Nondecoupling of Maximal Supergravity from the Superstring

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We consider the conditions necessary for obtaining perturbative maximal supergravity in d dimensions as a decoupling limit of type II superstring theory compactified on a $(10 - d)$ torus. For dimensions $d = 2$ and $d = 3$, it is possible to define a limit in which the only finite-mass states are the 256 massless states of maximal supergravity. However, in dimensions $d \geq 4$, there are infinite towers of additional massless and finite-mass states. These correspond to Kaluza-Klein charges, wound strings, Kaluza-Klein monopoles, or branes wrapping around cycles of the toroidal extra dimensions. We conclude that perturbative supergravity cannot be decoupled from string theory in dimensions ≥ 4. In particular, we conjecture that pure $\mathcal{N} = 8$ supergravity in four dimensions is in the Swampland.

There has recently has been some speculation that four-dimensional $\mathcal{N} = 8$ supergravity might be ultraviolet finite to all orders in perturbation theory [1–3]. If true, this would raise the question of whether $\mathcal{N} = 8$ supergravity might be a consistent theory that is decoupled from its string theory extension. A related issue is whether $\mathcal{N} = 8$ supergravity can be obtained as a well-defined limit of superstring theory. Here we argue that such a supergravity limit of string theory does not exist in four or more dimensions, irrespective of whether or not the perturbative approximation is free of ultraviolet divergences.

In this Letter, we will study limits of type IIA superstring theory on a $(10 - d)$-dimensional torus T^{10-d} for various d. One may regard the following analysis as analogous to the study of the decoupling limit on Dp-branes (the limit where field theories on branes decouple from closed string degrees freedom in the bulk) for various p [4,5]. The decoupling limit on Dp-branes is known to exist for $p \leq 5$. On the other hand, subtleties have been found for $p \geq 6$, where infinitely many new world-volume degrees of freedom appear in the limit. This has been regarded as a sign that a field theory decoupled from the bulk does not exist on Dp-branes for $p \geq 6$. We will find similar subtleties for type IIA theory on $T^{10-d} \times \mathbb{R}^d$ for $d \geq 4$.

It will be sufficient for our purposes to consider the torus T^{10-d} to be the product of $(10 - d)$ circles, each of which has radius R. Numerical factors, such as powers of 2π, are irrelevant to the discussion that follows and therefore will be dropped. In ten dimensions, Newton's constant is given by

$$G_{10} = g^2 \ell_s^8,$$

where ℓ_s is the string scale and g is the string coupling constant. Thus, the effective Newton constant in d dimensions is given by

$$G_d \equiv \ell_d^{d-2} = \frac{G_{10}}{R^{10-d}} = \frac{g^2 \ell_s^8}{R^{10-d}}, \quad (1)$$

where ℓ_d is the d-dimensional Planck length, so that

$$g = \frac{R^{5-(d/2)}}{\ell_s^4} \ell_d^{(d/2)-1}. \quad (2)$$

We are interested in whether there is a limit of string theory that reduces to maximal supergravity, which is defined purely in terms of the dynamics of the 256 states in the massless supermultiplet. In other words, we are interested in the limit in which all of the excited string states, together with the Kaluza-Klein excitations and string winding states associated with the $(10 - d)$ torus, decouple. A necessary condition for this to happen is that these states are all infinitely massive compared to the d-dimensional Planck scale ℓ_d. (In this limit, the string length ℓ_s provides a regularization scale for supergravity. Thus, if string amplitudes depend sensitively on ℓ_s, it can be taken as evidence for ultraviolet divergences in supergravity. This is seen explicitly, for example, in the one-loop four graviton amplitude, which is ultraviolet divergent in nine dimensions. The corresponding string expression is finite, and its low-energy limit is sensitive to the presence of these massive states with momenta $\sim 1/\ell_s$.) This is achieved by taking

$$\frac{1}{R}, \quad \frac{1}{\ell_s}, \quad \text{and} \quad \frac{R}{\ell_s^2} \gg \frac{1}{\ell_d}, \quad (3)$$

with ℓ_d fixed. This is compatible with keeping g fixed for $d < 6$. If the extra states do decouple, then the surviving states are the 256 massless states of maximal supergravity, which is $\mathcal{N} = 8$ supergravity when $d = 4$.

