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CAPITAL GAINS AND THE ECONOMIC THEORY OF CORPORATE FINANCE

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CAPITAL GAINS AND THE ECONOMIC THEORY OF CORPORATE FINANCE

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The dependence of one agent's actions upon those of another constitutes a fundamental departure point for much of received economic theory. Apart from a deterministic setting the presence of uncertainty implies a dependence on the probable actions of other agents; that is, the ultimate behavior of an individual is to a certain extent a consequence of his beliefs concerning the behavior of other agents. While the difficulty associated with formulating even crude conjectures of this nature is overwhelming, actual informational demands are even greater as from the dependence of agent A's actions on his beliefs concerning agent B's actions, it follows directly that agent B's actions are dependent on his beliefs concerning agent A's beliefs relative to his (agent B's) actions as well, ad infinitum. Consider the following quote from Keynes [7]:

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> ...professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practise the fourth, fifth and higher degrees.

The above passage indicates that economic decision making in an uncertain setting may entail assessments of what average opinion may be.* The classical method of circumventing the problem is to eliminate deviations between individual and average beliefs by positing identical subjective beliefs or "homogeneous expectations." In a static setting this may be an appropriate resolution but when dynamic considerations are introduced it becomes necessary to go one step further. The fundamental problem is that current period actions, while not affecting current period outcomes, may often affect future outcomes; e.g., when an individual purchases a security he is concerned not only with dividend income, but also with potential capital gains (losses). While dividends

*The consequences of this issue have not been devoid of discussion in the literature as exemplified by the distinction drawn by Starr [15] concerning ex ante and ex post Pareto optimality.

this period are independent of the expectations of other investors, next period security prices are dependent on such expectations and concommitant market transactions. In this context the avoidance of ex ante mistakes requires, in terms of the above quote from Keynes, the ability to correctly engage in subsequent rounds of anticipating "what average opinion thinks average opinion to be." While this is a phenomena which plagues many equilibrium concepts, we explore its implications in the context of the theory of corporate finance and stockmarket equilibrium.

I. MODIGLIANI AND MILLER

The classic paper in the economics of corporate finance is of course the first Modigliani and Miller paper, "The Cost of Capital, Corporation Finance and the Theory of Investment" [10]. This paper argues that the market value of any firm is determined by the "risk class" to which the firm belongs, and is independent of the debt-equity ratio of the firm.

The notion of a risk class of firms is central to the MM argument and may be defined as follows. Let $X_{i\theta}$ denote the earnings of firm i in state of nature θ. Each investor has a subjective probability distribution over θ. Firms i and j belong to the same risk class for a given investor if their earnings are perfectly correlated over states of nature so that

$$X_{i\theta} = \alpha_{ij} X_{j\theta} \quad \text{for all } \theta,$$

where α_{ij} is a constant independent of θ.

The MM paper is a little vague as to just what is assumed concerning the agreement of beliefs among investors. We will adopt the strongest of assumptions in order to highlight the difficulties that arise under even this case. So we postulate that investors are in unanimous agreement as to the subjective probability distribution over θ and as to the earnings of each firm in each state of nature θ. It follows that every investor agrees as to the identity of the firms in any risk class k.

MM assume that bonds are default risk free and that individual investors can borrow or lend at the risk free interest rate that applies to corporate bonds. Then the basic MM proposition is the following.

<u>MM Proposition</u>. The market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate ρ_k appropriate to its class.

Let $\bar{X}_j$ denote the expected earnings of firm j and let M_j denote the market value of firm j. $M_j = p_j E_j + D_j$, where p_j is the price per share of firm j's stock, E_j is the number of shares of firm j's stock outstanding, and D_j is the face value of bonds issued by firm j.

The MM Proposition asserts that

$$M_j = p_j E_j + D_j = \bar{X}_j/\rho_k$$

for any firm j belonging in risk class k.

The argument to establish this proposition is the following. Suppose firms 1 and 2 are in the same risk class and, for simplicity, assume each has the same earnings stream ($\alpha_{12} = \alpha_{21} = 1$), and hence the the same expected return $\bar{X}$. Suppose firm 1 is financed entirely by stock while firm 2 has bonds in its capital structure. $M_1 = p_1 E_1$ is the value of firm 1 and $M_2 = p_2 E_2 + D_2$ is the value of firm 2. Suppose $M_2 > M_1$.

