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ON THE FORMAL THEORY OF INSPECTION
AND EVALUATION IN PRODUCT MARKETS

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ABSTRACT

This paper builds a formal theory of consumer behavior under imperfect information when goods are described by multiple characteristics which vary in their degree of "observability." An optimal strategy for the consumer is shown to exist. In general, this strategy is shown to involve both inspection (sampling to observe general characteristics of goods) and evaluation (consumption of goods to observe specific characteristics). Comparative statics of the optimal strategy are also analyzed.

1. INTRODUCTION

The general importance of drawing a distinction between information concerning characteristics of goods which are directly observable and information concerning characteristics which cannot be observed without actual "consumption" in some appropriate sense has just begun to be realized by economists. Labor economists have implicitly been making use of just such a distinction for some time. For example, researchers on labor mobility in the early 50's such as Reynolds [1951] emphasized that high job turnover by young workers could be explained in part as a kind of job-shopping. More recently, Pencavel [1972] has used the notion that all characteristics of jobs cannot be observed by simply sampling firms as the basis for an equilibrium model of job quits, and Wilde [1976] has explored the nature of optimal search strategies when jobs are characterized by multiple characteristics, some of which are directly observable and others which cannot be observed without actually taking the job.

Besides these applications to the quit-rate problem (which emphasize the supply side of the labor market), the distinction between different kinds of information can be used to analyze other problems, both in the labor market and in other markets characterized by imperfect and costly information. Stiglitz [1975] applies it to the economic theory of screening and education, and Nelson [1970] uses it in a unique approach to consumer behavior in product markets.

Nelson partitions goods into two classes, search goods and experience goods. The distinction between the two classes is drawn in terms of the consumer's preferred mode of evaluating the potential stream of utility yielded by purchasing the good. For search goods the evaluation occurs prior to purchase; that is, a decision to purchase the good is based only on price (and other directly observable characteristics), not on any unobservable qualitative features of the product. By contrast, experience goods are those for which the primary information process used to evaluate the potential utility of a purchase is actual consumption of the good.

This paper builds a theory of consumer behavior under imperfect information when goods are described by multiple characteristics, some of which are directly observable and others which cannot be observed without actually consuming the good.

The fundamental assumption in the paper is not that goods, per se, can be partitioned into classes according to their informational properties, but rather that characteristics of goods can be so partitioned. That is, two classes of characteristics are defined, general and specific. General characteristics are those which can be observed without actually consuming the good, and specific characteristics are those which cannot be observed without consumption. Information concerning general characteristics will be called general information and information concerning specific characteristics will be called specific information. And finally, investment in general information will be called search or inspection, as suggested by

Hirshleifer [1973], and investment in specific information will be called evaluation.¹

While these definitions of general and specific information seem analogous to Nelson's definitions of search and experience goods, there is a fundamental difference; Nelson focuses on goods as total packages of characteristics while the concepts of specific and general information focus directly on the individual characteristics. To see the difference, suppose a good is described by a vector of characteristics (x_1, x_2, \dots, x_n) . Nelson assumes each value x_i is potentially observable prior to purchase and that the consumer must choose between paying a "search cost" and observing all x_i prior to purchase or paying the purchase price and observing all x_i by direct consumption. Clearly this all or nothing property is strong. For example, consider the case of experience goods. Since each good is evaluated by direct consumption, the good must actually be purchased, so at least the purchase price is observed. But this cannot, by definition, effect the decision as to which good to buy. For experience goods, Nelson assumes "consumers either sample at random from among all brands or from those brands in the price range the consumer deems appropriate for himself." This implies that the consumer either ignores price altogether or has perfect information about price and is imperfectly informed only about qualitative characteristics.

There are several ways to avoid these logical difficulties. In keeping with the spirit of Nelson's definitions, one could assume each characteristic x_i is potentially observable by paying a characteristic-specific search cost, c_i . Now the consumer draws goods

at random and observes at least price. Subsequently he can either buy the good, observe more characteristics prior to purchase, or reject the good and draw a new observation. This is essentially the kind of problem analyzed by MacQueen [1962], and while it preserves the flavor of Nelson's approach, it also makes the significant shift in emphasis from goods to characteristics. In a sense, Nelson's search goods and experience goods are special cases in this model; search goods are those for which all characteristics (x_1, x_2, \dots, x_n) are necessarily observed prior to purchase (at cost equal to the sum of all c_i , $i=1, \dots, n$), and experience goods are those for which only price is observed prior to purchase (and, furthermore, all prices are acceptable).

