1. INTRODUCTION

Commencing with Harberger's [1962] classic paper, a number of studies have analyzed the incidence of taxation in the context of a deterministic, two-sector, two-factor general equilibrium model. Recently, R. N. Batra [1975] and R. A. Ratti and P. Shome [1977a, 1977b] have reexamined the robustness of these deterministic results for the case in which production uncertainty is incorporated into the model. By using "entrepreneurial" models in which the firm is assumed to maximize the expected utility of profits, they find that the incidence of taxes depends on the preferences and probability assessments of the entrepreneur, and in general, the deterministic results no longer obtain.

Most firms, however, are not owned by a single individual, and Batra and Ratti and Shome do not indicate how appropriate their results are for other ownership forms. In particular, their models do not utilize any form of risk-sharing arrangements such as those available through the securities markets. In the presence of a stock market, it will be shown that the deterministic results of Harberger continue to hold for the firm in their economy if the firm has publicly-traded securities and acts in the best interests of its shareholders. With this shareholders' interests criterion and the Batra-Ratti-Shome model, the securities market is sufficient to separate the production decisions of the firm from the portfolio-consumption decisions of shareholders. In a related analysis, Baron and Forsythe [1979] focus on the role of the securities market in establishing unanimity among shareholders about the value maximization criterion for firms. Here, the emphasis is on the impact of taxes on production and factor rewards. Because of separation and the Harberger assumption that aggregate demand is unaffected by the tax rate, the equilibrium in the securities, output, and factor markets has the same qualitative properties as in a deterministic model, and the standard propositions regarding the incidence of taxation continue to hold. If we alter the Harberger assumption that there is no direct tax effect, we will show that a sufficient condition for his results to continue to hold is that all individuals exhibit nondecreasing absolute risk aversion. For expositional purposes only, the analysis will be limited to the study of the effect of the corporate income tax,

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2 These include Johnson [1956], Mieszkowski [1967], Wells [1955].
3 An analysis of the conditions needed for this separation may be found in Baron [1979].
but in the final section the results for other forms of taxation, such as those considered by Mieszkowski [1967], will be shown to extend also to the stochastic economy considered here.

2. THE MODEL AND EQUILIBRIUM

2.1. Firms. Following the specification of Harberger, a two-factor model is considered in which $X_1$ is produced in the corporate sector and $X_2$ is the non-corporate sector. Production takes place under conditions of perfect competition, full employment, inelastic factor supplies, and irreversible factor intensities. The quantities of capital and labor employed in the $j$-th sector are denoted by $K_j$ and $L_j$, respectively, and $K$ and $L$ are the total fixed supplies of each factor.

The output of the corporate sector is subject to uncertainty with a production function of the form

$$X_1 = \alpha F_1(K_1, L_1),$$

where $\alpha$ is a random variable, $\alpha \geq 0$, representing exogenous and uncertain influences affecting output. The output of the noncorporate sector is assumed to be deterministic and given by

$$X_2 = F_2(K_2, L_2).$$

Although each sector is assumed to be composed of many firms, only a representative firm in each sector will be analyzed in order to simplify the notation. The production functions $F_j, j=1, 2$, are assumed to be linear homogeneous and concave so that

$$F_j(K_j, L_j) = L_j f_j(k_j), \quad j = 1, 2,$$

where $k_j$ is the capital-labor ratio in sector $j$, $f'_j > 0$, and $f''_j < 0$.

It is assumed that firms make their input decisions at the beginning of the period, prior to the realization of $\alpha$, by contracting for labor at the competitive wage rate $w$, and by financing their capital purchases by selling bonds, $B_j = K_j$, $j=1, 2$, which yield a deterministic gross rate of return $r_j (r_j \geq 1)$ determined in a securities market. At the completion of trades in the securities and factor markets, the contracted levels of inputs are employed and an outcome of $\alpha$ is realized, as is output. The market clearing price in the corporate sector output market depends on $\alpha$ and hence is uncertain at the time input decisions are made.

