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DEMOCRATIC EXPLOITATION OF A NON-REPLENISHABLE RESOURCE

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I. INTRODUCTION

In a recent article, Neher (1976) suggests an interesting democratic process for allocating a scarce, renewable natural resource among different generations. The procedure is characterized by a) continuous voting, b) one person--one vote, c) unsophisticated voting, and d) simple majority rule, and determines the length of the optimal exploitation plan in a society of overlapping generations. Allocations resulting from this plan are revealed to be unjust in the "Rawlsian" sense as the selfishness of living voters is reflected in current decisions.

We adopt the same procedure to determine the allocation of a nonrenewable resource under democratic exploitation. With continuous democratic voting, plans are continuously revised and resources are forever being consumed at a rate which is a constant proportion of the total resource available. As before, democratic exploitation of the resource is unjust in the Rawlsian sense: succeeding generations are given smaller allocations of the resource.

A natural question concerns the conditions under which the democratic allocation coincides with the socially optimal allocation based on a utilitarian principle of justice. The implicit treatment of future generations under democratic exploitation becomes explicit upon observing the intertemporal weighting of generational utilities which induces a benevolent central planner to adopt the democratic allocation. As expected, declining weights discount the welfare of more distant generations with the discount rate reflecting the degree of inequity implicit in the democratic allocation.

II. DEMOCRATIC ALLOCATIONS

Following Neher we consider a society with overlapping generations. Individuals desire to allocate the resource so as to maximize utility over their expected lifetime. For simplicity, we assume utility is only derived from consuming the resource; the resource has no intrinsic value in itself. Assuming that people weight utility equally in all periods (see Fisher (1930, p. 84)) and that there is decreasing marginal utility of consumption, each individual plans lifetime consumption according to the rule:

$$c(t) = \frac{1}{T} Y(0) \text{ for } t \in [0,T] \tag{1}$$

where $c(t)$ is the rate of resource consumption at time t, $Y(t)$ is the resource remaining at time t, and T is the individual's life span.

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Individuals of various ages prefer different consumption plans based on their life expectancy. To resolve generational conflicts, society votes for an optimal depletion horizon, $\bar{T}$, according to eher's procedure, and resource use is planned at a constant rate over that interval. But since voting is continuous and consumption plans are constantly revised, the exhaustion date for the resource is repeatedly postponed. With continuous planning revision, society ultimately follows the plan described by the rule

$$c(t) = \frac{Y(t)}{\bar{T}}, \quad t\varepsilon[0,\infty] \tag{2}$$

so that planned current consumption is proportional to the existing resource supply and the resource is never depleted. The inequity in the allocation is clear; future generations have less to consume, the welfare of succeeding generations being implicitly discounted by those alive today.

III. CENTRALLY PLANNED ALLOCATIONS

One way to analyze the welfare properties of the democratic allocation is to ask if there exists a set of weights attached to generational utilities that would induce a benevolent central planner to select the democratic allocative rule of Eq. (2). Assume the planner chooses the path, c, and the horizon, τ, so as to maximize social welfare W, given by

$$W = \int_0^\tau \alpha(t)u(c(t))dt + \int_\tau^\infty \alpha(t)u(0)dt \tag{3}$$

subject to

$$Y(0) - \int_0^\tau c(t)dt \geq 0 \tag{4}$$

where $\alpha(t) > 0$ is the relative weight attached to the welfare of persons alive at time t and u(c(t)) is the social welfare of consumption at time t, with $u'(0) = \infty$, $u' > 0$, $u'' < 0$ and $\lim_{c\to 0} u(c) = u(0)$. The second term of the right hand side of Eq. (3) represents the utility of those persons having none of the resource to consume.[1]