The point of departure of this paper is to consider goods for which a subset of the characteristic-specific search costs are infinite; that is, this paper considers consumer demand for products which possess some characteristics that cannot be observed prior to consumption. The following example illustrates the consumer's problem. Consider a good which can be described by two characteristics, price and quality. Price is directly observable but quality can only be observed via consumption of the good. Assume there is some variance across goods with respect to both characteristics, and the consumer knows the distribution of both (note the distributions may not be independent). Then a consumer will search until he finds an acceptable price, after which he makes a purchase and evaluates the quality of the good he has bought. Now it is quite possible that he be disappointed with his evaluation of the good and, upon

his next purchase of the good, return to the market with the intent of searching once more for an acceptable price, repeating the evaluation process until he finds an acceptable price-quality combination.

The layout of the paper is as follows. The next section defines the specific search environment for this problem. Section three then sets up the dynamic programming problem faced by the consumer, derives the relevant functional equation, and establishes conditions under which there is a unique reservation price. Section four develops some specific properties of the optimal policy for the general case. In particular, it is shown that one of the following situations applies for any given level of search costs: (1) all prices and all qualities are acceptable; (2) all prices are acceptable but some quality levels will be rejected, acceptable quality is a function of observed price; (3) some prices are rejected as well as some quality levels, acceptable quality remains a function of price; or (4) only the lowest price is acceptable, some quality levels are rejected. Corollary to this categorization are the observations that there are no search goods in the sense Nelson uses that term (if price matters then quality must matter for some prices) and experience goods exist only when search costs (with respect to price observation) are zero. Section 5 considers a special model which allows comparisons with respect to changes in the lifetime of the good being purchased. It is argued there that an increase in durability lowers both the reservation price and reservation qualities causing a shift away from evaluation and towards inspection. A final section discusses and summarizes the results obtained.

2. BASIC ASSUMPTIONS OF THE MODEL

The problem analyzed in this paper is set in the product market. Goods vary with respect to both price and some non-price characteristic, quality. Consumers are imperfectly informed about the specific price/quality pairs offered by specific firms. For a given cost they can, however, draw samples from the market distribution and observe price prior to consumption. Formal assumptions defining the search environment follow.

- A1) Each good is described by a pair (p, q) where p is price and q is quality. Price is directly observable and quality can only be observed via actual consumption of the good.
- A2) $U(p, q)$ is the total net value to the consumer of purchasing a good with price p and quality q . Each good has a lifetime of s periods. Assume $U(p, q)$ is continuous, differentiable, and bounded on its domain with $\partial U(p, q) / \partial p < 0$ and $\partial U(p, q) / \partial q > 0$.²
- A3) Let $\phi(p, q)$ be the joint p.d.f. defining the market distribution of p and q , where $p \in [p_a, p_b]$ and $q \in [q_a, q_b]$. Assume the consumer knows $\phi(p, q)$ with certainty.

- A4) The cost of drawing an observation at random from $\phi(p,q)$ and observing price is c_s . The cost of returning to a specific firm and repurchasing a given good is $c_r \leq c_s$. c_s and c_r are measured in the same units as $U(p,q)$.
- A5) The consumer can sample as many observations from $\phi(p,q)$ as he likes at the beginning of each period but demands at most one unit of the good per period.³
- A6) The consumer desires to maximize the discounted stream of utility of consumption, net of search costs. Sampling is without recall, the horizon is infinite, and the discount rate is $0 < \beta < 1$.