Ratti and Shome [1977a] recognize that the price is uncertain but, in order to avoid dealing with price uncertainty, they assume a small country for which product prices are given by world markets. Batra does not make the small country assumption, yet assumes that the output price is not random. In noting this, Ratti and Shome [1977b] suggest that when assuming a large country, the price which equates expected demand to expected supply should be used. As will be demonstrated, these assumptions are unnecessary, since an uncertain price does
not affect the standard tax incidence results for the model considered here.\footnote{This result depends importantly on the form of the production function since it is linear in $\alpha$.}

When the output market clears, factors are paid their wages, and the after-tax earnings are then distributed to shareholders in proportion to their holdings. Finally, it is assumed that all commitments to factor inputs are met and that there is no risk of default on the bond obligations.\footnote{The same results will obtain if there is default risk but no bankruptcy costs, as demonstrated in Baron [1976].} Letting $p(\alpha)$ denote the price of output in the first sector expressed in terms of the price in the second sector, the after-tax earnings $\Pi_1(\alpha)$ of the corporate sector may be expressed as

$$\Pi_1(\alpha) = (1 - t) \left[ p(\alpha) \alpha F_1(K_1, L_1) - r_1 B_1 - w L_1 \right]$$

$$= (1 - t) \left[ p(\alpha) \alpha L_1 f_1(k_1) - L_1 (r_1 k_1 + w) \right],$$

where $t$ is the corporate income tax rate. The tax system is assumed to be such that the corporate tax involves full loss offset. The earnings of the noncorporate sector are given by

$$H_2 = F_2(K_2, L_2) - r_2 B_2 - w L_2 = L_2 f_2(k_2) - L_2 (r_2 k_2 + w).$$

Adopting the view of Harberger [p. 215] that the corporation income tax is one "which strikes the earnings of capital in the corporate sector, but not in the noncorporate sector," the return on the debt as well as the equity of a firm in the first sector is subject to the tax. In this case, the appropriate equilibrium condition in the bond market is

$$\frac{(1 - t) r_1}{r_2} \equiv 1.$$

Thus, the after-tax return to the equity of the corporate sector can be rewritten as

$$\Pi_1(\alpha) = (1 - t) \left[ p(\alpha) \alpha L_1 f_1(k_1) - w L_1 \right] - r k_1 L_1.$$

It should be clear from this formulation that the Harberger assumption requires that interest payments are not deductible as usually is assumed in the finance literature. If interest were deductible, the after-tax return to equity in sector one would be

$$\Pi_1(\alpha) = (1 - t) \left[ p(\alpha) \alpha L_1 f_1(k_1) - w L_1 - r k_1 L_1 \right],$$

since equilibrium in the bond market would require

$$r_1 = r_2 \equiv 1.$$

To parallel the Harberger analysis, the specification given by (2) is used throughout the remainder of the paper. If interest payments are deductible, however, the imposition of a corporate income tax is neutral, since it has no effect on the equilibrium in this model, and for realizations of $\alpha$ for which profits are positive (negative), the corporate income tax is exactly a lump-sum tax (subsidy) as Stiglitz
2.2. **Consumers.** At the beginning of the period, consumers are assumed to make portfolio decisions and to allocate their labor and capital to firms, while at the end of the period they purchase commodities using their factor payments plus their share of the profits distributed by firms. At the end of the period consumer i’s consumption problem, conditional on \( \alpha \), is

\[
\text{maximize } U^i(c^i_1, c^i_2)
\]

subject to

\[
p(\alpha^0)c^i_1 + c^i_2 \leq I^i(\alpha^0),
\]

where \( U^i(c^i_1, c^i_2) \) is an ordinal, concave utility function for the two commodities and \( I^i(\alpha^0) \) is the income of consumer \( i \) when \( \alpha^0 \) is the realization of \( \alpha \).

Consumer \( i \) is assumed to be initially endowed with fixed amounts of labor \( \bar{L}^i \) and capital \( \bar{K}^i \) which may be hired by firms at prices \( w \) and \( r \), respectively. Each consumer \( i \) is also endowed with a portfolio consisting of ownership shares, \( \bar{\gamma}^i_1 \), of the corporate sector firms. A consumer may sell his shares in the securities market at the market price \( V_1 \) and may purchase new shares \( \gamma^i_1 \) and bonds \( \gamma^i_2 \). Since the noncorporate sector does not include publicly-traded firms, it is assumed that consumers receive a fixed share, \( \gamma^i_2 \), of their profits. The income available for consumption is given by

\[
I^i(\alpha) = \gamma^i_1 \Pi^i(\alpha) + \gamma^i_1 \Pi^2 + rb^i + wL^i.
\]

Each consumer is assumed to have a subjective probability assessment of \( \alpha \) which may be represented by the absolutely continuous, distribution function \( G^i(\alpha) \). At the beginning of the period each consumer solves the portfolio problem

\[
\text{maximize } E^iu^i(I^i(\alpha), p(\alpha))
\]

subject to

\[
\gamma^i_1 V_1 + b^i \leq \gamma^i_1 V_1 + \bar{K}^i,
\]

where \( u^i(I^i(\alpha), p(\alpha)) \) obtained from (3) is consumer \( i \)’s indirect utility function which is assumed to be strictly concave in \( I^i(\alpha) \), and \( E^i \) denotes the expectation operator.

