. The tax does not directly affect any consumer's claims to units of the first output.
13. Because of the constant returns to scale technology, the value of equity capital does not enter these expressions, since the equilibrium market value of each firm is zero. This can be seen by multiplying (7) by $L_1$ and comparing the resulting expression with that for $V_1$. Similarly, multiplying (9) by $L_2$ indicates the profit of the noncorporate sector firm is zero.

14. Such would be the case if uncertainty enters into the noncorporate sector and there are no risk-sharing markets in that sector. Since no certainty equivalent price can be established which the noncorporate firms may use in planning their decisions, the standard tax incidence results will depend upon the preferences and expectations of the managers of the noncorporate firms.

REFERENCES