Consider an investor holding S_2 shares of firm 2 representing α percent of the outstanding shares ($S_2 \equiv \alpha E_2$). Let $Y_{2\theta}$ denote income from his holdings in state of nature θ. Then

$$Y_{2\theta} = \alpha(X_{2\theta} - rD_2)$$

where r is the interest rate. But, by assumption $X_{2\theta} = X_{1\theta}$ for every θ, hence let $X_\theta = X_{1\theta} = X_{2\theta}$ so that

$$Y_{2\theta} = \alpha(X_\theta - rD_2).$$

Suppose the investor sells his shares in firm 2 and borrows αD_2 dollars to invest the proceeds in the shares of firm 1. Let S_1 denote his new holdings of firm 1's stock. Then

$$p_1 S_1 = \alpha(p_2 E_2 + D_2).$$

Income from this new portfolio in state θ is denoted by $Y_{1\theta}$, where

$$Y_{1\theta} = \frac{S_1}{E_1} X_\theta - \alpha r D_2$$

$$= \frac{\alpha M_2}{p_1 E_1} X_\theta - \alpha r D_2 = \alpha\left(\frac{M_2}{M_1} X_\theta - rD_2\right).$$

with $M_2 > M_1$ (and with $X_\theta > 0$), then $Y_{1\theta} > Y_{2\theta}$, hence each investor will find it profitable to sell shares of firm 2 and acquire shares of firm 1, until $M_2 = M_1$. A similar argument applies if $M_1 > M_2$.

There are several problems with this argument, all well known. First, if there is default risk then in general, the MM proposition fails. This relies on the work of Smith [13] and Stiglitz [16]. Second, if the borrowing (or lending) rate of

the investor differs from that for any firm, then generally the result fails. Again see Stiglitz [16], who regards this as perhaps the most restrictive condition of the MM proposition. Finally, the argument is generally invalid when capital gains are taken into account. We want to expand on this last point.

The MM argument implicitly assumes that investors are concerned only with income from their portfolios and are not concerned about the market values of the stocks and bonds they hold. Once wealth as well as income becomes a matter of interest for the investor, the riskless arbitrage argument fails. Specifically, suppose our investor holds S_2 shares of firm 2, $M_2 > M_1$ and the investor believes $\bar{X}_1 = \bar{X}_2 = \bar{X}$ and that firms 1 and 2 are in the same risk class. He knows that by transferring into firm 1's stock he can raise his income in every state of nature, just as in the MM situation. Furthermore, suppose every investor has the same beliefs. Does this guarantee a shift out of firm 2's stock into firm 1's stock?

Unfortunately there is no such guarantee and the reason is that identified by Keynes. Just because I think firm 2 is overvalued is no guarantee that everyone else thinks it is overvalued. From the point of view of an individual investor it is clearly conceivable that everyone else in the market regards firms 1 and 2 as being in different risk classes, and that the overvalued firm is 1 rather than 2. If this were the case, then it would still make sense from the point of view of income to buy firm 1, but not from the point of view of contemplated capital gains. So far as we can see, there is no way to eliminate this problem -- the market simply does not provide any

investor assurance that his perception of risk classes is that which everyone else perceives. Moreover, things are worse than this, since even if everyone agrees, everyone must know that everyone agrees, and so forth, if truly riskless arbitrage is to occur.

The same problem arises in the homemade leverage literature; we present a somewhat more formal statement of the difficulty.

II. THE HOMEMADE LEVERAGE THEOREM

Generally, we will follow the approach in Smith [13, 14], but we note various generalizations and/or modifications of his approach in the existing literature. There is a single firm which issues bonds and stocks, and we examine the decision problem of a typical investor. We use the notation:

θ = earnings per dollar of assets of the firm

r = contractual interest rate on bonds

D = number of \$1 bonds issued by the firm

E = number of shares issued by the firm

p_o = price per share at the beginning of the period

$M = p_o E + D$ = capitalization of the firm

$\mu = \frac{D}{p_o E}$ = debt-equity ratio of the firm

S = number of shares purchased by the investor

B = number of bonds purchased by the investor

W_o = initial wealth of the investor

W = wealth of the investor at the end of the period

p_1 = price per share at the end of the period.

