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THE STOCK MARKET AND THE THEORY OF THE FIRM UNDER PRICE UNCERTAINTY

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Numerous attempts have been made to incorporate uncertainty into the neoclassical theory of the firm. The initial work of Arrow and Debreu first accomplished this by assuming a complete set of contingent claim markets. In their model, a stock market permitted an optimal sharing of risks and firms acted in the best interests of their stockholders by maximizing their stock market value. The major criticism of this work stems from the fact that all risks are insurable and that further investigation was required to understand an economy where risks are shared incompletely.

The move to a model which does not possess the equivalent of a complete set of contingent claim markets encountered a major difficulty in attempting to specify a firm's objective. In a model with incomplete markets, consumers no longer necessarily impute the same set of contingent claim prices and there will generally be disagreement on the determination of an optimal production plan based on value maximization. To circumvent this problem various studies[1] assumed that firms maximize the expected utility of profits. As Radner (1970) has pointed out, this approach essentially begs the question since there is no attempt to relate the behavior of the firm to the preferences of its owners. It is instead an expression of "the divorce of ownership from management." The fact that firms make risk averse decisions need not imply that firms are expected utility maximizers. Indeed, by using the appropriate contingent claim prices, value-maximizing firms will make decisions which reflect the risk preferences of their stockholders.

The purpose of this paper is to reexamine the criterion of value maximization when price uncertainty is present. Whereas we wish to work within a perfectly competitive framework, we will make one alteration to the usual set of competitive assumptions. Since we wish to focus on behavior in an economy where risks may only be shared incompletely, we will restrict certain forms of entry into the capital market. Indeed, if we would allow unrestricted entry, we would expect to find that the market would soon possess the equivalent of a complete set of contingent claim markets. Individuals could easily accomplish this by trading in the separate contingent claim components of any existing stream of returns. To prevent this unpackaging, we will allow only a specific type of entry. Namely, we will allow entering firms to offer return functions from a predetermined set of functions. This set will include only those return functions which are initially present in the economy. This assumption precludes purely financial entry, but makes all existing technologies for the production of returns freely accessible.

With entry restricted in this fashion, we will show that entry into the capital market will take place until there are as many firms offering independent patterns of returns across states of nature as there are random prices in the economy. Even though a complete sharing of risks generally will not be possible when this condition is satisfied, we will show that value-maximizing firms will make Pareto optimal production decisions by relying upon prices inherent in the stock market.[2]

The crucial point that establishes this is that while individuals impute different sets of contingent claim prices, they must all impute the same certainty-equivalent prices. In this context, a certainty-equivalent price is the amount an individual would pay for an asset which returns the random price. It will then be shown that if purely financial entry is also permitted, then the above conclusions will continue to hold. What our restricted form of entry serves to highlight is that our conclusions are not a consequence of permitting a form of complete risk-sharing.

Further, if the return function of a firm in each possible state of the world is simply the profit of that firm in each state we will show that the decision rules which result are simply analogs of the familiar rules which are obtained for a profit-maximizing competitive firm under certainty. For example, statements such as "firms should choose input levels such that the value of the marginal product of a factor equals the factor price" become "firms should choose input levels such that the certainty-equivalent value of the marginal product of a factor equals the certainty-equivalent factor price."

I. THE MODEL

We begin with a two-period, state preference model by assuming that there is only a single commodity which is available for consumption now, c_0, or which may be invested into firms in order to provide for consumption later if state of the world θ occurs, c_θ, $\theta = 1,\ldots,S$. Also assume that there are I individuals in this economy, each possessing an endowment of the commodity, $\bar{c}^i$, and a portfolio consisting of fractions, $\bar{s}^i_j$, of each firm j. Each firm must choose the level of its decision variables, x^j, which determines the values of the firms, $V_j(x)$, and the returns each firm offers next period if state of the world θ occurs, $r_{j\theta}(x^j)$.

A. Investors

Each individual is assumed to maximize his utility of consumption both now and later by choosing his portfolio of security holdings. We do not require that the individual be an expected utility maximizer but instead assume that he acts so as to maximize a more general utility function $U^i(c^i_0, c^i_1, \ldots, c^i_S)$ subject to the budget constraints

$$c^i_0 + \sum_{j=1}^{N} V_j(x)s^i_j \leq \bar{c}^i + \sum_{j=1}^{N} V_j(x)\bar{s}^i_j$$

$$c^i_\theta \leq \sum_{j=1}^{N} r_{j\theta}(x^j)s^i_j \qquad \theta = 1,\ldots,S$$

$$c_0 \geq 0, \; c_\theta \geq 0 \qquad \theta = 1,\ldots,S.$$

Assuming the utility function of each individual is sufficiently regular, and allowing unlimited short sales, the first-order conditions are given as

$$V_j(x)U^i_0 = \sum_{\theta=1}^{S} U^i_\theta r_{j\theta}(x^j) \qquad j = 1,\ldots,N \qquad (1)$$

where

$$U^i_0 = \frac{\partial U^i}{\partial c^i_0} \quad \text{and} \quad U^i_\theta = \frac{\partial U^i}{\partial c^i_\theta} \qquad \theta = 1,\ldots,S,$$

denote individual i's marginal utility for consumption now and for consumption later if state of the world θ occurs, respectively. Each individual imputes a set of contingent claim prices for consumption in state of the world θ as

$$\rho^i_\theta = \frac{U^i_\theta}{U^i_0}, \qquad \theta = 1,\ldots,S, \; i = 1,\ldots,I.$$

