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MARKETS AND ENVIRONMENTAL MANAGEMENT  
WITH A STORABLE POLLUTANT

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## ABSTRACT

Lee (4) investigates possibilities where pollutants may be stored for a period of time and later released into the environment when adverse effects are minimal. The treatment and storage of pollutants before their release into the environment is a crucial part of many abatement programs. Surprisingly, emission charges will not induce optimal abatement when storage is possible. This occurs because the firms' response to the dynamic tax is indeterminant. We suggest alternative controls, whereby rights to emit pollutants are sold competitively and demonstrate that markets provide incentives for the optimal generation-storage-emission of pollution by firms. In deriving this result an important difference between markets and taxes is revealed. With markets there is still indeterminacy at the firm level, but the aggregate response of all firms is dictated by market forces that insure pollution is reduced by some desired amount.

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## INTRODUCTION

In considering the decentralized control of pollution and other external diseconomies, one usually regards the use of Pigouvian effluent taxes or the implementation of a pollution rights market as being roughly equivalent. While the systems differ in administrative details, they both attempt to internalize the external effects of a firm's activities by charging for the marginal costs of pollution either through taxes or pollution rights prices.

In this paper we observe an important potential difference between taxes and markets for pollution control. We examine an instance where taxes won't work to reduce pollution because the firms' reactions to the tax is indeterminant. In the same instance, however, if pollution rights are allocated through a market, (as described by Dales [1968]) optimal control is achieved. There is still indeterminacy at the firm level, but the aggregate response of all firms is dictated by market forces that insure pollution is reduced by some desired amount.

The instance of tax failure to control pollution we examine is borrowed from a recent article by Lee [4] appearing in this journal.

Lee investigates possibilities where pollutants may be stored for a period of time and later released into the environment when adverse effects are minimal. The analysis is significant because of the practical importance of storage possibilities and because pollution models with storage have not been analyzed. Most important though is Lee's conclusion that when storage is possible, optimal emission control is not achieved using a time sequence of fixed per-unit effluent charges set equal to the marginal social cost of pollution. Two problems occur with fixed charges: First, firms undervalue storage capacity and may build facilities that are too small because they do not capture all of the social benefits from storing pollutants.<sup>1</sup> Second, and more important, according to Lee's equation (25), which describes the optimal sequence of taxes, time intervals  $[\bar{t}, \bar{t}]$  exist such that

$$\bar{C}(t_1)e^{-rt_1} - \alpha \int_{t_1}^{t_2} e^{-rt} dt = \bar{C}(t_2)e^{-rt_2} \quad \text{for } \bar{t} \leq t_1 < t_2 \leq \bar{t},$$

where  $\bar{C}(t)$  is the fixed per-unit effluent charge set equal to the social marginal cost of emissions, and  $\alpha$  is the unit pollution storage cost. According to this equation, the present value cost of emitting a pollutant at time  $t_1$  is the same as storing the pollutant for later release at time  $t_2$ . The optimal emissions sequence can be induced using charges  $\bar{C}(t)$ , but an indeterminacy exists in that the firm is indifferent to various emission paths, and there is no assurance the socially optimal one will be chosen. These indeterminacies arise because of storage which allows for the "smoothing" of emission costs over time, such that the present value marginal cost of emissions in each period is the same.

In general, taxes won't work to control emissions whenever it is desirable to release some stock of pollutants gradually over time. The discounted emissions costs (including effluent charges), to the firm must be the same over time, otherwise all pollutants will be discharged in the period with the lowest cost. Consequently, indeterminacies exist at the firm level and there is no assurance that a time sequence of effluent charges will induce the optimal sequence of aggregate emissions.

In fact, the same indeterminacy arises (as demonstrated below) in using effluent charges to achieve any set of emissions. Baumol and Oates [1,2] show, in a static world, that whatever the standard of environmental quality the public authority selects, effluent charges can realize that standard at the least cost to society. But what is the role of the "standards" approach in a dynamic world where pollutants can be stored? Because of indeterminacies Pigouvian taxes are ineffective instruments. Can other monetary incentives induce desirable effluent controls?