While we adopt a one period planning horizon for the investor in the interest of notational simplicity, the firm is assumed to be an ongoing entity. This is in contrast to the Smith [13] and Stiglitz [16]

papers,* where it is implicitly assumed that liquidation takes place at the end of the single period.

We assume there is no default risk, again for notational simplicity and to highlight the special problems associated with capital gains. Thus the bonds of the firm pay guaranteed interest and so the investor holds no cash in his portfolio.

The basic budget constraint is then

$$W_o = B + p_oS, \tag{1}$$

and wealth at the end of the period is given by

$$W = (1+r)B + [\theta(D+p_oE) - rD]\frac{S}{E} + p_1S. \tag{2}$$

The first term on the RHS of (2) is the value of the investor's holdings of bonds at the end of the period. The second term identifies the investor's share of equity earnings during the period, assumed to be paid out in the form of dividends at the end of the period, and the final term is the market value of the investor's shares at the end of the period. Rewriting (2) we have

$$W = (1+r)B + [\theta(1+\mu) - r\mu]p_oS + p_1S. \tag{3}$$

*In Stiglitz [17] a state preference dynamic programming approach is used. Stock prices are deterministic and bond prices are random until maturity date when bonds are paid off in full. There is zero default probability and complete agreement of expectations on this.

Where $\mu = D/E$. In contrast to Smith and Stiglitz, there are two random variables, θ and p_1, with joint p.d.f. $f(\theta,p_1)$.

The investor chooses S and B to maximize expected utility of terminal wealth W subject to the budget constraint. Let V denote expected utility, where

$$EU(W) = V = \int_0^\infty\int_{-\infty}^\infty U(W)f(\theta,p_1)d\theta dp_1.$$

Let $L = V + \lambda[W_o - B - p_oS]$. Then at an interior maximum we have

$$\frac{\partial L}{\partial B} = (1+r)EU'(W) - \lambda = 0 \tag{1}$$

$$\frac{\partial L}{\partial S} = E\{U'(W)[(\theta(1+\mu) - r\mu)p_o + p_1]\} - \lambda p_o = 0 \tag{2}$$

$$\frac{\partial L}{\partial \lambda} = W_o - B - p_oS = 0 \tag{3}$$

Consider the effect on V of a change in the firm's debt-equity ratio μ:

$$\frac{dV}{d\mu} = \frac{\partial V}{\partial \mu} + \frac{\partial V}{\partial B}\frac{dB}{d\mu} + \frac{\partial V}{\partial S}\frac{dS}{d\mu}.$$

From (3), $\frac{dB}{d\mu} = -p_o\frac{dS}{d\mu}$, while from (1) and (2), $\frac{\partial V}{\partial B} = \lambda$, $\frac{\partial V}{\partial S} = \lambda p_o$. Hence, $\frac{dV}{d\mu} = \frac{\partial V}{\partial \mu}$, an instance of the usual envelope theorem.

Specifically,

$$\frac{\partial V}{\partial S} = E\{U'(W)(\theta - r)\}p_o S + \int_0^\infty \int_{-\infty}^\infty U(W)\,\frac{\partial f}{\partial \mu}\,dp_1 d\theta,$$

taking as our general case that μ is a parameter of $f(\theta, p_1)$.

In the special case examined by Smith and Stiglitz, $p_1 = p_o$ (it is known with certainty that the terminal market price per share is p_o), and $f_\mu = 0$. Thus $\frac{\partial V}{\partial \mu} = EU'(W)(\theta - r)p_o S$, while from (2)

$$\frac{\partial L}{\partial S} = EU'(W)\{[\theta(1+\mu) - r\mu + 1]p_o\} - \lambda p_o = 0$$