At this point we should note that if there is not the equivalent of a complete set of contingent claim markets, the first-order conditions provide us with a set of N equations in S(>N) unknowns, ρ^i_θ. Thus, there is no reason that individuals will necessarily impute the same set of contingent claim prices.[3]

B. Firms

We will begin by assuming that there are N firms in the economy, and since we wish to examine the effects of price uncertainty we will assume that each firm's return function if of the form

$$r_{j\theta}(x^j) = w(\theta)f^j(x^j), \qquad \theta = 1,\ldots,S, \quad j = 1,\ldots,N, \qquad (2)$$

where $w(\theta)$ is a 1 x K vector of prices which occur in state θ and $f^j(x^j)$ is a K x 1 vector of state-independent decision functions of firm j. Note that if some of the prices are nonstochastic, the functional form given by (2) may be altered by letting $w_k(\theta) = w_k$ for some k.

We further assume that no random price is perfectly correlated with any set of other random prices. To see that this assumption is not particularly restrictive, suppose that $w_k(\theta)$ is perfectly correlated with $\{w_1(\theta),\ldots,w_{K-1}(\theta)\}$. In this case there exist constants $a_1,\ldots,a_{K-1}$, such that

$$w_K(\theta) = \sum_{j=1}^{K-1} a_j w_j(\theta), \qquad \theta = 1,\ldots,S,$$

and the price $w_K(\theta)$ may be replaced by this sum in (2).

Let us next consider the value of firm j which offers these returns. Using any arbitrary individual's set of contingent claim prices, this value is given by

$$\begin{aligned} V_j &= \sum_{\theta=1}^{S} \rho_\theta^i \, r_{j\theta}(x^j) \\ &= \sum_{\theta=1}^{S} \rho_\theta^i \sum_{k=1}^{K} w_k(\theta) f_k^j(x^j) \\ &= \sum_{k=1}^{K} f_k^j(x^j) \sum_{\theta=1}^{S} \rho_\theta^i w_k(\theta). \end{aligned} \qquad (3)$$

The usual difficulty that arises here is that, if there are fewer firms than possible states of the world which may occur, then production decisions of firms are not invariant to the set of contingent claim prices used. This problem need not arise, however, if individuals agree upon each certainty-equivalent price given by

$$\tilde{w}_k = \tilde{w}_k^i = \sum_{\theta=1}^{S} \rho_\theta^i w_k(\theta) = \sum_{\theta=1}^{S} \rho_\theta^j w_k(\theta) = \tilde{w}_k^j \quad k = 1,\ldots,K, \qquad (4)$$

$$i,j = 1,\ldots,I,$$

where $\tilde{w}_k^i$ is the value individual i would pay for an asset with the same state distribution of returns $w_k(\theta)$ as the random price k. In the event

of agreement upon these imputed values, we would argue that the firm should use these prices, which are inherent in the stock market, to make its value-maximizing decisions. Thus firm j would perceive its value as

$$V_j = \sum_{k=1}^{K} \tilde{w}_k f_k^j(x^j),$$

and acting as a price taker, would determine its decisions from the first-order conditions given by

$$\frac{\partial V_j}{\partial x^j} = \sum_{k=1}^{K} \tilde{w}_k \frac{\partial f_k^j(x^j)}{\partial x^j} = 0.$$

For an arbitrary set of preferences in the economy, all individuals will impute the same certainty-equivalent prices if and only if there are as many firms that offer independent patterns of returns across states of the world as there are random prices. From (1)

$$V_j(x) = \sum_{\theta=1}^{S} \rho_\theta^i r_{j\theta}(x^j) = \sum_{k=1}^{K} f_k^j(x^j)\tilde{w}_k^i, \qquad j = 1,\ldots,N.$$

and if there are fewer firms than prices then there are K - N + 1 linearly independent certainty-equivalent prices vectors, $\tilde{w}^i$, which may be imputed.[4] Thus only when N = K will (4) be satisfied.