Let us now consider the spectrum of nonperturbative superstring excitations in this limit. First, consider a Dp-brane wrapping a p cycle of the torus. The mass of such a state in d dimensions is

$$M_p = \frac{R^p}{g\ell_s^{p+1}} = \frac{R^{p+(d/2)-5}}{\ell_s^{p-3}}\ell_d^{1-(d/2)}. \qquad (4)$$

When $d \le 5$, we also need to consider a NS5-brane wrapping a 5 cycle. This has a mass given by

$$M_{\text{NS5}} = \frac{R^5}{g^2\ell_s^6} = \frac{\ell_s^2}{R^{5-d}}\ell_d^{2-d}. \qquad (5)$$

In order to obtain the pure supergravity theory with 32 supercharges in d dimensions, these nonperturbative states also need to decouple, so their masses must satisfy $M_p, M_{\text{NS5}} \gg 1/\ell_d$. In the case of $d = 4$, the nonperturbative Bogomol'nyi-Prasad-Sommerfield (BPS) particle spectrum also includes Kaluza-Klein monopoles, which are discussed in the next paragraph.

Before studying the limit in any dimension d, we will discuss what to expect on general grounds. A Kaluza-Klein momentum state and a wrapped string state have masses $1/R$ and R/ℓ_s^2, respectively, and they are half-BPS objects that carry a single unit of a conserved charge. In d dimensions, their magnetic duals are $(d - 4)$-branes. The BPS saturation condition together with the Dirac quantization condition implies quite generally that the mass m of a BPS particle and the tension $\mathcal{T}$ of its magnetic dual $(d - 4)$-brane are related by

$$m\mathcal{T} \sim \frac{1}{G_d} = \frac{1}{\ell_d^{d-2}}. \qquad (6)$$

Applying this to $d = 4$, we immediately conclude that there is no limit in four dimensions where we can keep all BPS particles heavier than the Planck scale. In particular, magnetic duals of Kaluza-Klein excitations, which are the well-known Kaluza-Klein monopoles, are BPS states with masses $\sim R/\ell_4^2 \to 0$. [If the torus has six independent radii R_i, the Kaluza-Klein monopole mass spectrum has the form $M^2 = \sum (n_i R_i/\ell_4^2)^2$.] Similarly, magnetic duals of wrapped strings are NS5-branes wrapping 5-cycles of T^6, and their masses go as $\ell_s^2/R\ell_4^2 \to 0$. Later, we will discuss implications of these light states. When $d \ge 5$, at least a subset of the BPS branes become tensionless in the limit (3).

By contrast, in three dimensions it is possible to define a limit where all BPS particles become infinitely massive simultaneously. In this case, magnetic duals of BPS particles are (-1)-branes, namely, instantons, and their Euclidean actions vanish in the limit. Thus, one would expect nonperturbative effects to be very large in three dimensions even though no singularity is apparent from the spectrum.

In two dimensions, there are no magnetic duals of BPS particles, and we expect that there is a smooth limit where all BPS particles are massive and instanton actions remain nonvanishing.

Now let us look at each case in more detail. When $d = 2$, the conditions we want to impose are

$$M_p = \frac{1}{R}\left(\frac{\ell_s}{R}\right)^{3-p} \quad \text{and} \quad M_{\text{NS5}} = \frac{1}{R}\left(\frac{\ell_s}{R}\right)^2 \to \infty. \qquad (7)$$

On the other hand, the string coupling constant is given by

$$g = \left(\frac{R}{\ell_s}\right)^4. \qquad (8)$$

Thus, the desired limit can be taken by sending $R \to 0$ while keeping the string coupling constant finite. In this limit, all particle masses are much higher than the Planck mass, except for the massless two-dimensional $\mathcal{N} = 16$ supergravity states [6]. However, Dp-brane and NS5-brane instantons wrapping T^8 have Euclidean actions proportional to $(\ell_s/R)^{3-p} \sim g^{p-3/8}$ and $(\ell_s/R)^2 \sim g^{-(1/4)}$, respectively. Though the actions all remain finite and nonzero in the limit, their effects are not uniformly suppressed for small g. Thus, the resulting theory may not have a weak coupling limit that is dominated by the perturbative contribution.

When $d = 3$, the conditions we need to impose are

$$M_p = \frac{1}{\sqrt{R\ell_3}}\left(\frac{\ell_s}{R}\right)^{3-p} \quad \text{and} \quad M_{\text{NS5}} = \frac{1}{\ell_3}\left(\frac{\ell_s}{R}\right)^2 \to \infty. \qquad (9)$$

Since we now have

$$g^2 = \frac{R^7}{\ell_s^8}\ell_3, \qquad (10)$$

we can rewrite (9) as

$$M_p = \frac{g^{p-3/4}}{R^{7-p/8}\ell_3^{p+1/8}} \quad \text{and} \quad M_{\text{NS5}} = \frac{1}{g^{1/2}R^{1/4}\ell_3^{3/4}} \to \infty. \qquad (11)$$