Thus

$$EU'(W)\{(\theta - r)(1+\mu) + (1+r)\} = \lambda.$$

But from (1), $(1+r)EU'(W) = \lambda$ so that

$$EU'(W)\{(\theta - r)\} = 0, \quad \text{hence } \frac{dV}{d\mu} = 0.$$

The assumptions that $p_1 = p_o$ and $f_\mu = 0$ thus lead to the homemade leverage theorem. That is, assuming $B > 0$, changes in the debt-equity ratio of the firm have no effect on the expected utility of the investor; any changes in the firm's leverage position can be exactly offset by anti-leveraging (to use Smith's term) on the part of the investor. In particular, assumi g a regular constrained interior maximum, it can be shown (see Smith [13]) that

$$\frac{dS}{d\mu} = -\frac{S}{1+\mu}, \quad \frac{dB}{d\mu} = \frac{p_o S}{1+\mu}.$$

Hence,

$$\frac{dW}{d\mu} = (1+r)\frac{dB}{d\mu} + [\theta(1+r) - r\mu + 1]p_o\frac{dS}{d\mu} + p_o S[\theta - r] = 0,$$

so that terminal wealth W is independent of the debt-equity ratio μ.

Needless to say, the assumptions that $p_1 = p_o$ and $f_\mu = 0$ are not trivial restrictions on the problem. It is true that the one period Smith and Stiglitz models based on liquidation of the firm at the end of the period formally bypass issues associated with randomness of p_1; and if p_1 is not a random variable so that $f = f(\theta)$, there is no reason to assume $f_\mu \neq 0$. Once one drops the one period model of the firm, it is just not possible to avoid the problems associated with the resale market for the firm's stock, and generally there is a nondegenerate joint probability distribution over p_1 and θ. The joint distribution $f(p_1, \theta)$ satisfies $f_\mu = 0$ only under stringent assumptions concerning attitudes of investors. Satisfying $f_\mu = 0$ amounts to saying that the investor assumes that in the aggregate other i vestors will ignore the firm's debt-equity ratio in arriving at their portfolio choices. As contrasted with the one period model where the homemade leverage theorem holds, now any individual investor has to concern himself not only with the relatively "objective" analysis of the earning potential of a firm but also with the "subjective" analysis of the attitudes and beliefs of other investors, as they are reflected in the resale market for the stock of the firm. The problem seems to be in part one of information transmitted. The fact that all other investors will ignore the debt-equity ratio in making portfolio choices, even if true, does not imply that $f_\mu = 0$ since f reflects the

beliefs of the given investor, not objective reality.

When $f_\mu \neq 0$ and $f(p_1,\theta)$ is nondegenerate, then generally the homemade leverage theorem fails. To see this note that when p_1 is random, (2) can be rewritten as

$$EU'(W)\{(\theta - r)(1+\mu)p_\circ + (1+r)p_\circ + (p_1 - p_\circ)\} = \lambda p_\circ.$$

so that

$$EU'(W)\{\theta - r\} = EU'(W)\{p_\circ - p_1\}/p_\circ(1+\mu).$$

Since

$$\frac{dV}{d\mu} = EU'(W)(\theta - r)p_\circ S + \int_0^\infty\int_{-\infty}^\infty U(W)f_\mu dp_1 d\theta,$$

$\frac{dV}{d\mu} = 0$ if and only if

$$\int_0^\infty\int_{-\infty}^\infty [U'(W)(p_\circ - p_1) + U(W)f_\mu p_\circ(1+\mu)]dp_1 d\theta = 0.$$

Note that even when $f_\mu = 0$, $\frac{dV}{d\mu} = 0$ only if $EU'(W)(p_\circ - p_1) = 0$.

One way of interpreting this is to say that in the face of a change in the firm's debt-equity position, the investor is not able to engage in the appropriate offsetting anti-levaraging in a completely risk free manner; arguments based on riskless arbitrage fail.

III. STOCKHOLDER UNANIMITY

We next consider the question of whether there is some unambiguous measure of "the interests of the stockholders" for a firm operating in a world of uncertainty. The "stockholder unanimity" literature (Diamond [3], Leland [8], Ekern and Wilson [4], and Forsythe [6]) asserts that under certain conditions such an unambiguous measure exists. However, once again, the basic propositions that have been derive rest upon the assumption that the firm will be liquidated at the end of the period.