C. <u>Entry</u>

As we discussed in the introduction, we will restrict entry into the capital market by requiring that any new firm must offer a state distribution of returns as generated by an already existing return function. More formally, let R^j be the Sx 1 vector valued return function possessed by firm j and define the initial set of return functions $\mathcal{R}$ as

$$\mathcal{R} = \{R^j(\cdot),\ j = 1,\ldots,N\}.$$

Then any new firm, say the (N+1)st, will offer a stream of returns given by $R^{N+1}(x^{N+1})$, where $R^{N+1}(\cdot) \in \mathcal{R}$. In essence, this assumes that all existing technologies for the production of returns are freely accessible and precludes any entry into the economy with a new technology.

We may now demonstrate that the economy postulated here will not be in equilibrium unless there are at least as many firms offering independent patterns of returns as there are random prices. This will be done by showing that unless this condition is not satisfied then there is a riskless profit that may be earned by some entrepreneur.[5] Consider the entry opportunities for the (N + 1)st firm. If there are at least two individuals, i and j, who impute different values to a stream of returns the firm can feasibly generate, then the entrepreneur can extract a riskless profit from them. Let V^i_{N+1} be the values imputed by the two individuals and, without loss of generality, suppose that $V^i_{N+1} > V^j_{N+1}$. Then the entrepreneur may choose $\varepsilon > 0$ and $\delta > 0$ such that $V^i_{N+1} - \varepsilon > V^j_{N+1} + \delta$ and there is some number of shares s_{N+1} that individual i will buy from the entrepreneur for $V^i_{N+1} - \varepsilon$ and individual j will sell short to the entrepreneur for $V^j_{N+1} + \delta$ such that he will earn a profit of

$$[V^i_{N+1} - \varepsilon - (V^j_{N+1} + \delta)]s_{N+1} > 0.$$

What remains to be shown is that there are at least two individuals which impute different values if the entering firm offers a stream of returns which is independent of those offered by existing firms. Since we have assumed that an arbitrary set of preferences are present in the economy, we are free to assume that all K - N + 1 linearly independent vectors of certainty-equivalent imputations, $\tilde{w}^i$, are present. Next, let us determine how many linearly independent decision function vectors, f, are possible such that all individuals impute the same value to these functions. The equations

$$\sum_{k=1}^{K} \tilde{w}^i_k f_k = V \qquad i = 1,\ldots,K - N + 1$$

provides us with a set of K - N + 1 equations in K unknowns. Thus there are N linearly independent decision function vectors for which all individuals will impute the same value, and if the (N + 1)st firm offers an independent pattern of returns, at least two individuals must impute different valuations to that firm.

II. A MUTUAL FUND INTERPRETATION

Let us begin by examining a trivial case in which (4) is satisfied. Suppose there exists a sufficient number of "futures" markets in which traders may speculate or hedge against future price fluctuations. In this case, there is an asset corresponding to each random price and all individuals will agree upon $\tilde{w}_k$, which may now be interpreted as the equilibrium value of a futures contract returning $w_k(\theta)$. In this model, any firm whose return function is given by (2) may be interpreted as a mutual fund with a portfolio consisting of $f^j_k(x^j)$ contracts returning the future price $w_k(\theta)$, k = 1,...,K. The value of a firm is given as

$$V_j = \sum_{k=1}^{K} \tilde{w}_k f^j_k(x^j),$$

which is precisely the sum of the values of the contracts it holds. If the firm does not believe that its actions will affect the equilibrium price of any "futures" contract, value maximization will lead to the decision rule given by (5).

Even in the absence of actual "futures" markets, we may continue to use this interpretation. To see this, reexamine the budget constraint of each individual. Recall that, for individual i, consumption in state of the world θ is given by

$$c^i_\theta = \sum_{j=1}^{N} r_{j\theta} s^j_j,$$

where s_j^i is individual i's ownership fraction of firm j. Using (2),

$$c_\theta^i = \sum_{j=1}^{N} w(\theta) f^j(x^j) s_j^i$$
$$= \sum_{j=1}^{N} \sum_{k=1}^{K} w_k(\theta) f_k^j(x^j) s_j^i$$
$$= \sum_{k=1}^{K} w_k(\theta) \sum_{j=1}^{N} f_k^j(x^j) s_j^i. \qquad (6)$$

Since the random prices are the only source of uncertainty in the model, let us refer to each random price as the return on a "fundamental" security. In this light, each firm j may be though of as a mutual fund which holds $f_k^j(x^j)$ of the k^{th} "fundamental" security and, from (6), individual i holds the quantity $\sum_{j=1}^{N} f_k^j(x^j) s_j^i$ of the k^{th} "fundamental" security. It is well known that when an individual may purchase any combination of the "fundamental" securities, his opportunity set is not smaller than the set when he is constrained to hold a portfolio consisting of N < K mutual funds.[6] When there are as many mutual funds as securities, then the individual is indifferent between being able to purchase the securities or only the mutual funds.