In the environment defined by these six assumptions, the consumer faces the following problem. He knows $\phi(p,q)$. By paying c_s he can draw an observation from this distribution, say (p_o, q_o) , but he observes only p_o . Information regarding q_o is provided by the conditional distribution of q given p_o , say $g(q|p_o)$. In the absence of any constraints on the distribution $\phi(p,q)$, there is no definite relationship between price and expected long-run utility from purchasing the good. For example, if there is a strong positive correlation between price and quality, then the consumer might reject low prices and search out higher ones. On the other hand, if this positive correlation is weak enough (or, in general, non-existent)

then low prices will be preferred to high prices. Another possibility is disjoint intervals of acceptable prices separated by ranges of unacceptable prices. Clearly, it will be necessary to put more structure on $\phi(p,q)$ to get strong results. For the sake of discussion, assume that p and q are positively correlated, but that low prices are preferred to high systematically. In this case, if p_o (the observed price) is low enough, then the consumer may decide to purchase the good. Otherwise he rejects it and draws another observation from $\phi(p,q)$. If he buys the good he receives $U(p,q)$ over the s period lifetime of the good.⁴ At the end of s periods the consumer can either pay c_r and repurchase (p_o, q_o) , or he can sample again from $\phi(p,q)$ at cost c_s . In general, there will be a positive probability of the consumer not being satisfied with q_o since there will be ranges of price for which the conditional expected utility of consumption is high enough to warrant purchase, allowing for the possibility of being disappointed. Given a little more structure on $\phi(p,q)$, it is possible to show that the optimal policy is a sequential-lexicographic process whereby the initial purchase decision is based on some critical value of p , and the decision whether to stay with a particular (p,q) combination is based on a critical value of q . That is, the optimal strategy is characterized by a pair $(p^*, q^*(p))$ such that at each stage of the decision process if

$$(1) \quad \begin{cases} p > p^*: & \text{sample again} \\ p \leq p^*: & \text{buy the good and if} \\ & \begin{cases} q < q^*(p): & \text{sample again after } s \text{ periods} \\ q \geq q^*(p): & \text{repurchase the good.} \end{cases} \end{cases}$$

3. THE OPTIMAL POLICY: EXISTENCE AND UNIQUENESS
OF A RESERVATION PRICE

The searcher's objective is to maximize the discounted stream of utility net of search costs from consuming the good. Let $V(p)$ be the expected value of this flow when a current observation of p has just been drawn. Define V_e as the expected value of $V(p)$ taken with respect to the marginal distribution of p ; that is,

$$(2) \quad V_e = \int_{p_a}^{p_b} V(p) f(p) dp,$$

where $f(p)$ is defined by

$$(3) \quad f(p) = \int_{q_a}^{q_b} \phi(p, q) dq.$$

If the consumer follows an optimal strategy in searching, $V(p)$ is defined by

$$(4) \quad V(p) = -c_s + \max\{V_e, B(p, V_e)\},$$

where $B(p, V_e)$ is the expected discounted flow of utility conditional on buying the good priced at p , when the expected net utility of search is V_e , assuming an optimal policy is pursued subsequent to the purchase. The logic of (4) is that the consumer must pay c_s to observe p . If p is an acceptable price then $V(p) = -c_s + B(p, V_e)$. If p is not acceptable then a new draw is taken from $\phi(p, q)$ and the

expected utility of searching, V_e , is received: i.e. $V(p) = -c_s + V_e$. Thus, the critical value of p is defined by $V_e = B(p^*, V_e)$. The immediate problem is to uncover conditions which guarantee such a p^* exists and is unique.

Consider $B(p, V_e)$. If the consumer purchases the good after observing p , then he receives $U(p, q)$ over the s -period lifetime of the good. But at the time he makes the decision to purchase, he hasn't observed q . The expected utility of consumption (net of price) during the first s periods is

$$(5) \quad E[U(p, q) | p] = \int_{\bar{q}(p)}^{\bar{\bar{q}}(p)} U(p, q) g(q | p) dq,$$

where $g(q | p)$ is the conditional p.d.f. of q given p , defined on $[\bar{q}(p), \bar{\bar{q}}(p)]$. At the end of s periods the consumer can either repurchase the good or draw a new sample from $\phi(p, q)$. The decision is clearly based on the observed value of q . If the consumer repurchases the good he receives $[U(p, q) - c_r]$ every s periods (if he repurchases once after observing q , he will never switch). If he samples again he receives V_e . Therefore, define $q^*(p, V_e)$ by