The maximization of (3) subject to (4) is a straightforward variational problem for which necessary conditions require that along the optimal path c,

$$\alpha(t)u'(c(t)) - \lambda = 0 \tag{5}$$

where λ is the multiplier attached to the stock constraint in (4),

$$\dot{\lambda} = 0 \tag{6}$$

and that

$$\lim_{t\to\tau} \alpha(t)[u(c(t))-u(0)-u'(c(t))c(t)] = 0 \tag{7}$$

If τ is finite then (7) becomes

[1] Unless we arbitrarily set u(0) = 0, the "scrap value" function must logically be included as part of W. The assumptions that $\lim_{c\to 0} u(c) = u(0)$ and $\lim_{c\to 0} u'(c) = \infty$ implicitly represent the planners' eithical attitudes towards depletion; for examples of alternative attitudes see Koopmans (1974) and Vousden (1973). Barry (1975) explores the implications of such ethical attitudes in a different context.

$$u(c(\tau))-u(0) = u'(c(\tau))c(\tau) \tag{7'}$$

which in view of the assumption $u'(0) = \infty$ implies $c(\tau) = 0$. However, by (5), $c(\tau) = 0$ and τ finite imply $c(t) = 0$ for all t, contradicting the optimality of c. Hence τ is infinite; all generations consume some of the resource.

Differentiating (5) with respect to time, and assuming $\alpha(t)$ is continuously differentiable, we have that along the socially optimal path

$$\frac{\dot{c}}{c} = \frac{\dot{\alpha}}{\alpha}\frac{1}{\eta(c)} \tag{8}$$

where $\eta(c) = -\frac{u''c}{u'}$ is the elasticity of marginal utility. Differentiating (2) with respect to time yields, for the democratic allocation

$$\frac{\dot{c}}{c} = -\frac{1}{\bar{T}} \tag{9}$$

In general there is a set of weights $\{\alpha(t)\}$ such that the socially optimal plan corresponds to the democratic allocation. Eq. (8) and (9) imply that the weights $\alpha(t)$ must satisfy

$$\frac{\dot{\alpha}}{\alpha} = -\frac{\eta(c)}{\bar{T}} \tag{10}$$

The explicit weight applied to future generations is declining and $\dot{\alpha}/\alpha$ is the rate of discount applied against individuals living in succeeding periods. In general, the rate at which future generational welfare is discounted will vary over time. Differentiating (10) with

respect to time we obtain

$$\frac{d}{dt}\left(\frac{\dot{\alpha}}{\alpha}\right) = -\frac{\dot{c}\eta'(c)}{\bar{T}} \tag{11}$$

so that

$$\frac{d}{dt}\left|\frac{\dot{\alpha}}{\alpha}\right| \lesseqgtr 0 \text{ as } \eta' \gtreqless 0 \tag{12}$$

Thus the discount rate is decreasing or increasing over time as the elasticity of marginal utility decreases or increases with c.

Consider the special case where u(c) has constant elasticity of marginal utility[2], $\bar{\eta}$. Eq. (10) simplifies to

$$\frac{\dot{\alpha}}{\alpha} = -\rho = -\frac{\bar{\eta}}{\bar{T}} \tag{13}$$

so that the absolute value of the discount rate[3], ρ, varies inversely with τ. Thus a society composed primarily of the elderly (young) voting to exhaust the resource over a short (long) time interval in effect discounts the welfare of future societies more (less) heavily[4].

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[2] The class of utility functions with constant elasticity of marginal utility includes only the functions $u(c) = \frac{c^{1-\alpha}}{1-\alpha}$ where $\alpha > 0$, $\alpha \neq 1$ and $u(c) = \ln u(c)$.

[3] In this case $\alpha(t) = e^{-\rho t}$

[4] Analogous statements can be made concerning the relationship between ρ and η. In particular Eq. (13) implies that for larger $\bar{\eta}$ a larger ρ is required to induce the democratic allocation. All else being equal, when $\bar{\eta}$ is large the central planner will allocate consumption more equally among generations since the decrease in marginal utility is relatively large for generations consuming more of the resource.

Theories of distributive justice are troublesome in their application to the allocation of non renewable resources. Equal distribution across all generations is difficult to imagine because of the finiteness of the resource. Democratic exploitation, without planning revision, results in a truncated equal distribution over T generations. With planning revision, some of the resource is consumed by all generations. Nevertheless, the distribution is "unjust," as the welfare of future generations is implicitly discounted.

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