The value of each "fundamental" security is given by substituting (2) into the first-order conditions to the investor's utility maximization problem as

$$V_j U_0^i = \sum_{\theta=1}^{S} U_\theta^i \sum_{k=1}^{K} w_k(\theta) f_k^j(x^j), \qquad j = 1,\ldots,N$$

or

$$V_j = \sum_{k=1}^{K} f_k^j(x^j) \sum_{\theta=1}^{S} \rho_\theta^i w_k(\theta)$$
$$= \sum_{k=1}^{K} f_k^j(x^j)\, \tilde{w}_k^i \qquad j = 1,\ldots,N \quad (7)$$

Only when there are as many independent mutual funds (firms) as "fundamental" securities, N = K, will all individuals, independent of their preferences, necessarily agree upon the value of these securities. This results since (7) provides us with a set of K equations in K unknowns, $\tilde{w}_k^i$. Since each individual is confronted with the same set of equations, they must necessarily agree upon each certainty-equivalent price.

It is instructive to reexamine Diamond's model in this framework. He assumes that there is a riskless asset which returns r and N risky assets, each of which returns

$$\pi_j(\theta) = f_j(\theta) g_j(k_j) - rk_j \qquad j = 1,\ldots,N.$$

where $f_j(\theta)$ is the multiplicative technological uncertainty which confronts firm j, $g_j(k_j)$ is the firm j's state-independent production function, and k_j is the amount of input used by firm j. There are N + 1 assets and N + 1 certainty-equivalent values,

$$\sum_{\theta=1}^{S} \rho_\theta^i f_j(\theta), \qquad j = 1,\ldots,N$$

and

$$\sum_{\theta=1}^{S} \rho_\theta^i,$$

to be imputed. Thus, the conditions outlined above are satisfied. Proceeding with Diamond's model, he normalizes the price of the riskless asset to be one which requires

$$\sum_{\theta=1}^{S} \rho_\theta^i = \frac{1}{r},$$

and the value of the firm is given by

$$V_j = g_j(k_j) \sum_{\theta=1}^{S} \rho_\theta^i f_j(\theta) - rk_j \sum_{\theta=1}^{S} \rho_\theta^i$$

$$= g_j(k_j)\tilde{f}_j - k_j.$$

Value-maximization requires that

$$\tilde{f}_j \frac{\partial g_j(k_j)}{\partial k_j} = 1,$$

which is precisely his result.[7]

In a competitive environment with entry as defined above, we would argue that there will always be agreement with regard to certainty-equivalent prices. To the contrary, suppose there is some disagreement, then some investor will perceive that a firm, say firm j, is making suboptimal decisions. This investor may freely replicate that firm by choosing $R^j(\cdot)$ from $\mathcal{R}$ and operate it at the level he desires. This process will continue until agreement is reached and, from above, we know that there will never need to be more firms than random prices.

Let us conclude by considering the implications of this free entry argument for a competitive industry. We will define an industry as the set of all firms which possess the same return function, say $R^j(\cdot)\varepsilon\mathcal{R}$. Until there are a sufficient number of firms to ensure agreement about the certainty-equivalent prices, each firm may operate at a different level. Once there is a sufficient number of firms, however, all firms will face the same set of prices and will operate at the same level. Hence, since all firms in an industry are then identical, there is essentially only one firm available in which individuals may invest and the entire argument must be repeated if firms wish to make subsequent decisions. This result is hardly surprising as it states that even in the presence of uncertainty, the number of firms in a competitive industry is indeterminate.

III. VALUE-MAXIMIZING DECISION RULES

As we have previously indicated, if firms take the certainty-equivalent prices as given, then value maximization requires that

$$\frac{\partial V_j}{\partial x^j} = \sum_{k=1}^{K} \tilde{w}_k \frac{\partial f_k^j(x^j)}{\partial x^j} = 0.$$

Let us consider the implications of this rule in the context of two special examples. Following Leland (1974), let us assume the return function for a competitive firm is specified as

$$p_j(\theta)q_j - C_j(q_j)$$

where the price at which the firm may sell its output, $p_j(\theta)$, is random, q_j is the output decision of firm j, and $C_j(q_j)$ is a known, nonstochastic, convex cost function. Assuming the existence of a riskless firm which earns a rate of return r ensures that all individuals will agree upon

$$\sum_{\theta=1}^{S} \rho_\theta^i p_j(\theta) = \tilde{p}_j$$

and

$$\sum_{\theta=1}^{S} \rho_\theta^i = \frac{1}{r}.$$

Thus the value-maximizing firm will make its output decision such that

$$\tilde{p}_j = \frac{1}{r} C_j'(q_j). \tag{8}$$

It can be immediately seen that the standard result for profit maximization under certainty that "price equals marginal cost" becomes, under uncertainty, that "the certainty-equivalent price equals the certainty-equivalent marginal cost." If we solved explicity for $\tilde{p}_j$ from (1), we would find that

$$\Sigma\rho_\theta^i \, p_j(\theta) q_j - C_j(q_j) \, \Sigma p_\theta^i = V_j$$

or that

$$\tilde{p}_j = \frac{C_j(q_j) + rV_j}{q_j},$$

and value maximization requires that the firm chooses its output such that average cost, computed as average variable cost plus the average cost of capital, equals marginal cost.