$$V_e = [U(p, q^*(p, V_e)) - c_r] + \beta^s [U(p, q^*(p, V_e)) - c_r] + \dots$$

or

$$(6a) \quad V_e = \sum_{i=0}^{\infty} \beta^{is} [U(p, q^*(p, V_e)) - c_r].$$

$q^*(p, V_e)$ is the quality level which makes the consumer indifferent between repurchasing the good $(p, q^*(p, V_e))$ and sampling again. Note that $\partial U(p, q) / \partial q > 0$ implies $q^*(p, V_e)$ is unique. But q is

bounded for any $p, q \in [\bar{q}(p), \bar{\bar{q}}(p)]$. Thus, $q^*(p, V_e)$ as defined by (6a) can be driven above $\bar{q}(p)$ when V_e is high and below $\bar{q}(p)$ when V_e is low. In the former case define $q^*(p, V_e) = \bar{q}(p)$ and in the latter $q^*(p, V_e) = \bar{\bar{q}}(p)$. Then when $q^*(p, V_e) = \bar{q}(p)$ it must be that

$$(6b) \quad V_e \geq \sum_{i=1}^{\infty} \beta^{is} [U(p, \bar{q}(p)) - c_r]$$

and when $q^*(p, V_e) = \bar{\bar{q}}(p)$ it must be that

$$(6c) \quad V_e \leq \sum_{i=1}^{\infty} \beta^{is} [U(p, \bar{\bar{q}}(p)) - c_r].^5$$

Let $G(q|p)$ the c.d.f. associated with $g(q|p)$. Then, since $\partial U(p, q)/\partial q > 0$, $q < q^*(p, V_e)$ implies the consumer resamples, and $q \geq q^*(p, V_e)$ implies the consumer repurchases. So the probability that the consumer samples again after s periods is $G(q^*(p, V_e)|p)$. The probability he repurchases the original good is $[1 - G(q^*(p, V_e)|p)]$. Define $W(p, V_e)$ as the discounted expected utility of consumption conditional on repurchasing the good when price p is originally observed (i.e. conditional on $q \geq q^*(p, V_e)$). Then $B(p, V_e)$ is defined by

$$(7) \quad B(p, V_e) = E[U(p, q)|p] + \beta^s \{G(q^*(p, V_e)|p) V_e + [1 - G(q^*(p, V_e)|p)]W(p, V_e)\}$$

where $W(p, V_e)$ is given as

$$(8) \quad W(p, V_e) = [1 - G(q^*(p, V_e)|p)]^{-1} \int_{q^*(p, V_e)}^{\bar{\bar{q}}(p)} \left\{ \sum_{i=1}^{\infty} \beta^{is} [U(p, q) - c_r] \right\} g(q|p) dq,$$

and $\bar{\bar{q}}(p)$ is the upper limit on the conditional distribution $g(q|p)$. Using (5), (7), and (8) in equation (4) the basic functional equation associated with the consumer's problem can be written as

$$(9) \quad V(p) = -c_s + \max \{V_e, E[U(p, q)|p] + \beta^s [G(q^*(p, V_e)|p) V_e + [1 - G(q^*(p, V_e)|p)]W(p, V_e)]\}$$

The immediate problem is to show that (9) has a unique solution. Establishing such a result is straightforward and is not sensitive to the precise form of $\phi(p, q)$. The following theorem is proved in the appendix.

Theorem 1: Given (A1) \rightarrow (A6)', there exists a unique, continuous, bounded solution to the functional equation (9).

In the present setting, theorem 1 is not as strong as it might seem. It only establishes that (i) for any observed p there is a determinate solution to accepting or rejecting the good and that (ii) V_e is unique and can thus be treated as a constant. Theorem 1 does not imply p^* is unique. Figure 1 illustrates the case where there are three values of p such that $B(p, V_e) = V_e$. Prices in the ranges $[p_a, p_1^*]$ and $[p_2^*, p_3^*]$ are acceptable but observations at prices in the ranges $[p_1^*, p_2^*]$ and $[p_3^*, p_b]$ are rejected.