As an alternative to effluent charges, we consider a system where rights entitling firms to emit in each period are sold in a competitive market. Total emissions are restricted to an acceptable or desirable level each period by limiting the supply of pollution rights. Unlike effluent charges, we find this system can produce the optimal generation-storage-emission program and prescribed standards are realized at least cost. Normally, except for administrative details, markets and emission charges act similarly to control pollution. With

markets, however, indeterminacy in the total response by all firms is eliminated by limiting the number of pollution rights issued.

#### A POLLUTION RIGHTS MARKET

To illustrate this result, consider Lee's model modified to allow for several polluting firms. Each firm,  $j$ , ( $j = 1, \dots, J$ ), generates and emits an environmental pollutant at the rates  $g_j(t)$  and  $e_j(t)$  respectively. The social benefit realized from the generation of pollution (perhaps in the form of increased production), is assumed to be entirely captured by the firm, and is given by  $B_j(g_j(t), t)$ . This function changes over time as abatement technology, product demand and production costs vary. Let  $\bar{e}(t) = \sum_j e_j(t)$  be aggregate emissions. Then the social cost of emissions is  $C(\bar{e}(t), t)$  which changes over time with variations in population density, environmental awareness, seasonal fluctuations in the environment, etc.

Pollutants may be stored during periods of high social emission costs to be released later when costs have decreased. At any time,  $t$ , the stock of pollutants held by firm  $j$  is  $G_j(t) - E_j(t)$  where  $G_j(t) = \int_0^t g_j(\tau) d\tau$  and  $E_j(t) = \int_0^t e_j(\tau) d\tau$ . The variable cost of storage given by  $\alpha(G_j(t) - E_j(t))$  is proportional to the stock of pollutants, where  $\alpha > 0$ . There is a total storage capacity,  $K$ , for the economy provided at a cost  $H(K)$  such that

$$0 \leq \sum_j G_j(t) - \sum_j E_j(t) \leq K \quad (1a)$$

In present value terms, the net social benefit from the generation, storage and emission of a sequence of pollutants for an economy with  $J$  firms is

$$V = \int_0^T \sum_j \{B_j(g_j(t), t) - \alpha(G_j(t) - E_j(t))\} e^{-rt} dt \quad (2)$$

$$- H(K) - \int_0^T C(\bar{e}(t), t) e^{-rt} dt$$

where  $r$  is the discount rate. The optimal sequence of generations and emissions is obtained by maximizing  $V$  with respect to  $K$  and the time sequences for  $g_j$  and  $e_j$  for all  $j$  subject to (1a) and the constraints.<sup>2</sup>

$$g_j(t), e_j(t), \forall j \quad (1b)$$

$$G_j(T) - E_j(T) = 0, \forall j \quad (1c)$$

Note that eq.(1a) indicates that firms store pollutants centrally. We might have made storage site specific, but this adds nothing substantive to the analysis. To eliminate the undervaluation problem and to insure adequate storage capacity, we will assume that the optimal capacity is publicly provided, perhaps with mandatory contributions from firms.

A description of optimal generation and emissions paths is unnecessary as the problem of maximizing  $V$  is similar to Lee's analysis. Also from Lee we know, effluent fees will not induce optimal abatement when storage is possible. Instead, consider a competitive market for emission and storage rights. Each period the pollution authority limits the supply of emission rights,  $r(t)$  to  $\bar{e}^{**}(t)$ , the optimal total emission level.<sup>3</sup> Thus

$$e_j(t) \leq r_j(t) \text{ for all } j, t.$$

and (3a)

$$\sum_j e_j(t) \leq \sum_j r_j(t) \leq \bar{e}^{**}(t)$$

where  $r_j(t)$  is the number of rights held by  $j$  at time  $t$ .

