TOWARD A THEORY OF LEGISLATIVE DECISION

John A. Ferejohn and Morris P. Fiorina
California Institute of Technology

and

Herbert F. Weisberg
Ohio State University

Published in Game Theory and Political Science,
Peter C. Ordeshook and Richard D. McKelvey, eds.
Recent developments in formal political analysis have spawned two seemingly related theories of democratic political processes. The more familiar of the two is the theory of electoral competition based on Downs' (1957) heuristics and greatly elaborated by Davis, Hinich and Ordeshook (1970), Kramer (1975), McKelvey (1976), and others. Somewhat less familiar (perhaps because the intellectual movement is less well integrated) is the theory of legislative decision which has grown from roots in game theory and the theory of social choice. Black (1958), Riker (1962), Plott (1967), Wilson (1969), Schwartz (1970), Kadane (1972), and several others have nurtured the rudimentary models which compose this theory.

While the two areas of theoretical work have developed separately, they share numerous elements. Both theories postulate the existence of a set of voters, N, each of whom has a preference relation, $R_i$, on a set of alternatives, X. The theory of electoral competition typically embeds the set of alternatives in some abstract space, but the basic theory does not depend on this embedding, so for the present we shall take X to be simply an unstructured set of alternatives. Often, restrictions are put on the preference relations of individuals, but these too are mostly inessential for establishing the basic connections between the theories. Thus, we shall rely on the ordinalist definition: $x$ is said to be preferred to $y$ by individual $i$ if and only if $xR_iy$ and not $yR_ix$. If both $xR_iy$ and $yR_ix$, $i$ is said to be indifferent.

In the theory of legislative behavior an alternative $x$ defeats a distinct alternative $y$ in a majority vote only if the number of individuals preferring $x$ to $y$ exceeds that preferring $y$ to $x$. A majority rule equilibrium (MRE) is a set of alternatives, E, with the property that each element is undefeated in a majority vote. A simple intuition suggests the MRE as a predictive concept: given a legislature in which a proposal, $x$, not in the MRE is on the floor, there is some coalition which has both the power to form under the rules of the legislature, and the incentive to form to pit some alternative $y$ against $x$. 


The theory of electoral competition contains two additional members of society -- the candidates. Candidates announce elements of X as platforms. A candidate announcing a platform x defeats a candidate announcing a platform y only if the number of voters preferring x to y exceeds that preferring y to x. The candidates are assumed to prefer victory to defeat. An electoral competition equilibrium (ECE) is a set of points C with the property that each point in C is not a losing platform. The behavioral motivation here is that if a candidate espouses a platform outside C the other candidate has the incentive and the ability to find another platform that will defeat the first candidate.

From the preceding exposition it should be evident that E = C when either set is nonempty. This oft-noted connection has led some to think that the relation between the theories is very close -- that they are substantively differing interpretations of the same formal structure. One should remember, however, that the sets C and E are nonempty only in special cases, and that existing models of legislative behavior and electoral competition are mute with respect to what will happen in the (usual) case when these sets are empty. Moreover, the two bodies of theory differ in an important element of their intuitive content. The basic mechanism in electoral competition is that there are two candidates competing for votes, whereas in a legislature the strategic activities are undertaken by the various coalitions.

Given these distinctive intuitive contents, some analysts make the natural suggestion that we formulate the theory of electoral competition as a two-person noncooperative game, and the theory of legislative decision as an n-person cooperative game. Such a line of attack logically need not produce disjunct theories, but in practice few connections exist. For example, attempts to discern whether the classical solution concepts of a mixed strategy equilibrium and a von Neumann-Morgenstern solution bear any resemblance (Perejohn, 1974; McKelvey and Ordeshook, 1976) reveal little overlap between these notions. This failure to establish a close tie between the more general theories of electoral competition and legislative decision reinforces the suggestion that the two theories might best develop independently.