There are two natural sets of sufficient conditions for p^* to be unique in this setting. One is $B(p_a, V_e) > V_e$ and $\partial B(p, V_e)/\partial p < 0$ for all p in $[p_a, p_b]$. The other conditions are $B(p_a, V_e) < V_e$ and $\partial B(p, V_e)/\partial p > 0$ for all p in $[p_a, p_b]$. In the

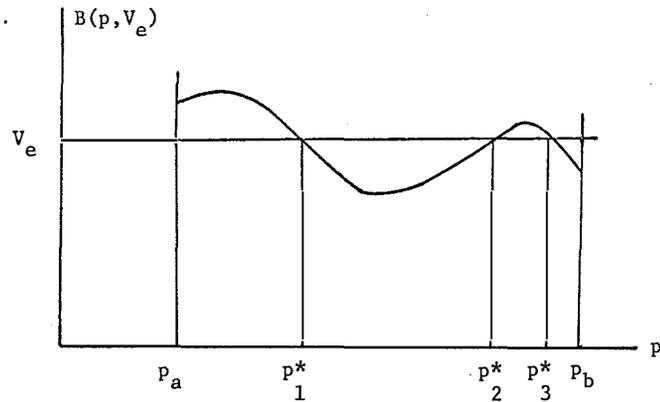


Figure 1: Optimal Policy Rule for Multiple Ranges of Acceptable Prices

former case low prices are acceptable and high prices are not, while in the latter case the opposite is true. Both of these cases are consistent with the notion that higher quality is associated with higher price. Here this translates as $\partial G(q|p)/\partial p < 0$; i.e. the probability that quality is less than some given q falls as price rises -- higher price implies that "on average" quality is higher. But it is the combination of price and quality which is important; the distribution of $U(p,q)$ is what matters to the consumer.

Let $\hat{q}(w,p)$ be the value of q which sets $U(p,\hat{q}) = w$. Then $\partial \hat{q}/\partial p = -U_1/U_2 > 0$. Define $\Psi(w|p)$ as the c.d.f. of $w = U(p,q)$ conditional on p . Then $\Psi(w|p) = G(\hat{q}(w,p)|p)$, and

$$(10) \quad \frac{\partial \Psi(w|p)}{\partial p} = \frac{\partial G(\hat{q}|p)}{\partial p} + g(\hat{q}|p) \frac{\partial \hat{q}}{\partial p}.$$

With $\partial G(\hat{q}|p)/\partial p < 0$ it is not possible to sign $\partial \Psi(w|p)/\partial p$ since both $g(\hat{q}|p)$ and $\partial \hat{q}/\partial p$ are positive. In general one would expect $\partial \Psi(w|p)/\partial p > 0$. That is, a higher price should denote less utility for any good within a generic class. This is the case, for example, when price and quality are independent. In the absence of an equilibrium model which generates $\Psi(p,q)$ endogenously, or of a more precise specification of what "quality" means, independence may be a reasonable working assumption, especially since the crucial property is not $\partial G(q|p)/\partial p < 0$ but rather $\partial \Psi(w|p)/\partial p > 0$. In the remainder of this section it is simply assumed $\partial \Psi(w|p)/\partial p > 0$.

Lemma 2: Assume $\partial \Psi(w|p)/\partial p > 0$ for all $p \in [p_a, p_b]$ and $q \in [q_a, q_b]$.

Then $\partial B(p, V_e)/\partial p < 0$ for all $p \in [p_a, p_b]$.

Proof: From (6), noting $\sum_{i=0}^{\infty} \beta^{is} = 1/1-\beta^s$, $B(p, V_e)$ can be written

$$(11) \quad B(p, V_e) = E[U(p, q) | p] + \beta^s \{v_e G(q^*(p, V_e) | p) + \frac{1}{1-\beta^s} \int_{q^*(p, V_e)}^{\bar{q}(p)} (U(p, q) - c_r) g(q | p) dq\}$$

$$\equiv E[U(p, q) | p] + H(p, V_e)$$

From (5),

$$(12) \quad E[U(p, q) | p] = \int_{\bar{q}(p)}^{\bar{q}(p)} U(p, q) g(q | p) dq .$$