The optimal total storage capacity,  $K^{**}$ , is publicly provided. Firms can purchase certificates,  $S_j(t)$  to store pollutants for one period at a variable cost,  $\alpha$ . Thus,

$$G_j(t) - E_j(t) \leq S_j(t)$$

and (3b)

$$\sum_j G_j(t) - \sum_j E_j(t) \leq K^{**}$$

Firms receive initial endowments of rights,  $\{r_{j0}(t)\}$ , and  $\{S_{j0}(t)\}$  which can be traded in the market.<sup>4</sup> Firms choose  $\{g_j(t)\}$ ,  $\{r_j(t)\}$ ,  $\{e_j(t)\}$  and  $\{S_j(t)\}$ <sup>5</sup> to maximize discounted profits, which include net sales of pollution and storage rights,

$$\Pi_j = \int_0^T [B_j(g_j(t), t) - \alpha(G_j(t) - E_j(t)) - p(t)(r_j(t) - r_{j0}(t)) - q(t)(S_j(t) - S_{j0}(t))] e^{-rt} dt \quad (4)$$

subject to (1a - 1c) and (3a - 3b), where  $p(t)$  and  $q(t)$  are the market price of emission and storage rights respectively.<sup>6</sup>

This variation of the Dales (3) pollution rights market provides for storage, and incorporates the Baumol and Oates (1971, 1975)

"standards approach" to establish total emission rights each period.<sup>7</sup>

Here with complete information, standards are set optimally. With incomplete information a sequence of "acceptable" standards,<sup>8</sup> are set.

A market equilibrium for rights with imposed standards,  $\bar{e}^{**}(t)$ , exists if there are prices  $\{p^*(t)\}$ , and  $\{q^*(t)\}$ , generations  $\{g_j^*(t)\}$ , and emissions  $\{e_j^*(t)\}$  such that

$$\left. \begin{aligned} 0 &\leq \sum_j G_j^*(t) - \sum_j E_j^*(t) \leq K^{**} & (1a) \\ g_j^*(t), e_j^*(t), K^{**} &\geq 0 & (1b) \\ G^*(T) - E^*(T) &= 0, & (1c) \end{aligned} \right\} \text{(generations and emissions are feasible)}$$

$g_j^*(t)$ ,  $e_j^*(t)$ ,  $r_j^*(t)$ , and  $S_j^*(t)$  is the solution to:

$$\left. \begin{aligned} \max \{ \Pi_j \} \\ g_j(t), e_j(t), r_j(t), S_j(t) \end{aligned} \right\} \text{(firms maximize profits)} \quad (5)$$

and

$$\left. \begin{aligned} \bar{e}^{**}(t) - \sum_j e_j^*(t) &\geq 0, p^*(t) \geq 0 & (6a) \\ p^*(t) [\bar{e}^{**}(t) - \sum_j e_j^*(t)] &= 0 \\ (K^{**} - \sum_j S_j^*(t)) &\geq 0, q^*(t) \geq 0 & (6b) \\ (K^{**} - \sum_j S_j^*(t)) q^*(t) &= 0 \end{aligned} \right\} \text{(market clearing condition)}$$

In equilibrium firms maximize profits subject to prices and the emissions constraints, and pollution rights and storage rights markets clear. In (6) the total rights sold cannot exceed the standards. Whenever there is an excess supply of certificates, prices are zero.

A market equilibrium has the desirable property summarized in Proposition 1: If an equilibrium exists satisfying conditions (1a-1c), (5), and (6a-6b) then (a) the resulting generation-storage-emission sequence is optimal and (b) along the equilibrium price path,  $p^*(t) = MC^{**}(t) = dC(\bar{e}(t))/de_j(t)$ .

Proof: (Part a) We could show directly that the maximizing conditions in (1) are identical to the profit maximizing conditions for the firm in equilibrium. A more enlightening proof is to show that in equilibrium firms maximize aggregate profits while meeting the emissions constraint  $\sum_j e_j(t) \leq \bar{e}^{**}(t)$ , and that therefore the sequences  $\{e^*(t)\}$  and  $\{g^*(t)\}$  are optimal. The same proof will apply in Proposition 2 to show in equilibrium firms maximize profits while meeting any given emissions constraint.