Adopting the notation of the previous section, consider a world in which there are N + 1 firms, each financed solely through stock (as is pointed out in the existing literature, this is not a substantive restriction so far as unanimity results are concerned). Firm j, j = 0, . . . , N + 1, has a profit function $\pi^j(d_j,z)$ where d_j is a decision variable for firm j, and z is a random variable.

Let S^j = number of shares of stock in firm j

S_i^j = shares of firm j owned by investor i

$\overset{\circ}{p}{}^j$ = initial price per share of firm j

$\overset{\circ}{V}{}^j$ = initial market value of firm j, where $\overset{\circ}{V}{}^j = \overset{\circ}{p}{}^j S^j$

p^j = end of period price per share of firm j

V^j = end of period value of firm j ($V^j = p^j S^j$)

End of period wealth for consumer i, W_i, is given by

$$W_i = \sum_{j=0}^{N} S_i^j\left(\frac{\pi^j}{S^j} + p^j\right),$$

that is, end of period wealth for consumer i equals earnings from his portfolio plus the end of period market value of the portfolio.

Then consumer i chooses $S_i \equiv (S_i^0, \ldots, S_i^N)$ to maximize $EU_i(W_i)$ subject to

$$\sum_{j=0}^{N} \overset{\circ}{p}{}^j (S_i^j - \bar{S}_i^j) = 0,$$

where $\bar{S}_i^j$ is the initial endowment of shares of firm j held by consumer i.

A <u>financial equilibrium</u> is then defined as a matrix of stock holdings $[\hat{S}_i^j]$ $i = 1, \ldots, M$, $j = 0, \ldots, N$ for a given decision vector $d = (d^0, \ldots, d^N)$ such that

(i) $\hat{S}_i \equiv (\hat{S}_i^0, \ldots, \hat{S}_i^N)$ maximizes $EU_i(W_i)$ subject to

$$\sum_{j=0}^{N} \overset{\circ}{p}{}^j (S_i^j - \bar{S}_i^j) = 0; \text{ and}$$

(ii) $$\sum_{i=1}^{M} \hat{S}_i^j = S^j \qquad j = 0, \ldots, N.$$

The maximization problem for consumer i can be formulated in terms of the Lagrangian

$$L_i = EU_i(W_i) + \lambda_i [\sum_{j=0}^{N} \overset{\circ}{p}{}^j (S_i^j - \bar{S}_i^j)]$$

with first order conditions

$$EU_i'(W_i)\{\frac{\pi^j}{S^j} + p^j\} - \lambda_i \overset{\circ}{p}{}^j = 0 \qquad j = 0, \ldots, N$$

$$\sum_{j=0}^{N} \overset{\circ}{p}{}^j (S_i^j - \bar{S}_i^j) = 0.$$

Alternatively, from the budget constraint,

$$S_i^0 = \frac{1}{p^0}(\sum_{j=0}^{N} \overset{\circ}{p}{}^j \bar{S}_i^j - \sum_{j=1}^{N} \overset{\circ}{p}{}^j S_i^j)$$

with

$$W_i = S_i^0(\frac{\pi^0}{S^0} + p^0) + \sum_{j=1}^{N} S_i^j(\frac{\pi^j}{S^j} + p^j).$$

(We will adopt the simplifying assumption that $p^0 = \overset{\circ}{p}{}^0$; that is, the initial and end of period prices of security 0 are the same and are known with certainty). Then we have

$$W_i = (\frac{\pi^0}{S^0} + p^0)\frac{1}{p^0}\sum_{j=0}^{N} \overset{\circ}{p}{}^j \bar{S}_i^j + \sum_{j=1}^{N} S_i^j\{\frac{\pi^j}{S^j} + p^j - (\frac{\pi^0}{S^0} + p^0)\frac{\overset{\circ}{p}{}^j}{\overset{\circ}{p}{}^0}\}; \text{ or,}$$

$$W_i = (\frac{\pi^0 + V^0}{V^0})\sum_{j=0}^{N} \overset{\circ}{p}{}^j \bar{S}_i^j + \sum_{j=1}^{N} S_i^j\{(\frac{\pi^j + V^j}{S^j}) - (\frac{\pi^0 + V^0}{V^0})\overset{\circ}{p}{}^j\}.$$