Since $p = 0, 2, 4, 6$ in type IIA theory, this can again be arranged by taking $R \to 0$ keeping g finite. [Note that, in type IIB theory, a wrapped D7-brane cannot be made heavy unless $g \gg 1$. This is not in contradiction with T duality, since g transforms under T duality in such a way that ℓ_p given by (1) remains invariant. T duality along one of the circles on T^{10-d} transforms the coupling $g \to g\ell_s/R$ so it diverges in the limit $R \to 0$ with the original coupling constant, given by (10), kept finite.] This is also compatible with the limit (3). Thus, all particle states develop large masses and may decouple, except for those in three-dimensional $\mathcal{N} = 16$ supergravity theory [7]. However, Dp-brane and NS5-brane instanton actions, which are given by $g^{p-3/4}(R/\ell_3)^{p+1/8}$ and $g^{-(1/2)}(R/\ell_3)^{3/4}$, vanish in the limit $R \to 0$ for any finite value of g. This means that nonperturbative effects are strong, and it may be difficult to determine the properties of the resulting three-dimensional supergravity.

In view of these observations, it is interesting that gravity theories formulated in terms of a finite number of fields are known to exist in two and three dimensions. In three

dimensions, the relation with Chern-Simons gauge theory [8] suggests that pure Einstein gravity is finite to all orders in perturbation theory. However, this theory has no propagating degrees of freedom, and it is not known whether there is a finite quantum gravity theory in three dimensions that includes propagating (scalar or spin-1/2) degrees of freedom. Such degrees of freedom are present, of course, in the examples considered here. The fact that we find limits of string theory compactifications with a finite number of such propagating degrees of freedom in these dimensions may be encouraging, though the implications of the nonperturbative instanton contributions need to be understood.

When $d = 4$, the conditions (3) necessary for the extra modes to have infinite masses are

$$M_p = \frac{1}{\ell_4}\left(\frac{\ell_s}{R}\right)^{3-p} \quad \text{and} \quad M_{\text{NS5}} = \frac{\ell_s^2}{R\ell_4^2} \to \infty. \quad (12)$$

Clearly, this cannot be realized simultaneously for all $p = 0, 2, 4, 6$. This is in accord with the general argument given earlier, since a wrapped Dp-brane and a wrapped D$(6 - p)$-brane are electric-magnetic duals. Similarly, the magnetic duals of Kaluza-Klein excitations and wrapped strings are Kaluza-Klein monopoles and wrapped NS5-branes, whose masses behave as R/ℓ_4^2 and $\ell_s^2/R\ell_4^2$, respectively. There are infinitely many such states since they have arbitrary integer charges. In the limit $R, \ell_s^2/R \to 0$, there is no mass gap, and the spectrum becomes continuous.

To understand the implications of these infinitely many light states, we note that, among the elements of the four-dimensional U-duality group, $E_7(\mathbb{Z})$ is the four-dimensional S-duality transformation that interchanges the 28 types of electric charge with the corresponding magnetic charges [9,10]. This duality is described by the following transformations of the moduli:

$$\text{S}: R \to \tilde{R} = \frac{\ell_4^2}{R} \quad \text{and} \quad \ell_s \to \tilde{\ell}_s = \frac{\ell_4^2}{\ell_s}. \quad (13)$$

Note that this transformation inverts the radius R in four-dimensional Planck units (in contrast to T duality, which inverts R in string units). Since g is related to R and ℓ_s by (2), this transformation acts as the inversion $g \to \tilde{g} = 1/g$, which maps BPS states into each other. For example, a wrapped Dp-brane is interchanged with a wrapped D$(6 - p)$-brane. Similarly, a Kaluza-Klein excitation is interchanged with a Kaluza-Klein monopole (whereas T duality would relate it to a wrapped F string). Thus, in the dual frame in which the compactification scale $\tilde{R} \to \infty$, the six-torus is decompactified. This explains the continuous spectrum in the limit (3). The fact that an infinite set of states from the nonperturbative sector become massless shows that the limit of interest does not result in pure $\mathcal{N} = 8$ supergravity in four dimensions. Rather, it results in ten-dimensional decompactified string theory with the string coupling constant inverted. This is true in both the type IIA and type IIB cases. The only way of avoiding this would be

to relax (3), in which case there would instead be extra finite-mass Kaluza-Klein or winding number states, which would therefore not decouple.