2.3. **Security and Factor Market Equilibrium.** It is assumed that firms act

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6 The reformulation given in this section is based on that given in Helpman and Razin [1978] and used in Baron and Forsythe [1979]. The optimal consumption is a function of \( p(\alpha) \) and \( I^i(\alpha) \) and hence indirectly a function of \( \alpha \).

7 If the noncorporate sector is viewed as being composed of institutions such as mutual companies or mutual savings associations, trading in ownership shares could be considered. Similarly, if farms are included in that sector, consumers could purchase or sell acreage or enter into sharecropping arrangements.

8 This analysis allows for noncorporate firms which may be wholly owned by a single individual. If individual \( i' \) owns such a firm, then \( \gamma^i_1 = 1 \) and \( \gamma^i_2 = 0 \) for \( i \neq i' \).

9 For a detailed derivation of this indirect utility approach, see Milne [1979].
in the best interest of their shareholders and in this model it can be shown that shareholders unanimously prefer that the firm maximizes its market value. Since uncertainty enters linearly into the returns of firms in the corporate sector, it is easy to show that the random component of the return, \( p(\alpha)x \), can be obtained by a linear combination of existing securities, i.e.,

\[
p(\alpha)x = \beta_1 \Pi_1(\alpha) + \beta_2 \Pi_2
\]

where

\[
\beta_1 = \frac{1}{(1 - t)L_1f_1'(k_1)} \quad \text{and} \quad \beta_2 = \frac{L_1[(1 - t)w + rk_1]}{(1 - t)L_1f_1'(k_1)\Pi_2}.
\]

Given this spanning property, the “price,” \( p^*/r \), of the random component \( p(\alpha)x \) of the return is the market certainty equivalent of the random variable \( p(\alpha)x \) discounted to the beginning of the period. Due to the multiplicative nature of uncertainty, the market certainty equivalent may be determined directly from the market value of the corporate sector firm, the inputs, and the factor prices.\(^{10}\)

By assuming that the input decisions of one firm have a negligible effect on the availability of inputs of other firms and that all consumers perceive that the profit and market value of a given firm is independent of the decisions of any other firm, it may be shown that all shareholders prefer that firms in the corporate sector maximize their market value\(^{11}\) given by

\[
V_1 = \frac{1}{r} \{(1 - t)p^*L_1f_1'(k_1) - L_1[(1 - t)w + rk_1] \}.
\]

The preferred input levels for firms maximize the values \( V_1 \) and \( \Pi_2 \) and satisfy\(^{12}\)

\[
\begin{align*}
(1 - t)p^*f_1'(k_1) - r &= 0 \\
(1 - t)(p^*f_1'(k_1) - w) - rk_1 &= 0 \\
f_2'(k_2) - r &= 0
\end{align*}
\]

and

\[
\begin{align*}
\text{The price } p^*/r \text{ is given by}
\frac{p^*}{r} &= \beta_1 V_1 + \frac{\beta_2 \Pi_2}{r} \\
&= \frac{r(V_1 + k_1L_3) + (1 - t)wL_1}{r(L_1f_1'(k_1)(1 - t))}
\end{align*}
\]

\(^{10}\) These conditions are derived by maximizing the consumer's expected utility at a securities market equilibrium. The derivation will not be presented here, since analogous conditions are derived in Baron and Forsythe [1979].

\(^{11}\) As Milne [1976, 1979] has shown, the reduction of the asset economy to one in which

(i) consumers solve the portfolio problem in (5)
(ii) corporate firms maximize their market value in (6)
(iii) noncorporate firms maximize their profits in (1),

is isomorphic to a certainty economy in which the price is \( p^* \).
An equilibrium in factor markets requires that the returns to factors be the same in both sectors, so

\begin{equation}
 p^*(1 - t)f'_1(k_1) = f'_2(k_2)
\end{equation}

and

\begin{equation}
 p^*(f_1(k_1) - k_1f'_1(k_1)) = f_2(k_2) - k_2f'_2(k_2).
\end{equation}

At an equilibrium resources are fully employed, so

\begin{equation}
 K = \sum_i K_i = L_1k_1 + L_2k_2
\end{equation}

\begin{equation}
 L = \sum_i L_i = L_1 + L_2.
\end{equation}

It is assumed that an equilibrium exists and that positive amounts of both commodities are produced.