Recent work by McKelvey (1976) raises additional doubts about the usefulness of a theory which encompasses both electoral competition and legislative decision. Presumably, as a starting point for developing a general theory we would examine any regularities which exist when \( E = C \) is empty. But McKelvey shows that if \( X \) is embedded in a Euclidean space, with individual preferences monotonic in Euclidean distance, then when \( E = C = \emptyset \), each alternative in \( X \) indirectly dominates
each other alternative in \( X \). Thus, no regularities may be observed when \( E = C = \emptyset \) -- both electoral and legislative processes might produce outcomes scattered over the entire set of alternatives. If this nihilistic possibility were a fact, the outcomes of empirically occurring legislative and electoral processes would depend primarily on the specific rules and procedures which characterize those processes, and on the sophisticated maneuverings of the participants in those processes. And, positive models of such processes would necessarily focus on what Shepsle (1978) calls "structure induced equilibria." Rather than general theories of electoral competition and legislative decision, we would have numerous specific theories differentiated by the institutional structure explicitly assumed by the theory.

While we believe that the explicit incorporation of particular rules and procedures into our models is an interesting and potentially profitable avenue of research, we are not yet completely convinced that the older approach has been milked dry. After all, McKelvey's result is a possibility result, no more. Theoretically outcomes may scatter over the entire policy space, but that is not to say that empirically they actually do so. In fact, what little evidence we have suggests the opposite. Consider the Fiorina and Plott (1978) experiments on committee decisions under majority rule. The principal finding of these authors is that the set \( E = C \) is a good predictor of legislative outcomes if it is non-empty. But even more interesting from our point of view is the experimental series in which \( E = C = \emptyset \). Rather surprisingly, the experimental outcomes continue to cluster, rather than scatter widely over the set \( X \). On the basis of their observations Fiorina and Plott speculate that some as yet unformulated theory exists which explains the outcomes which occur in the absence of an MRE (ECE), but specializes to the latter when it exists. Granted, one experimental series is hardly conclusive, but taken together with independent experimental findings of the Carnegie group (McKelvey, Ordeshook and Winer, 1978) it suggests that there are regularities in legislative decision processes even when the MRE is empty (with no comparable experimental results for the electoral competition case we cannot say whether analogous regularities exist).

The remainder of this paper attempts to extend the theory of legislative decision to the case of an empty MRE. Since experimental evidence motivated us to undertake this effort we will first describe the experimental outcomes in greater detail with emphasis on what we see as possible regularities in the outcomes. We will briefly note why existing theories do not account for these regularities. Then we will sketch a new theory which makes predictions for majority rule agenda-free legislative processes. Having done so, it is a simple task to outline the theoretical predictions for a series of future
experiments. Until such evidence is in, however, our arguments must remain provisional and tentative.

1. EXPERIMENTAL EVIDENCE

The experiments considered in this section took the form of five-person committees operating under majority rule. The set of alternatives consisted of points in the plane, where individual preference over such points was monotonic in Euclidean distance from an actor's ideal point (for details see Fiorina and Plott). Subjects were allowed to debate proposals but were not permitted to exchange monetary information about their payoffs.

Figure 1 shows the experimental outcomes of the Fiorina-Plott communication series in the case where $E = C = (39,68)$. As is evident, the MRE and the mean of the experimental results are very close. Moreover, the scatter of outcomes is quite small — we are not just observing the mutual cancellation of large deviations.