But using $\Psi(w | p) = G(\hat{q}(w, p) | p)$ and defining $\bar{w}(p) = U(p, \bar{q}(p))$, and $\bar{w}(p) = U(p, \bar{q}(p))$, (12) becomes

$$(13) \quad E[U(p, q) | p] = \int_{\bar{w}(p)}^{\bar{w}(p)} w \Psi(w | p) dw .$$

Integrating by parts,

$$(14) \quad E[U(p, q) | p] = \bar{w}(p) - \int_{\bar{w}(p)}^{\bar{w}(p)} \Psi(w | p) dw,$$

and differentiating with respect to p ,

$$(15) \quad \frac{\partial E[U(p, q) | p]}{\partial p} = - \int_{\bar{w}(p)}^{\bar{w}(p)} \frac{\partial \Psi(w | p)}{\partial p} dw < 0 .$$

Now consider the second half of (11). Using the definition of $q^*(p, V_e)$ and substituting $\Psi(w | p)$ for $G(q | p)$ gives

$$(16) \quad H(p, V_e) = \beta^s \left\{ v_e \Psi \left(v_e (1-\beta^s) + c_r | p \right) + \left(\frac{1}{1-\beta^s} \right) \int_{v_e (1-\beta^s) + c_r}^{\bar{w}(p)} (w - c_r) \Psi(w | p) dw \right\} .$$

Integrating (16) by parts

$$H(p, V_e) = \beta^s \left\{ v_e \Psi \left(v_e (1-\beta^s) + c_r | p \right) + \left(\frac{1}{1-\beta^s} \right) (w - c_r) \Psi(w | p) \Big|_{v_e (1-\beta^s) + c_r}^{\bar{w}(p)} \right.$$

$$\left. - \left(\frac{1}{1-\beta^s} \right) \int_{v_e (1-\beta^s) + c_r}^{\bar{w}(p)} \Psi(w | p) dw \right\}$$

Collecting terms gives $H(p, V_e)$ as

$$(17) \quad H(p, v_e) = \left(\frac{\beta^s}{1-\beta^s} \right) \left\{ \left(\bar{w}(p) - c_r \right) - \int_{v_e(1-\beta^s) + c_r}^{\bar{w}(p)} \Psi(w|p) dw \right\} .$$

Differentiating (17) with respect to p and combining the result with (15),

$$(18) \quad \frac{\partial B(p, v_e)}{\partial p} = \frac{\partial E[U(p, q)|p]}{\partial p} + \frac{\partial H(p, v_e)}{\partial p} \\ = - \left\{ \int_{\bar{w}}^{\bar{w}(p)} \frac{\partial \Psi(w|p)}{\partial p} dw + \frac{1}{1-\beta^s} \int_{v_e(1-\beta^s) + c_r}^{\bar{w}(p)} \frac{\partial \Psi(w|p)}{\partial p} dw \right\} .$$

Thus, $\partial \Psi(w|p)/\partial p > 0$ implies $\partial B(p, v_e)/\partial p < 0$.

Lemma 3: If $\partial \Psi(w|p)/\partial p > 0$ and $c_s > 0$ then $B(p_a, v_e) > v_e$.

Proof: The proof of this lemma is by contradiction. Suppose $B(p_a, v_e) \leq v_e$. Then, since $\partial B(p, v_e)/\partial p < 0$ for all $p \in [p_a, p_b]$, it must be that $B(p, v_e) < v_e$ for all $p \in [p_a, p_b]$. Thus $\max\{v_e, B(p, v_e)\} = v_e$ and $V(p) = -c_s + v_e$ for all $p \in [p_a, p_b]$. But then integrating over p gives $v_e = -c_s + v_e$ which implies $c_s = 0$. This contradicts $c_s > 0$. Thus $B(p_a, v_e) > v_e$.

Theorem 4: If $\partial \Psi(w|p)/\partial p > 0$ then either $B(p, v_e) > v_e$ for all

$p \in [p_a, p_b]$ or there exists a unique $p^* \in [p_a, p_b]$ such that $B(p^*, v_e) = v_e$. $p^* = p_a$ if and only if $c_s = 0$.