Some additional work is required to derive the output market clearing condition in this model. To accomplish this, it is useful to think of a firm as producing a bundle of "outputs" defined across states of the world. In this model there are two such outputs: the first provides the consumer with \( p(\alpha^0)x^0 \) units of income if the realization of \( \alpha \) is \( \alpha^0 \); the second provides one unit of income independent of the realization of \( \alpha \). Substituting (6) into the budget constraint of the consumer's problem in (5) and rearranging terms, it can be seen that

\begin{equation}
 \frac{P^*}{r} [(1 - t)\gamma L_1f_1(k_1)] + \frac{1}{r} \{-\gamma L_1[(1 - t)w + rk_1] + rb^I\}
\end{equation}

\begin{equation}
 \leq \frac{P^*}{r} [(1 - t)\gamma L_1f_1(k_1)] + \frac{1}{r} \{-\gamma L_1[(1 - t)w + rk_1] + rK^I\}.
\end{equation}

Thus, consumer \( i \) is endowed with \( z_1 = (1 - t)\gamma L_1f_1(k_1) \) units of output one and \( \bar{z}_1 = \{-\gamma L_1[(1 - t)w + rk_1] + rK^I\} \) units of output two and the consumer purchases \( z_1^I = (1 - t)\gamma L_1f_1(k_1) \) of output one and \( \bar{z}_1^I = \{-\gamma L_1[(1 - t)w + rk_1] + rb^I\} \) units of output two. Thus, the reformulated budget constraint becomes

\begin{equation}
 \frac{1}{r} [p^*z_1^I + \bar{z}_1^I] \leq \frac{1}{r} [p^*\bar{z}_1^I + \bar{z}_1^I].
\end{equation}

Given the reformulation in (15), the demand function for the first output can be derived given a distribution of the tax proceeds. The tax on the corporate sector may be interpreted as a payment of \( tL_1f_1(k_1) \) units of the first output and \( -twL_1 \) units of the second output from that sector to the government. The government is assumed to distribute a portion \( \eta^I \) of each output to individual \( i \), where \( \sum_i \eta^I = 1 \), so that the tax is fully distributed. Individual \( i \)'s expected indirect utility function may be expressed in terms of the two outputs as

\begin{equation}
 E'u' (p(\alpha)xz_1^I + z_2^I + w\bar{L}_1^I + \bar{\gamma} I_2 + \eta^I tL_1(p(\alpha)xf_1(k_1) - w), p(\alpha)),
\end{equation}
which is to be maximized subject to (15). The demand function for the first output is a function $D^i$ which may be expressed as

$$z^i = D^i(p^*, t | \eta^i)$$

and aggregate demand $D$ is given by

$$D(p^*, t | (\eta^i)) = \sum_i D^i(p^*, t | \eta^i)$$

where $(\eta^i)$ is the vector of distribution shares over individuals.\footnote{The demand for the first output determines the demand for the second output as a consequence of full employment.}

The assumption made by Harberger is that the distribution of the tax revenue is such that the aggregate demand function depends only on $p^*$ and hence can be expressed as $D^*(p^*)$. The supply of output one is $\sum_i z_1^i = L_1 f_1(k_1)$, so in equilibrium

$$D^*(p^*) - L_1 f_1(k_1) = 0.$$  

The equilibrium in this model is thus characterized by the system of five equations (11)–(14) and (18) in five unknowns $k_1$, $k_2$, $L_1$, $L_2$, and $p^*$, which can be analyzed in the standard manner to obtain Harberger's results.\footnote{Harberger evaluated his results at $t=0$.}

If the aggregate demand function depends directly on $t$, the analysis of the effect of the corporate tax is more complicated. To identify the direct effect of the tax, solve (15) as an equality for $z_1^i$, substitute into (16) and differentiate with respect to $z_1^i$ to obtain the first-order condition

$$E u^i_t(p(x) - p^*) = 0$$

where $u^i_t$ denotes $\frac{\partial u_i^j}{\partial I^j(x)}$. Total differentiation yields

$$\frac{dz^i}{dt} = -\eta^i E u^i_{11}(p(x) - p^*) L_1 (p(x) - p^*) f_1(k_1) - w)$$

where the denominator is negative due to the strict concavity of the indirect utility function. To sign the numerator, we know from (8), that

$$wL_1 = p^* L_1 f_1(k_1) - \frac{rk_1 L_1}{1 - t}$$

so that (19) may be written as

$$\frac{dz^i}{dt} = -\eta^i L_1 f_1(k_1) - \frac{\eta^i r k_1 L_1}{(1 - t)} \frac{E u^i_{11}(p(x) - p^*)}{E u^i_{11}(p(x) - p^*)^2}.$$  