Figure 2 differs from Figure 1 only in that the player whose ideal point is at $(39,68)$ in Figure 1 is moved southeastward to $(51,59)$. This, of course, destroys the MRE and opens up the possibility for McKelvey's theorem to operate. But does it? What we see is still a fairly tight scatter of experimental outcomes, although not so tight as in the case of Figure 1. Is there any information in this scatter, or does its interest lie
merely in the fact that it is not as broad as it might be?
The mean of the outcomes \((45.62)\) has no obvious significance. One fact of interest is that the outcomes tend to cluster around but not lie within the centrally located set, \(M\), the minimax set (to be discussed in the next section).
A related fact is that outcomes tend to lie along the contract curves between pairs of committee members rather than to lie within the regions these curves delineate. Admittedly, such observations are casual and based on a small number of observations. But consider now the experimental series conducted independently at CMU. Figure 3 illustrates the experimental outcomes reported by Mc Kelvey, Ordeshook and Winer. While the configuration of ideal points is qualitatively different from that used in the Caltech experiments (no ideal point in the interior of the convex hull of the ideals), the observations previously made continue to hold. Outcomes cluster around but do not enter the central set, \(M\), and they lie along the contract curves between pairs of committee members. Additionally, the outcomes seem to form several tiny "clusters," something less evident in the Caltech experiments, but perhaps also true depending on the perception of the reader.

These two sets of experiments were independently designed and conducted. Given this fact and the relatively small number of observations, we are impressed that any regularities whatsoever should emerge from the findings,
sufficiently impressed to consider any theory which accounts for these regularities. Do such theories presently exist?

2. THE ADEQUACY OF EXISTING THEORIES

The natural building blocks for a theory of legislative behavior would seem to be the solution concepts from the theory of cooperative games, Simpson's (1969) minimax set, and the competitive solution (CS) of McKelvey, Ordeshook and Winer. We will not dwell on the game theoretic solution concepts; their shortcomings are discussed in McKelvey, Ordeshook and Winer. Briefly, in addition to the well-known problems of nonuniqueness and nonexistence, solution concepts like the V-solution and the Bargaining set perform poorly in the experiments previously discussed as well as in additional unpublished experiments conducted by Plott.

Simpson has proposed a rather different predictive concept, the minimax set. We can define it as follows. Let \( n(yPx) \) be the number of votes for \( y \) against \( x \). Then let \( v(x) = \max n(yPx) \). Finally let \( M = \{x^*eX \mid v(x^*) = \min_{y \in X} v(x)\} \). Given finite \( N \), \( M \) always exists. Indeed the sets, \( M \), in Figures 1 and 2 are the minimax sets. They are easy to compute and "centrally" located. Further, as the number of voters increases, provided they are scattered evenly over the space, the size of \( M \) shrinks (Kramer, 1977).
Finally, if $E$ is nonempty, $M \subseteq E$.

Kramer has provided an interesting though somewhat implausible motivation of $M$ for the case of electoral competition. Assume there are two candidates competing for votes who both obey the following rule: When the opposing candidate adopts platform $x \in X$, adopt a platform $z \in X$ with the property that $N(zPx) = v(x)$. That is, both candidates are sequential vote maximizers. Kramer shows that assuming a stable policy space and stable voter preferences, the outcome of this process "converges" to $M$ as successive elections take place.

A less dynamic motivation is as follows. If $x \in X$ then even if it can be beaten by some other element $y$, the size of the minority coalition supporting $x$ is at a maximum. Thus, it might be more difficult to locate such a $y$ or to organize it to defeat $x$. This motivation is admittedly less elegant than Kramer's but it is developed in the context of the legislative situation.

There are various difficulties with the minimax set. It can include a relatively large subset of the set of alternatives, although this problem fades as $N$ increases. More importantly from our standpoint, the existing experimental observations, while often "near" $M$, tend to cluster on its boundaries. Thus, we suspect that $M$ is too broad, that a sharper concept might exist. In particular, can we formulate a concept that isolates the boundaries of $M$?

Most recently McKelvey, Ordeshook and Winer have proposed a new solution concept — the "competitive solution." The latter is a set of proposals and supporting coalitions constructed as follows. Each coalition puts forward exactly one alternative which may be considered its official proposal. A collection of such proposals is a competitive solution if each individual is indifferent as to which coalition he joins. Work on this concept is in an intermediate stage and, at present, it is not clear whether the competitive solution generally exists. In the five-person legislative experiments conducted by the Carnegie group the CS works well as a predictor of the experimental results. But more theoretical work must be done before a fair evaluation of the CS can be given. Although we admire its aesthetics, we have found it difficult to work with and doubt that a general existence theorem can be obtained for broad classes of interesting games, including the larger spatial games. Thus, we are not yet prepared to put all our theoretical eggs in this basket.