Alternatively, suppose the return function of a firm is specified as

$$p(\theta) f^j(x^j) - \sum_{k=1}^{K} w_k(\theta) x_k^j$$

where $f^j(x^j)$ is a concave production function of the firm, x_k^j is the amount of factor k used by the firm, $p(\theta)$ is the random output price and $w_k(\theta)$, $k = 1,\ldots,K$, are random input prices. Once there are a

sufficient number of firms in the industry to guarantee agreement about the certainty-equivalent prices, the value-maximizing decision rule becomes

$$\tilde{p} \frac{\partial f^j(x^j)}{\partial x_k^j} = \tilde{w},$$

or that the certainty-equivalent factor price equals the certainty-equivalent value of the marginal product of that factor. As a special case of this functional form, let us reconsider Diamond's model which was outlined above. If we interpret his multiplicative technological uncertainty factor, $f_j(\theta)$, as a random output price we find

$$\tilde{f}_j = \frac{V_j + k_j}{g_k(k_j)},$$

or that the certainty-equivalent is just the average market value of output. This results since $V_j + k_j$ is the sum of the market value of profits plus the market value of factor payments. Thus his result that the firm should choose its level of capital so that

$$1 = \tilde{f}_j \frac{\partial g_j(k_j)}{\partial k_j} = \frac{V_j + k_j}{g_j(k_j)} \frac{\partial g_j(k_j)}{\partial k_j}$$

in essence equates the firm's value of its marginal product with the price of a unit of capital.

IV. CONSTRAINED PARETO EFFICIENCY

Prior to examining the efficiency properties of the equilibrium obtained under value maximization, it is essential to point out that the best we can hope for here is constrained Pareto efficiency. This results since we are imposing the institutional constraint of a stock market by allowing only a redistribution of claims to each firm's returns and the number and opportunities of firms are given. As Diamond has previously pointed out this is tantamount to requiring that each individual's consumption pattern across states of the world be a linear combination of firms' return patterns.

With this stock market constraint in mind the constrained Pareto-efficiency problem may be stated as: for arbitrary $\lambda^i \geqq 0$, solve the problem

$$\text{maximize} \sum_{i=1}^{I} \lambda^i U^i(c_0^i, \ldots, c_S^i)$$

$$\text{subject to} \quad \sum_{i=1}^{I} c_0^i + \sum_{i=1}^{I}\sum_{j=1}^{N} V_j(x^j)s_j^i \leqq \sum_{i=1}^{I} \bar{c}^i + \sum_{i=1}^{I}\sum_{j=1}^{N} V_j(x^j)\bar{s}_j^i \qquad (\mu)$$

$$c_\theta^i \leqq \sum_{j=1}^{N}\sum_{k=1}^{K} w_k(\theta) f_k^j(x^j) s_j^i, \qquad \theta = 1,\ldots,S, \quad i = 1,\ldots,I, \qquad (\rho_\theta^i)$$

$$\sum_{i=1}^{I} s_j^i \leqq 1 \qquad j = 1,\ldots,N, \qquad (\gamma_j)$$

where the dual variables associated with the constraint are given in parentheses. The conditions for consumption efficiency are given by

$$\lambda^i U_0^i - \mu = 0, \qquad i = 1,\ldots,I, \qquad (9)$$

$$\lambda^i U_\theta^i - \rho^i = 0, \qquad i = 1,\ldots,I, \quad \theta = 1,\ldots,S, \qquad (10)$$

and

$$-\mu V_j + \sum_{\theta=1}^{S} \rho_\theta^i w_k(\theta) f_k^j(x^j) - \gamma_j = 0 \qquad i = 1,\ldots,I \quad j = 1,\ldots,N. \qquad (11)$$

and, for production efficiency, by

$$\mu \sum_{k=1}^{N} \frac{\partial V_k}{\partial x^j} \sum_{i=1}^{I} (\bar{s}_k^i - s_k^i) + \sum_{\theta=1}^{S}\sum_{i=1}^{I} \rho_\theta^i \sum_{k=1}^{K} w_k(\theta) \frac{\partial f_k^j(x^j)}{\partial x^j} s_j^i = 0 \qquad (12)$$

$$j = 1,\ldots,N.$$

To verify the conditions for consumption efficiency, (11) requires that

$$\sum_{\theta=1}^{S} \rho_\theta^i w_k(\theta) f_k^j(x^j) = \sum_{\theta=1}^{S} \rho_\theta^h \sum_{k=1}^{K} w_k(\theta) f_k^j(x^j), \quad j = 1,\ldots,N, \quad h,i = 1,\ldots,I.$$