One may regard our results on the limit of superstring compactification on T^{10-d} as examples illustrating the conjectures formulated in Refs. [11,12] on the geometry of continuous moduli parametrizing the string landscape. It is generally believed that, in four or more dimensions, not all field theories with interactions containing at most two space-time derivatives can be realized as low-energy effective theories of consistent quantum gravity theories with finite Planck scale. The set of theories which cannot be derived from consistent quantum gravity theories was called the "Swampland" in Ref. [11] (in contrast to the "Landscape," which parametrizes theories which do have such UV completions). Recently, several proposals have been made as to how to characterize the Swampland. Among these conjectures are the statements that, if a theory has continuous moduli, there are points in the moduli space that are infinitely far away from each other, and an infinite tower of modes becomes massless as a point at infinity is approached [12]. Since the limit considered in this Letter corresponds to a point in the moduli space of string compactifications at infinite distance from a generic point in the middle of moduli space, the conjectures predict that an infinite number of particles become massless in the limit. For $d = 4$, we have found that among such particles are Kaluza-Klein monopoles, i.e., Kaluza-Klein modes on T^6 in the dual frame in the limit $\tilde{R} \to \infty$. On the other hand, the moduli space of pure $\mathcal{N} = 8$ supergravity also contains infinite distance points, but it does not take account of new light particles appearing near these points. If the BPS particles required by string theory were included, one would have string theory and not $\mathcal{N} = 8$ supergravity. (One can imagine an alternative history in which type II superstring theory and M theory were discovered by properly interpreting the BPS solitons of $\mathcal{N} = 8$ supergravity.) Thus, the conjectures of Ref. [12] imply that the $\mathcal{N} = 8$ supergravity is in the Swampland. Similarly, there are many superstring compactifications with $\mathcal{N} < 8$ supersymmetry, and discarding stringy states in these compactifications results in further supergravity theories in the Swampland.

It is interesting to see how scattering amplitudes behave in the limit (3). Consider a four-dimensional graviton scattering amplitude where the graviton momenta are below the four-dimensional Planck scale. According to (1) and (2), the ten-dimensional Planck length ℓ_{10} is given by

$$\ell_{10} = g^{1/4}\ell_s = R^{3/4}\ell_4^{1/4}. \quad (14)$$

After the S-duality transformation (13), the limit $R \to 0$ turns into $\tilde{R} \to \infty$. Thus, we have $\tilde{\ell}_{10} = \tilde{R}^{3/4}\ell_4^{1/4} \to \infty$ in ten dimensions. Since $\tilde{\ell}_{10} \ll \tilde{R}$, the extra dimensions decompactify, and the theory is effectively ten-dimensional. Furthermore, if we take this limit keeping the graviton

momenta fixed (in units of the four-dimensional Planck mass), the scattering process becomes trans-Planckian. Generically, we expect that it will involve formation and evaporation of virtual black holes in ten dimensions.

The original motivation of this work was to investigate the relation between superstring theory and $\mathcal{N} = 8$ supergravity to see, in particular, under what conditions supergravity might be ultraviolet finite. What we have found is that in four or more dimensions ($d \geq 4$) there is no limit of compactified superstring theory in which the stringy effects decouple and only the 256 massless supergravity fields survive below the four-dimensional Planck scale. This is true whether or not there are ultraviolet divergences in supergravity perturbation theory. Of course, there is a well-defined procedure for extracting UV finite four-dimensional scattering amplitudes from perturbative string theory. This involves taking $g \rightarrow 0$ first, before taking the limit (3). However, this procedure does not keep ℓ_4 fixed, and therefore it does not correspond to the limit considered in this Letter.

It might be instructive to compare the situation to that of the conifold limit of Calabi-Yau compactified type II superstring theory studied by Strominger [13]. In that case, certain terms in the low-energy effective theory that are independent of the string coupling constant g, due to the decoupling of vector and hypermultiplet fields, can be computed in string perturbation theory. One can estimate the singularity of these terms by using the fact that a brane wrapping a vanishing cycle describes a nonperturbative BPS particle that becomes massless in the conifold limit. If one could identify analogous terms in $\mathcal{N} = 8$ supergravity, one could transform the Feynman diagram computation in four-dimensional supergravity into a corresponding computation in ten dimensions, which might give insight into the question of ultraviolet finiteness.

The situation is qualitatively different in two and three dimensions ($d = 2, 3$), where all nonsupergravity states develop masses larger than the Planck scale in the limit (3), and therefore they can decouple. In these cases, only the 256 massless supergravity states survive, and a self-contained quantum gravity theory may well exist decoupled from string theory. We have found, however, that in the $d = 3$ case there are instantons with zero action, which give rise to large nonperturbative contributions. In the $d = 2$ case, the instanton actions do not vanish in the limit (3), but not all of them are small when g is small. Therefore, the amplitudes may not be dominated by the perturbative contribution in this case, too.

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