3. A THEORY OF AGENDA-FREE LEGISLATURES

The experimental legislatures operated by Fiorina and Plott have no preset agenda. Any member can offer an amendment to the proposal on the floor at any time, although all voting between motions is formal and pairwise. The initial
proposals, typically made in ignorance, tend to be casual suggestions of individual maxima or various "symmetrical" points, e.g. (50,50) or (100,75). But after some information has been exchanged through votes and debate, the process of proposal generation not surprisingly settles down into a reflection of the dynamics of majority-building. This process is more truncated than one might expect. We know that for any proposal on the floor numerous majority preferred alternatives exist, but one seldom observes more than a few of these actually put up for a vote. Moreover, at some point the committee will accept a relatively successful motion on the floor even though many alternative proposals not yet considered could defeat it. Boredom? Perhaps. But consider an alternative more systematic notion.

We say that proposal $x \in \mathcal{X}$ is vulnerable to a coalition $c \in \mathcal{N}$ if there is a $y \in \mathcal{X}$ such that everyone in $c$ prefers $y$ to $x$ and $c$ is a majority. Equivalently, $x$ is vulnerable to a majority coalition if it is not contained in the Pareto set of that coalition. We let $c(x)$ denote the collection of coalitions to which $x$ is vulnerable. If $c(x) = \emptyset$ we say $x$ is invulnerable. Generally, however we can associate with each $x \in \mathcal{X}$ a real number $c(x) = |c(x)|$ which will be called the vulnerability of $x$. We may note the following facts:

**Proposition 1:** $c(x) = 0 \iff x \in E$

**Proposition 2:** $L = \{x \in \mathcal{X} \mid \overline{c}(x) = \min_{z \in \mathcal{X}} c(z)\}$ is nonempty.

The first proposition simply notes the obvious fact that the MRE is the set of invulnerable points. The second proposition notes the equally obvious but somewhat more interesting fact that some point(s) in any collective decision situation will be less vulnerable than all other points, although this point(s) will not have 0-vulnerability when an MRE fails to exist.

One can use the notion of vulnerability to generate several solution concepts. As yet we are agnostic about which is the best candidate for future development. The basic idea underlying any of the possibilities, however, is the following. If $x \in \mathcal{X}$ is on the floor, it is vulnerable to each coalition in $c(x)$. Ceteris paribus, we would expect to observe $x$ defeated by an alternative proposal, $y$, the larger the size of $c(x)$. That is, the more coalitions contained in $c(x)$, the lower the odds that $x$ will be the collective choice. As mentioned, there are various ways to use this intuitive expectation. Perhaps the simplest is to note that the set $\{c(x) \mid x \in \mathcal{X}\}$ can be totally ordered as follows:

$$c(x) \geq c(z) \iff \overline{c}(x) \geq \overline{c}(z)$$

An infimum of the $\geq$ relation is obtained on the set $L$ in proposition 2, so that one could take $L$, the least
vulnerable set, as the solution concept.

Alternatively, one could investigate a probabilistic hypothesis about the association between vulnerability and legislative outcomes. More precisely, we can examine the prediction that the "likelihood" of a given outcome depends directly on its vulnerability. In the case of Figure 4 (a partial reproduction of Figure 2) such a probabilistic model might identify point d as most likely, followed by the other three vertices of the minimax set \{a,b,c\}, followed then by the union of the open line segments \((ab) \cup (bd) \cup (cd) \cup (ac)\) which bound the minimax set, which, we might add ties (in terms of vulnerability) points on contract curves relatively "far" from the minimax set. Given that existing experimental observations typically fall near the boundary of the minimax set, we regard a probabilistic vulnerability theory as promising. Figure 4 reveals that even those points which fall far from the boundary of the minimax set still tend toward the locally least vulnerable points -- the contract curves between committee members always have vulnerability at least as low as the regions they bound. All in all, Figure 4 shows a clear tendency for outcomes to cluster near the less vulnerable points even though such points comprise only a tiny fraction of the points in the space.