By (9) and (10), this requires

$$\frac{\sum_{\theta=1}^{S} U_\theta^i w_k(\theta) f_k^j(x^j)}{U_0^i} = \frac{\sum_{\theta=1}^{S} U_\theta^h \sum_{k=1}^{K} w_k(\theta) f_k^j(x^j)}{U_0^h} \qquad j = 1,\ldots,N, \quad h,i = 1,\ldots,I. \qquad (13)$$

It is easy to see that this is satisfied for our model in equilibrium by examining the first-order condition, (1). Paralleling Diamond, we may interpret the return pattern of a firm over states of the world as a joint product and (13) requires that the marginal rate of substitution for these joint products must be equated across consumers. If there are as many firms as prices (K = N), the state-independent decision functions may be eliminated from (13), and the consumption efficiency result requires that

$$\tilde{w}_k^i = \frac{\sum_{\theta=1}^{S} U_\theta^i w_k(\theta)}{U_0^i} = \frac{\sum_{\theta=1}^{S} U_\theta^h w_k(\theta)}{U_0^h} = \tilde{w}_k^h, \qquad j = 1,\ldots,N, \quad h,i, = 1,\ldots,I, \tag{14}$$

or that all investors must impute the same set of certainty-equivalent prices.

Further, since $\sum_{i=1}^{I} s_j^i = \sum_{i=1}^{I} \bar{s}_j^i = 1$, the condition for production efficiency, (12) reduces to

$$0 = \sum_{k=1}^{K} \frac{\partial f_k^j(x^j)}{\partial x^j} \sum_{i=1}^{I} \sum_{\theta=1}^{S} \rho_\theta^i w_k(\theta) s_j^i$$

$$= \mu \sum_{k=1}^{K} \frac{\partial f_k^j(x^j)}{\partial x^j} \sum_{i=1}^{I} \frac{\sum_{\theta=1}^{S} U_\theta^i w_k(\theta)}{U_0^i} s_j^i, \text{ by (9) and (10),} \tag{15}$$

$$= \mu \sum_{k=1}^{K} \tilde{w}_k \frac{\partial f_k^j(x^j)}{\partial x^j}, \text{ by (14), if } K = N, \text{ and } \sum_{i=1}^{I} s_j^i = 1. \tag{16}$$

This requirement coincides with our value-maximizing decision rules, (7).[8]

If there are fewer firms than random prices, the production efficiency condition (15) may still be obtained from value maximization

if the firm uses $\sum_i \rho_\theta^i s_j^i$ as the appropriate shadow price for its returns in state θ. Although this result has been previously obtained by Drèze (1974), there is one major difficulty that arises since the firm will generally be unable to find these prices by relying on information implicit in the stock market. Instead, it requires that all investors must reveal their own contingent claim prices to the firm, but in the absence of some incentive compatible mechanism, they generally will not reveal truthfully.

V. EXPECTED UTILITY MAXIMIZATION

As we discussed in the introduction, many studies have assumed that firms maximize the expected utility of their future prices. Due to this, we wish to establish a set of conditions which will result in Pareto efficient production decisions under this criterion. We will assume that the profit of the firm in each possible state of the world is the return it offers its shareholder in each state which, by (2), is $w(\theta)f^j(x^j)$. Suppose the firm wishes to maximize $EU^j(w(\theta)f^j(x^j))$, where the utility function and probability assessments are those of (say) the manager. Without any further restrictions, the firm will choose levels of its decision variables so as to satisfy

$$EU_\theta^j\, w(\theta) \frac{\partial f^j(x^j)}{\partial x^j} = 0, \tag{17}$$

where U_θ^j is the marginal utility of profits in state θ. By itself, it is unlikely that (17) will coincide with the condition for production efficiency since there is no reason to believe that the firm imputes the "correct" set of certainty-equivalent prices.

We can overcome this difficulty by allowing the firm to hold a portfolio of securities which may include some of the firm's own shares which it retains as treasury stock.[9] Letting z_i^j be the proportion of the shares of firm i purchased by firm j, the profit, $\Pi_j(\theta,x^j)$, that accrues to its shareholders is given by

$$(1 - z_j^j)\,\Pi_j(\theta,x^j) = [w(\theta)f^j(x^j) + \sum_{k\neq j}(r_k(\theta,x^k) - iV_k)\,z_k^j - z_j^j iV_j]$$

where i is the riskless rate of interest. The firm may be thought of as borrowing $z_k^j V_k$ at the riskless rate of interest in order to purchase the shares of firm k. It then must repay the amount $iz_k^j V_k$ from its profits. If the firm now maximizes its expected utility of profits with respect to (x^j,z^j), the necessary optimality conditions are

$$EU_\theta^j(r_\ell(\theta,x^\ell) - iV_\ell) = EU_\theta^j(w(\theta)f^\ell(x^\ell) - iV_\ell) = 0 \qquad \ell \neq j \tag{18}$$

$$EU_\theta^j[(w(\theta)f^j(x^j) - z_j^j iV_j) - (1-z_j^j)(iV_j)] =$$

$$EU_\theta^j[w(\theta)f^j(x^j) - iV_j] = 0 \tag{19}$$

$$EU_\theta^j(w(\theta)\frac{\partial f^j(x^j)}{\partial x^j} - \sum_\ell z_\ell^j i\frac{\partial V_\ell}{\partial x^j})/(1-z_j^j) = 0 \tag{20}$$