(i) Rewriting (2) in terms of  $\Pi_j$  in (4) we find that  $V$  evaluated at prices  $p^*(t)$  is given by

$$\begin{aligned} V &= \sum_j \Pi_j - \int_0^T C(\sum_j e_j(t), t) e^{-rt} dt \\ &+ \int_0^T [p^*(t) (\sum_j e_j(t) - \sum_j r_{jo}(t))] e^{-rt} dt \quad (1') \\ &+ \int_0^T q^*(t) (\sum_j S_j(t) - \sum_j S_{jo}(t)) e^{-rt} dt \\ &- H[K] \end{aligned}$$

Suppose we know  $K^{**}$  and  $e_j^{**}(t)$  for all  $j$  and  $t$ . Then, optimal  $V$ , denoted by  $V^{**}$ , is given by

$$\begin{aligned} V^{**} &= \max_{g_j(t), e_j(t), K} \{ \sum_j \Pi_j \\ &+ \int_0^T q^*(t) (\sum_j S_j(t) - \sum_j S_{jo}(t)) e^{-rt} dt \quad (1'') \\ &+ \int_0^T p^*(t) (\sum_j e_j^{**}(t) - \sum_j r_{jo}(t)) e^{-rt} dt \\ &- H[K^{**}] \end{aligned}$$

where  $K$  and  $e_j$ , which are normally control variables, have been replaced with their known optimal values. The last two terms in (1'') are constant when evaluated at  $\sum_j e_j^{**}(t)$  and  $K^{**}$ .

Thus we can say the controls  $\{g_j^{**}, e_j^{**}, K^{**}\}$  maximize  $\phi = \sum_j \Pi_j + \int_0^T q^*(t) (\sum_j S_j(t) - \sum_j S_{jo}(t)) e^{-rt} dt$  subject to the constraints

$\sum_j e_j(t) = \sum_j e_j^{**}(t)$  and  $K = K^{**}$ . We now need to show that when firms

maximize profits in equilibrium, they also maximize  $\phi$  subject to the constraints.

(ii) From (5), firms maximize  $\Pi_j$ , evaluated at  $p^*(t)$  and  $q^*(t)$ . At given prices, firm profits are independent of each other, and aggregate profits are maximized by maximizing individual profits. Thus,  $\{g_j^*, e_j^*\}$  maximizes  $\sum_j \Pi_j$  subject to  $\sum_j e_j(t) \leq \bar{e}^{**}(t)$  and  $K = K^{**}$ . But,  $\{g_j^*, e_j^*\}$  maximizes  $\sum_j \Pi_j$  subject to  $\sum_j e_j(t) = \bar{e}^{**}(t)$  and  $K = K^{**}$  as well. From (6b)  $\sum_j S_j(t) = K^{**}$  whenever  $q^*(t) > 0$ . Thus  $\int_0^T q^*(t) [\sum_j S_j(t) - \sum_{j_0} S_{j_0}(t)] e^{-rt} dt$  is also maximized in equilibrium, and therefore  $\{g_j^*, e_j^*\}$  maximizes  $\sum_j \Pi_j + \int_0^T q^*(t) [\sum_j S_j(t) - \sum_{j_0} S_{j_0}(t)] e^{-rt} dt$  subject to  $\sum_j e_j(t) = \sum_j e_j^{**}(t)$  and  $K = K^{**}$ .

(iii) From (i) and eq. (1'') we have  $V^{**} = \max_{g_j, e_j, k} \phi + c$ ,

subject to the constraints, where  $c$  is a constant. But (ii) implies that the controls  $\{e_j^*, g_j^*, k^{**}\}$  maximize  $\phi$  subject to the constraints, and thus  $\{e_j^*, g_j^*, k^{**}\}$  is optimal.

(Part B) Proceed by comparing profit maximizing conditions in equilibrium with maximization conditions for (2) to verify that  $p^*(t) = MC^{**}(t)$ .

Part B of Proposition 1 implies that with both the market and the tax system firms pay a price equal to the marginal cost of emissions in order to pollute. From the maximization in (5)

$$e^{-rt_1} p^*(t_1) - \alpha \int_{t_1}^{t_2} e^{-rt} dt = e^{-rt_2} p^*(t_2); \quad (7)$$

$$\bar{t} \leq t_1 < t_2 \leq \bar{\bar{t}}$$

whenever the constraints in 1a and 1b are not active. Thus in equilibrium the discounted cost of emitting a pollutant over  $[\bar{t}, \bar{\bar{t}}]$  is the same regardless of the sequence of discharges, and firms are indifferent to various emission paths. However, there is a subtle but important difference between the tax and market system. With taxes, firms choose a level of discharges subject to a fixed charge. In the market, total discharges are set. Prices are determined that equate the demand for and supply of pollution certificates and interminacies are illuminated by restricting the supply of certificates each period.