Proof: Clearly with $B(p_a, v_e) > v_e$ and $\partial B(p, v_e)/\partial p < 0$ then either $B(p, v_e)$ cuts v_e in $[p_a, p_b]$ or $B(p, v_e) > v_e$ for all $p \in [p_a, p_b]$. If $B(p, v_e)$ intersects v_e then p^* is uniquely defined by $B(p^*, v_e) = v_e$. If, however, $B(p, v_e) > v_e$ for all $p \in (p_a, p_b)$ then define $p^* = p_b$. In this case search costs are so high as to make all prices acceptable, at least prior to observing q .

To show $p^* = p_a$ if and only if $c_s = 0$, consider the following argument. For $p < p^*$, $V(p) = -c_s + B(p, v_e)$ and for $p \geq p^*$, $V(p) = -c_s + v_e$. Thus

$$(19) \quad v_e = -c_s + \left\{ \int_{p_a}^{p^*} B(p, v_e) f(p) dp + [1 - F(p^*)] v_e \right\} .$$

Rearranging (19),

$$(2) \quad c_s = \int_{p_a}^{p^*} [B(p, v_e) - v_e] f(p) dp .$$

But $v_e = B(p^*, v_e)$. Thus,

$$(21) \quad c_s = \int_{p_a}^{p^*} [B(p, v_e) - B(p^*, v_e)] f(p) dp .$$

$f(p)$ is the marginal distribution of p and is strictly positive on $[p_a, p^*]$. $[B(p, V_e) - B(p^*, V_e)] > 0$ for $p \in [p_a, p^*]$. Thus, $p^* = p_a$ if and only if $c_s = 0$.

p^* is the reservation price. The optimal strategy for the consumer in this problem is analogous to simple search models; the consumer draws observations from $\phi(p, q)$ (effectively draws of p from $f(p)$) until he finds one less than p^* . In the simple problem he would buy the homogeneous good at price p in all subsequent periods.

Here, because goods are heterogeneous with respect to quality, he buys the good just once, until he can evaluate q . If q is less than $q^*(p)$ then he draws again from $\phi(p, q)$, but p^* does not change. Figure 2 illustrates the case where $\partial B(p, V_e) / \partial p < 0$ and $p^* \in (p_a, p_b)$ is unique.

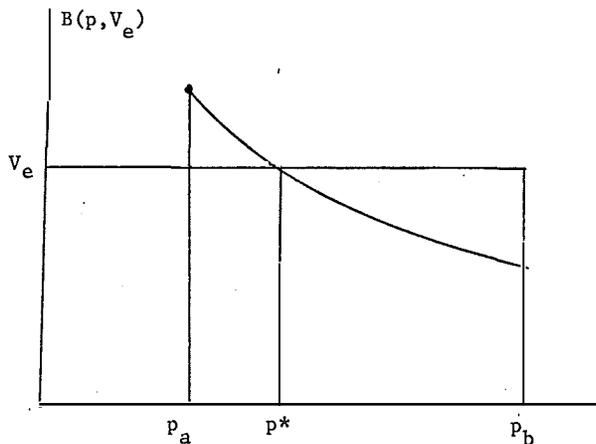


Figure 2: Unique p^* when $\partial B(p, V_e) / \partial p < 0$.

4. PROPERTIES OF THE OPTIMAL POLICY

The optimal policy of the consumer is summarized in two variables and two equations.

$$(22a) \quad V_e = B(p^*, V_e),$$

$$(22b) \quad c_s = \int_{p_a}^{p^*} [B(p, V_e) - B(p^*, V_e)] f(p) dp.$$

With some manipulation of these two equations, comparative statics are reasonably straightforward. In particular, $dp^*/dc_s > 0$; as search costs rise so does the reservation price. This is not surprising. More interesting results concern the relationship of acceptable prices to acceptable quality levels.

Comment 5: $p^* = p_b$ does not imply $q^*(p) = \bar{q}(p)$ for all $p \in [p_a, p_b]$, where $\bar{q}(p)$ is the lower limit on the conditional p.d.f. $g(q|p)$.