If there are as many firms as random prices, (18) and (19) ensure that the firm will impute the same certainty-equivalent prices as each investor, i.e.

$$\sum_{k=1}^{K}(\sum_{\theta=1}^{S}\rho_\theta^j\, w_k(\theta))f_k^\ell(x^\ell) = \sum_{k=1}^{K}\tilde{w}_k f_k^\ell(x^\ell) = V_\ell,\ \ell = 1,\ldots,N. \tag{21}$$

Here, the firm's contingent claim prices are defined as

$$\rho_\theta^j = \frac{U_\theta^j \eta_\theta^j}{iEU_\theta^j},$$

where η_θ^j is the firm's subjective probability that state of the world θ will occur.

If we further assume that the firm uses its certainty-equivalent prices to forecast changes in the values of all firms, then differentiating (21) and using $\tilde{w}$ yields

$$\frac{\partial V_\ell}{\partial x^j} = \sum_{k=1}^{K}\sum_{\theta=1}^{S}\rho_\theta^j\, w_k(\theta)\frac{\partial f_k^\ell(x^\ell)}{\partial x^j} = \begin{cases} 0 & \text{for } j \neq \ell \\ \sum_{k=1}^{K}\tilde{w}_k\dfrac{\partial f_k^j(x^j)}{\partial x^j} & \text{otherwise.}\end{cases} \tag{22}$$

Substituting this condition in (20) will cause the firm to chose the level of its decision variables such that

$$iEU_\theta^j\frac{\partial V_j}{\partial x^j} = 0.$$

As long as there are private shareholders ($z_j^j < 1$), the firm will choose to maximize the value of the firm and, by (22), it will make Pareto-efficient decisions.

We should point out that the assumption that the firm uses its imputed certainty-equivalent prices to forecast value changes is perhaps very strong. Whereas value-maximizing firms will find these prices inherent in the stock market and exhibit price-taking behavior with respect to them, there is no reason to believe that utility-maximizing firms need to perceive that these prices, as they have imputed them, will remain unchanged as they vary their production decisions. Furthermore, if we wish this criteria to hold for arbitrary sets of random prices, an even stronger condition would be needed. Namely, the firm's set of imputed contingent claim prices should be invariant to changes in its decision variables. Thus its utility function must satisfy

$$0 = \frac{\partial \rho_\theta^j}{\partial x^j} = \frac{\iota U_{\theta\theta}^j \eta_\theta \frac{\partial \Pi_j(\theta, x^j)}{\partial x^j} EU_\theta - U_\theta^j \eta_\theta \iota EU_{\theta\theta} \frac{\partial \Pi_j(\theta, x^j)}{\partial x^j}}{[\iota EU_\theta]^2}, \quad \theta = 1,\ldots,S \qquad (23)$$

where $U_{\theta\theta}^j = \frac{\partial^2 U}{\partial \Pi_j^2}$. Rewriting (22) gives

$$-\frac{U_{\theta\theta}^j}{U_\theta^j} \frac{\partial \Pi_j(\theta, x^j)}{\partial x^j} = -\frac{EU_{\theta\theta}}{EU_\theta} \frac{\partial \Pi_j(\theta, x^j)}{\partial x^j} \qquad \theta = 1,\ldots,S.$$

Since the right hand side of this equation is invariant with θ, this reduces to the requirement that the product of the firms marginal profit in a state with its measure of absolute risk aversion in that state must

be constant across states of the world. In other words, for a given profit function, a firm's utility function will satisfy (23) only if it satisfies

$$-\frac{U_{\theta\theta}^j}{U_\theta^j} \frac{\partial \Pi_j(\theta, x^j)}{\partial x^j} = -\frac{U_{\phi\phi}^j}{U_\phi^j} \frac{\partial \Pi_j(\phi, x^j)}{\partial x^j}, \quad \theta, \phi = 1,\ldots,S.$$

VI. CONCLUSIONS

As I have discussed elsewhere, the decision rules that result from value maximization are identical to those derived in the recent literature on stockholder unanimity.[10] The contributors to this literature reject the notion of value maximization, however. In a sense, they are formally correct since the decisions which are made do not maximize the *actual* market value of the firm. Indeed, in an economy with incomplete markets, what we have termed as certainty-equivalent prices may vary with a change in a firm's decision variable. It is precisely this point which allows Stiglitz (1972) to conclude the value-maximizing decisions are not efficient. On the other hand, we would argue that a firm must rely on the information implicit in the stock market and act as a price taker with respect to the certainty-equivalent prices. In this case, the firm may be thought of as acting to maximize its *perceived* value.