The first term in (20) is negative and thus if the second terms is nonpositive, each individual's demand for units of output one decreases with increases in the tax rate. Using the method of Arrow [1971, p. 119] it can be shown that if an individual's measure of absolute risk aversion is nondecreasing then
and thus, from (20), individual demand for \( z_i \) will vary inversely with the tax rate. However, Arrow also has shown that the expression in (21) is individual \( i \)'s income effect with regard to changes in the risky output \( z_i \), and so the assumption of nondecreasing absolute risk aversion implies that the risky output is an inferior good. Thus, with decreasing absolute risk aversion it must be the case that income effects are small in order to obtain the desired result that \( \frac{dz_i}{dt} < 0 \). Further, it should be noted that this result is stronger than required since, in fact, what we wish to assume is that

\[
D_2 = \sum \frac{dz_i}{dt} < 0.
\]

In the case of a direct tax effect, the market clearing condition may be written as

\[
D(p^*, t_i(q_i)) = \sum f_i(k_i)
\]

and the equilibrium of this model may now be analyzed by examining this equation along with equations (11)-(14). As shown in Baron and Forsythe [1981] an increase in the corporate income tax increases the wage paid to labor and decreases the return to capital. This result is qualitatively the same as Harberger who assumes that there is no direct tax effect and that the equilibrium is evaluated at \( t = 0 \).

3. DISCUSSION

Batra and Ratti and Shome find that when firms maximize the expected utility of profits, the results of Harberger and Mieszkowski fail to obtain. For example, Batra concludes that Harberger's principal result turns on the behavior of firm's relative and absolute risk aversion, since the factor returns in his model are dependent upon the utility functions and probability assessments of firms. If the securities of a firm are traded and production is subject to multiplicative uncertainty, the securities market establishes a certainty equivalent price that firms can use in planning their inputs in a manner directly analogous to that in a deterministic model. Using Harberger's assumptions, the certainty equivalent price separates production decisions from a consumer's consumption-portfolio decisions, so the reduced form of the model analyzed here is isomorphic to a certainty economy. In fact, the system of equations, (11)-(14) and (18), are identical to those analyzed by Harberger, and hence, the results he obtains under certainty also hold under uncertainty.\(^{15}\) Contrary to Batra’s conclusion, if the corporate sector is capital intensive relative to the noncorporate sector, then in proportion to its share of national income capital will bear a greater burden of the corporate

\(^{15}\) For a full derivation of these results see Baron and Forsythe [1981].
income tax than labor. Furthermore, the analysis of partial factor taxes also is straightforward in this model and, with the methodology developed here, Mieszkowski's results can be shown to extend to this stochastic economy.

Studies of firm behavior under uncertainty that represent the objectives of firms in terms of the preferences and expectations of a decision maker, either an entrepreneur or a manager, will necessarily conclude that those preferences and expectations influence production decisions unless a market is present that prices out the uncertainty in the model. The tax incidence results of Batra and Ratti and Shome are thus applicable to firms owned and operated by a single entrepreneur but not to publicly-traded firms that are managed in the interests of their shareholders. An alternative justification for the expected utility maximization objective of a firm is that it is descriptive of managerial decision-making when ownership is separated from the control of a firm. Even in the case of a manager who maximizes an arbitrary expected utility function, however, Baron and Forsythe have shown that separation obtains if the firm trades its own shares through treasury purchases. The conclusions of deterministic theory are then applicable.

To argue that uncertainty compromises the results of deterministic theory in a model in which the uncertainty enters in a linear manner thus requires rather special assumptions about the owner or manager of the firm. In a more general model, however, the necessary separation may not result. The correspondence between the uncertainty model considered here and the deterministic model results because the return vector (across states) of a corporate sector firm is spanned by the return vectors of the securities traded in the stock market. When the technology of the firm is such that this spanning property is not satisfied, shareholders will no longer, in general, be in agreement with respect to their preferences for the decisions of a firm, and hence there is no unambiguous objective for the firm to pursue. In this case there is little guidance as to how the firm should make its decisions and hence no framework in which the incidence of taxes can be investigated.

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Such would be the case if uncertainty enters into the noncorporate sector and there are no risk-sharing markets in that sector. Since no certainty equivalent price can be established which the noncorporate firms may use in planning their decisions, tax incidence results will depend on the preferences and expectations of the managers of the noncorporate firms.