Hence at a constrained maximum of expected utility for consumer i we have

$$EU_i'(W_i)\frac{\partial W_i}{\partial S_i^j} \equiv EU_i'(W_i)\{(\frac{\pi^j + V^j}{S^j}) - (\frac{\pi^0 + V^0}{S^0})\overset{\circ}{p}{}^j\} = 0$$

$$j = 1, \ldots, N.$$

It might be well at this point to note that in the unanimity literature it is usual to assume that end of period firm values $V^j \equiv p^j S^j$

are nonrandom; in fact they are taken as given constants (liquidation values) independent of such variables as d^j, the $j^{\underline{th}}$ firm's decision variable. In the general case of course, $p = (p^0, \ldots, p^N)$ as well as $\pi = (\pi^0, \ldots, \pi^N)$ are random variables with joint distribution $\phi^i(p,z)$, (recall that $\pi^j = \pi^j(d^j,z)$ where z is a random variable).

Hence $EU_i = \int_p \int_z U_i(W_i)\phi^i(p,z)dpdz$. In general, $d = (d^0, \ldots, d^N)$ is an argument of ϕ^i, that is, $\phi^i = \phi^i(p,z;d)$.

To turn to the stockholder unanimity theorems, the idea behind such theorems is to identify conditions on the financial environment (preferences, endowments, etc.) such that, for any firm j, all stockholders of j unanimously agree as to the desirability or lack of desirability of any change in firm j's decision variable d^j.

Hence all stockholders of firm j agree unanimously as to the optimal level at which d^j should be set. When unanimity holds, the rule for managing a firm under uncertainty becomes as unambiguous as the profit maximization rule that holds under certainty.

Consider then the effect of a change in d^j on the value of expected utility for consumer i:

$$\frac{dEU_i}{dd^j} = EU_i'(W_i)\{\frac{\partial W_i}{\partial d^j} + \sum_{j=1}^{N} \frac{\partial W_i}{\partial S_i^j}\frac{\partial S_i^j}{\partial d^j}\} + \int_p \int_z U_i(W_i)\frac{\partial \phi^i}{\partial d^j}dpdz$$

$$= EU_i'(W_i)\frac{\partial W_i}{\partial d^j} + \int_p \int_z U_i(W_i)\frac{\partial \phi^i}{\partial d^j}dpdz$$

by the usual envelope theorem.

Let $r = \frac{\pi^0}{V^0}$ so that

$$W_i = (r+1)\sum_{j=0}^{N} \overset{\circ}{p}{}^j \bar{S}_i^j + \sum_{j=1}^{N} S_i^j\{(\frac{\pi^j + V^j}{V^j} - (r+1)\overset{\circ}{p}{}^j\}.$$

Then

$$\frac{\partial W_i}{\partial d^k} = \sum_{j=0}^{N} \overset{\circ}{p}{}^j \bar{S}_i^j \frac{\partial r}{\partial d^k} + (r+1)\sum_{j=0}^{N} \bar{S}_i^j \frac{\partial \overset{\circ}{p}{}^j}{\partial d^k} - \sum_{j=1}^{N} S_i^j[(r+1)\frac{\partial \overset{\circ}{p}{}^j}{\partial d^k} + \overset{\circ}{p}{}^j\frac{\partial r}{\partial d^k}]$$

$$= \frac{\partial r}{\partial d^k}\{\sum_{j=0}^{N} \overset{\circ}{p}{}^j \bar{S}_i^j - \sum_{j=1}^{N} \overset{\circ}{p}{}^j S_i^j\} + (r+1)\{\sum_{j=0}^{N} \bar{S}_i^j \frac{\partial \overset{\circ}{p}{}^j}{\partial d^k} - \sum_{j=1}^{N} S_i^j \frac{\partial \overset{\circ}{p}{}^j}{\partial d^k}\}.$$

Take the special case where endowments are at a financial equilibrium so that $\bar{S}_i^j = S_i^j$ for all i, j. Then we have

$$\frac{\partial W_i}{\partial d^k} = \bar{S}_i^0[p^0\frac{\partial r}{\partial d^k} + (r+1)\frac{\partial p^0}{\partial d^k}].$$

We can now state an unanimity theorem:

<u>Stockholder Unanimity Theorem</u>: Assume that (1) profits for firms other than k are independent of d^k for $k = 0, \ldots, N$; (2) the joint pdf $\phi^i(p,z;d)$ is independent of d; (3) initial endowments are at a financial equilibrium. Then stockholders of firm k are unanimous in their evaluation of the optimum level of d^k for $k = 0, \ldots, N$.