This same difficulty arises when we consider expected utility maximizing firms. In this case, the certainty-equivalent prices are imputed by the firm and requiring then to act as a price taker with respect to these imputed prices appears to be an even stronger assumption.

FOOTNOTES

* I wish to express my thanks to David Cass, Walter Dolde, and Milton Harris who served as my thesis committee. Any remaining errors are my responsibility.

1. See Baron, Leland (1972), and Sandmo, for example.
2. This was previously pointed out by Diamond (1967).
3. Unless, of course, we place restrictions on the set of preferences in the economy. We do not care to do this in this study.
4. This results since any non-homogeneous system of N linear equations in K unknowns possesses N - K + 1 linearly independent solutions.
5. This argument is in the same spirit as that given by Satterthwaite (1977).
6. See Cass and Stiglitz (1970).
7. See Diamond's equation (25). Since he assumes the investor maximizes his expected utility of future consumption, the imputed contingent claim prices are given by

$$\rho_\theta^i = \frac{U_i'(c_i(\theta))\pi_i(\theta)}{rEU_i'(c_i\theta)},$$

where $\pi_i(\theta)$ is investor i's subjective probability that state of the world θ will occur.

8. It may be shown that in the unconstrained Pareto efficiency problem that the production decisions continue to be efficient; however, the consumption decisions are not. In general, the consumption decisions will be efficient (in an unconstrained sense) only when there exists the equivalent of a complete set of contingent claim markets. For a fuller discussion of this, see the author.

9. This approach is a generalization of one used in Baron and Forsythe (1976).

10. See Ekern (1974), Ekern and Wilson (1974), and Leland (1973, 1974).

REFERENCES

Arrow, K. J. "The Role of Securities in the Optimal Allocation of Risk Bearing." Review of Economic Studies 31 (April 1964); 91-96.

Baron, D. P. "Price Uncertainty, Utility, and Industry Equilibrium in Pure Competition." International Economic Review 11 (October 1970): 463-80.

Baron, D. P. and Forsythe, R. "An Addendum on the Role of Forward Markets and the Planning of Exports," Working Paper, Northwestern University (1976).

Cass, D. and Stiglitz, J. E. "The Structure of Investor Preferences and Asset Returns, and Separability in Portfolio Allocations: A Contribution to the Pure Theory of Mutual Funds." Journal of Economic Theory 2 (June 1970): 122-60.

Debreu, G. Theory of Value. New York: John Wiley & Sons, 1959.

Diamond, P. A. "The Role of a Stock Market in a General Equilibrium Model with Technological Uncertainty." American Economic Review 57 (September 1967): 759-76.

Drèze, J. H., "Investment Under Private Ownership: Optimality, Equilibrium and Stability," in _Allocation Under Uncertainty: Equilibrium and Optimality_, J. H. Drèze, Ed. New York: John Wiley & Sons, (1974), 53-70.

Ekern, S. "Some Aspects of Firms' Decision Making in an Economy with Incomplete Capital Markets." _Swedish Journal of Economics_ 76 (March 1974): 117-30.

Ekern, S. and Wilson, R. "On the Theory of the Firm in an Economy with Incomplete Markets." _Bell Journal of Economics_ 5 (Spring 1974): 171-80.

Forsythe, R. "Essays in the Theory of Incomplete Markets." PhD dissertation, Carnegie-Mellon University, 1975.

Leland, H. E. "Theory of the Firm Facing Uncertain Demand." _American Economic Review_ 62 (June 1972): 278-91.

__________. "Production Theory and the Stock Market." _Bell Journal of Economics_ 5 (Spring 1974): 125-44.

__________. "Capital Asset Markets, Production and Optimality: A Synthesis." Technical Report No. 115, Institute for Mathematical Studies in the Social Sciences, Stanford University, December 1973.

Radner, R. "Problems in the Theory of Markets Under Uncertainty." _American Economic Review_ 60 (May 1970): 454-60.

Sandmo, A. "On the Theory of the Competitive Firm Under Price Uncertainty." _American Economic Review_ 61 (March 1971): 65-73.

Satterthwaite, M.A. "On Stockholder Unanimity Towards Changes in Production Plans." Discussion Paper No. 293, Center for Mathematical Studies in Economics and Management Science, Northwestern University. August 1977.

Stiglitz, J.E. "On the Optimality of the Stock Market Allocation of Investment." _Quarterly Journal of Economics_ 86 (February 1972): 25-60.