The outcomes of the Carnegie experiments would look very good from the standpoint of the theory of vulnerability were it not for the fact that an alternative theory (CS) does even better. In Figure 5 the vertices of the minimax
set are the least vulnerable points, followed by the union of the open line segments connecting them. The experimental outcomes fall midway along such line segments — near points of second lowest vulnerability in other words. We will say more about this situation below. For now, we make the observation that the Carnegie game is less stable than the Caltech game in the sense that with \( N \) constant at five, the Caltech game possesses a single point with lower vulnerability than the five equi-vulnerable points of the Carnegie game.

Before proceeding to a discussion of some formal properties of the L theory of vulnerability we wish to point out several features of the concept of vulnerability. First, like the minimax set, the notion of vulnerability diverges sharply from the solution concepts of game theory and voting theory. The latter invariably pose either-or questions: does it exist or not? In contrast, least vulnerable points and vulnerability orderings always exist. The vulnerability concept poses a question of degree — "how stable," rather than one of kind — "stable or unstable." When a 0-vulnerable point(s) exists, a high degree of stability characterizes the legislative situation. But ceteris paribus a legislative situation possessing a j-vulnerable point(s) may well exhibit greater predictability than one possessing a k-vulnerable point(s) (where \( 0 < j < k \)) although the prevailing theories fail to differentiate between the latter two situations, lumping both together as "unstable."
A second point concerns the scope of application of the theories of vulnerability. Existing solution theories presume a world of perfect or free information, infinitely discriminating players and only one game in town. In the real world we find poor and/or costly information, thick indifference curves, and a rich menu of decisions so that time and resources sunk in one decision means a lessened ability to capitalize on other potentially attractive opportunities. The theory of vulnerability presumes this real world context: No MRE may exist, but there may be numerous points which are stable "for all practical purposes."

Given such a motivation it is natural to expect the theory of vulnerability to apply most convincingly to decisions involving numerous players deciding among numerous alternatives. To illustrate this point consider Figures 6a-6c. Figure 6a is a three-person committee in a two-dimensional policy space. The individual ideal points are the least vulnerable points in the game (1-vulnerable), but we would not expect experimental outcomes to fall at these points. Why? There are only four winning coalitions in this situation — few enough to run through sequentially — and it is perfectly obvious which coalition can upset each 1-vulnerable point. Such conditions facilitate exhaustive, fine-tuned negotiations such as those formalized in the CS. Similarly, consider Figure 6b which illustrates a (small)finite alternative case. L consists of proposals

<table>
<thead>
<tr>
<th>Voter 1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_3</td>
<td>X_4</td>
<td>X_5</td>
<td>X_6</td>
<td>X_7</td>
<td>X_2</td>
<td>X_2</td>
</tr>
<tr>
<td>X_4</td>
<td>X_5</td>
<td>X_6</td>
<td>X_7</td>
<td>X_3</td>
<td>X_3</td>
<td>X_7</td>
</tr>
<tr>
<td>X_5</td>
<td>X_6</td>
<td>X_7</td>
<td>X_3</td>
<td>X_4</td>
<td>X_4</td>
<td>X_6</td>
</tr>
<tr>
<td>X_1</td>
<td>X_1</td>
<td>X_1</td>
<td>X_1</td>
<td>X_1</td>
<td>X_1</td>
<td>X_5</td>
</tr>
<tr>
<td>X_2</td>
<td>X_2</td>
<td>X_2</td>
<td>X_2</td>
<td>X_2</td>
<td>X_5</td>
<td>X_1</td>
</tr>
<tr>
<td>X_6</td>
<td>X_7</td>
<td>X_3</td>
<td>X_4</td>
<td>X_5</td>
<td>X_6</td>
<td>X_4</td>
</tr>
<tr>
<td>X_7</td>
<td>X_3</td>
<td>X_4</td>
<td>X_5</td>
<td>X_6</td>
<td>X_7</td>
<td>X_3</td>
</tr>
</tbody>
</table>
x₁ and x₂, but given that there are only five alternative proposals to check (all of which defeat x₁ and x₂ by some coalition) we doubt that L would be a good predictor of such a committee's decision.