Realistically, the optimal sequence of emissions cannot be computed. The costs of different emission levels at various times and the ability of firms to abate and store pollutants are generally unknown. Instead, "acceptable" emission levels varying with the severity of pollution damage over time could be set. The proof to part (a) of Proposition 1 is easily modified to establish.

Proposition 2: For given emission constraints  $\sum_j e_j(t) \leq \bar{e}(t)$  and storage constraints  $\sum_j G_j(t) - \sum_j E_j(t) \leq \tilde{K}$ , the generations and emissions corresponding to a market equilibrium are efficient; total profits are maximized subject to the emission and storage constraint.

On the other hand, indeterminacies still arise in using fixed charges to achieve given emission standards. Taxes inducing firms to meet standards exist.<sup>9</sup> However, firms will not necessarily choose the desired sequence, since others are equally preferred under the tax program.

#### CONCLUSION

Propositions 1 and 2 imply a pollution rights market and may succeed in some circumstances where effluent charges fail to induce optimal pollution control. However, market controls have certain limitations:

1. In our analysis, pollution affects only the consuming sector. If pollution externalities exist between firms, the market will not meet aggregate pollution standards at least cost. Firms facing a uniform price for pollution certificates will ignore the external effects of emissions on other firms.
2. Trade in pollution rights may not exist because of thinness in markets and high administration and enforcement costs.

3. Only the equilibrium properties of the market have been examined. We need to know if and how the market reaches an equilibrium, and what the disequilibrium properties of the market are.

The relative advantages of various controls need to be assessed before advocating a particular program. In theory, a multi-period pollution rights market can induce an optimal or acceptable storage-emissions plan. And to some extent, market mechanisms already exist in some state and federal pollution controls. Certain solid wastes, air- and water-borne pollutants are controlled by requiring firms to obtain discharge licenses. This system could operate as a market for pollution rights, having firms bid on a limited supply of licenses.

## FOOTNOTES

1. With a fixed fee system, firms benefit from storage by temporarily avoiding discharge taxes. The tax equals the benefit to society from reducing emissions by one more unit. However, marginal benefits decrease with greater reductions in emissions so that the firm does not capture some of the benefits from infra-marginal reductions. Because of this the firm may construct no storage capacity when positive capacity is optional.
2. We arbitrarily impose the terminal storage constraint  $G_j(T) - E_j(T) = 0$  for simplicity.
3. Only total optimal emissions,  $\Sigma e_j^{**}(t)$ , and not the emissions for each firm need to be announced.
4. The authority can change allocations of initial rights  $\{r_{j_0}(t)\}$  and  $\{S_{j_0}(t)\}$  to affect total profits without changing the firm's choice of  $\{g_j(t)\}, \{e_j(t)\}$ .
5. A bracket  $\{\cdot\}$  denotes a time path for a variable.
6. If  $r_j(t) - r_{j_0}(t) > (<) 0$ , the firm is a net purchaser (seller) of pollution rights, with the same convention holding for storage rights.
7. Note that both a market in emission rights and storage rights is needed. Lee shows that when the optimal storage capacity is provided, there will be instances when the storage capacity constraint is binding. Thus storage capacity is a scarce good which must be allocated optimally like emission rights. Generally, multiple controls are needed to achieve multiple objectives, and in this case two markets are required to allocate two types of goods. For an interesting discussion of the use of policy instruments to achieve multiple goals see Smith and Russell (5) and Smith (6).
8. See Baumol and Oates (1,2) for a discussion of setting "acceptable" standards.
9. These taxes equal the market equilibrium prices for certificates. In equilibrium, Eq. (7) holds over certain intervals. Thus indeterminacies arise as the discounted cost of emitting a pollutant is invariant with the sequences of discharges.

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## SYMBOLS

- $\alpha$  - "alpha"  
 $\tau$  - "tau"  
 $\pi$  - "pi"  
{ } - "brackets"