Proof:

$$\frac{dEU_i}{dd^k} = EU_i'(W_i)\frac{\partial W_i}{\partial d^k} + \int_p \int_z U_i(W_i)\frac{\partial \phi^i}{\partial d^k} dpdz.$$

By (2), the expression under the integrals is zero, while by (1) and (3),

$$\frac{\partial W_i}{\partial d^k} = \bar{S}_i^0 [p^0 \frac{\partial r}{\partial d^k} + (r+1)\frac{\partial p^0}{\partial d^k}].$$

Clearly the expression inside the brackets is independent of i, hence with $U_i' > 0$ for every i,

$$\text{sign } \frac{dEU_i}{dd^k} = \text{sign } \frac{dEU_t}{dd^k}$$

for any i, t such that $\bar{S}_i^\circ > 0$, $\bar{S}_t^\circ > 0$. The optimal level of d^k is then such that

$$p^0 \frac{\partial r}{\partial d^k} + (r+1)\frac{\partial p^0}{\partial d^k} = 0,$$

identical for all stockholders.

A more sophisticated version of the stockholder unanimity theorem appears in Eckern and Wilson, and in Forsythe. In those two papers, an Arrow-Debreu formulation of uncertainty in terms of states of the world is presented. Buying a share of firm j then is equivalent to buying a contingent claim contract with known payoffs (depending on d^j) for each possible state of the world. Then "there will be

stockholder unanimity with respect to the actions of an individual firm, independent of investors' subjective probability distributions and utility functions, if any proposed change in the firm's state-distribution of returns does not alter the set of state-distribution of returns available to each individual in the economy" (Forsythe, p. 2). This more complicated "spanning condition" replaces condition (3) above; no longer is it necessary that initial endowments be equilibrium endowments. However, conditions (1) and (2) still stand; in particular, sensitivity of capital gains to firm decisions generally causes problems for the stockholder unanimity results.

Once again the basic problem is that of insulating the investor from the market. When the Eckern-Wilson "spanning condition" holds (and if future prices of stocks are independent of d), then there is riskless arbitrage possible to permit investors to offset any proposed changes by firm managers. But the riskless arbitrage is only possible if changes in firm decision variables leave $\psi_k^i = 0$ where

$$\psi_k^i = \int_p \int_z U_i(W_i)\frac{\partial \phi^i}{\partial d^k}(p,z;d)dpdz.$$

In other words, when changes in decision variables (e.g., the debt/equity ratio of a firm or some other decision variable) have nontrivial impacts on the probability distribution over end of period prices generally there is no way to offset this in a completely risk free manner by portfolio choices.

IV. EXTENSIONS AND IMPLICATIONS

The observations above are not unique to the cases considered. Extensions apply to the capital asset price model of Sharpe [12], Lintner [9], and Mossin [11]. While Brownlee and Scott [2] allow random stock prices, they insist on identical expectations; actually the requirements are greater in that all individuals must know that expectations are identical and independent of financial policies. This point is made clear by Fama [5] who separates homogeneous expectations from given investments strategies so that "[a]lthough decisions to be made in the future are unknown, the rules that firms use to make current and future investments are given. In addition, investment decisions are made independently of how the decisions are financed." In a classic paper Arrow [1] is able to accommodate differing subjective beliefs but at the cost of an implicit assumption that keeps the consumer captive in the market and perhaps may prevent him from engaging in a lottery which he finds desirable.

The observations in this paper solve no problem but perhaps serve to highlight it. The need for a theory at expectations formation is essential to the concept of equilibrium in a dynamic world subject to uncertainty.

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