In contrast, consider Figure 6c which represents an eleven-person committee (still fairly small as real legislatures go). This situation has a set of 9-vulnerable points, and various other points of low vulnerability. In absolute terms such points can be upset by numerous (i.e. nine) coalitions, but there are 1,024 winning coalitions in the game (462 of which are minimal winning). Looking at Figure 6c it may still be fairly obvious to us which nine coalitions can upset which point, but in a real world confused legislature will it be so obvious? We think not. As N increases (and as the complexity of the policy decision increases) upsetting points such as those identified in Figure 6c becomes exceedingly difficult. We might add that the comprehensive balancing act presumed by the CS becomes increasingly implausible as well. Thus, it is with tongues only partially in cheek that we propose the theory of vulnerability as a theory of "large" legislatures.

5. CURRENT RESEARCH

We are now engaged in several lines of inquiry. First we have investigated the connection between the least-
vulnerable set and other nonequilibrium solution concepts such as the minimax set. In general, no set inclusion relation exists even though both sets tend to be "centrally" located vis-à-vis the collection of voters. There appears to be some relationship in a spatial setup if the dimensionality of the space is small relative to the number of voters.*

Second, we have been trying to formalize a stochastic theory of vulnerability. A variety of Markov processes governing transitions from one proposal to others turn out to have the same ergodic sets. Moreover, these sets arise elsewhere in the theory of social choice and the theory of voting bodies. We are also trying to characterize the limiting distributions of these stochastic processes. In particular we would like to know what kinds of concentration properties hold for these distributions, in order to see if we can obtain stochastic convergence theorems that are analogous to Kramer's (1977) deterministic convergence result (since writing this paper we have made some progress along the lines discussed in this paragraph. See Ferejohn, Fiorina and Packel, 1978).

Finally, consider a spatial game with Type I preferences. It appears to us that it is possible to calculate the maximum value for L for a game of any N and given dimensionality (d). As one would expect, this value increases with N and with d. At least for games of small dimensionality, however, L increases far less rapidly than the number of winning coalitions in the game. To illustrate, a five-person spatial game in two dimensions has at worst a 3-vulnerable point(s). Given that there are sixteen winning coalitions in the game, 3-vulnerable points are subject to overthrow by 19 percent of the possible winning coalitions. An eleven-person game in two dimensions has at worst a 9-vulnerable point(s). But given that such a game has 1,024 winning coalitions, the 9-vulnerable points are subject to upset by less than 1 percent of the possible winning coalitions. We believe that real world legislatures customarily consider proposals composed of few components -- far fewer than the number of legislators (this is the whole purpose of germaneness rules and other restrictions on amendments). Thus, we suspect that real legislatures have numerous "near-cores," points which are very difficult, although not impossible to upset. Our conjecture, if proven, would lead to an empirical expectation quite the opposite of that arising from McKelvey's theorem. We are as yet unable to derive a general formula which would give vulnerability as a function of N and d. Recent work by Schofield (1978) may bear some relationship to this effort.

* For example, Figures 4 and 5 appear to represent the general case in two-dimensional spatial contexts with an odd number of voters (L may fall within M if the number of voters is even).
REFERENCES


Kadane, J. B., "On Division of the Question," Public Choice, 13 (1972), 47-54.