Consider the following counterexample. Assume $B(p_b, V_e) = V_e$. The claim is that $q^*(p_b) > \bar{q}(p_b)$. Assume the opposite. Then by definition

$$(23) \quad V_e \leq \frac{U(p_b, \bar{q}(p_b)) - c_r}{1 - \beta^s}$$

By assumption, though, V_e is given as

$$(24) \quad v_e = E[U(p_b, q) | p_b] + \beta^s \left\{ v_e G[q^*(p_b, v_e) | p_b] + \left(\frac{1}{1-\beta^s} \right) \int_{q^*(p_b, v_e)}^{\bar{q}(p_b)} [U(p_b, q) - c_r] g(q | p_b) dq \right\}.$$

Using $q^*(p_b) = \bar{q}(p_b)$, this can be written as

$$(25) \quad v_e = \frac{E[U(p_b, q) | p_b] - \beta^s c_r}{1 - \beta^s}$$

From (23) we have, then, that $q^*(p_b) = \bar{q}(p_b)$ implies

$$(26) \quad U(p_b, \bar{q}(p_b)) \geq E[U(p_b, q) | p_b] + (1 - \beta^s) c_r$$

Clearly there is a contradiction here since $\bar{q}(p_b)$ is the minimum quality associated with price p_b . Even if c_r is zero, (26) cannot be satisfied unless the conditional p.d.f. $g(q | p_b)$ is degenerate.

Comment 5 says that even if all prices are acceptable, some quality levels may not be acceptable. In effect, it demonstrates that when search costs are high the good may be an "experience" good; the consumer ignores price and samples until a satisfactory quality level is found (notice the "search cost" here is not associated with observing quality prior to purchase). The quality level which is acceptable is not independent of price, though. That is, if $p^* = p_b$ then $q^*(p)$ may exceed q_a and in any case is still sensitive to the observed price -- there is no "reservation" quality independent of price.

Theorem 6: $p^* \in (p_a, p_b)$ implies $q^*(p^*) > \bar{q}(p^*)$ where $\bar{q}(p^*)$ is the lower limit on the conditional p.d.f. $g(q | p^*)$.

Proof: Assume $q^*(p^*) = \bar{q}(p^*)$. Then, by definition,

$$(27) \quad v_e \leq \frac{U(p^*, \bar{q}(p^*)) - c_r}{1 - \beta^s}$$

But $v_e = B(p^*, v_e)$. When $q^*(p^*) = \bar{q}(p^*)$, $B(p^*, v_e)$ is

$$(28) \quad B(p^*, v_e) = \frac{E[U(p^*, q) | p^*] - \beta^s c_r}{1 - \beta^s}$$

Thus, (27) and (28) imply

$$(29) \quad U(p^*, \bar{q}(p^*)) \geq E[U(p^*, q) | p^*] + (1 - \beta^s) c_r$$

As long as $g(q | p^*)$ is non-degenerate, this leads to a contradiction, even when $c_r = 0$. Thus, $q^*(p^*) > \bar{q}(p^*)$.

Theorem 6 says that quality is never irrelevant when price matters. That is, if it pays to set a reservation price strictly less than p_b , then at that reservation price (and in a neighborhood of it) it pays to reject some quality levels. The theorem does not imply $q^*(p) > \bar{q}(p)$ for all $p \leq p^*$. For example, at p_a it may be optimal to accept all quality levels rather than resample. In one sense theorem 6 is very strong: it implies there are no "search" goods regardless of the level of search costs. In fact, if $c_s = 0$, then $p^* = p_a$, but $q^*(p_a)$ is not necessarily equal to $\bar{q}(p_a)$. The

following corollary demonstrates that if $c_r \rightarrow 0$ as $c_s \rightarrow 0$ (certainly the case when $c_s \geq c_r$ for all c_s and c_r), then $q^*(p_a) < \bar{q}(p_a)$.

Corollary 7: Let $c_s = 0 = c_r$. Then $\bar{q}(p_a) < q^*(p_a) = q^*(p^*) < \bar{q}(p_a)$.

Proof: $\partial\Psi(w|p)/\partial p > 0$ implies that p_a is the optimal price and $g(q|p_a)$ is the optimal distribution from which to draw quality samples. Thus $c_s = 0$ implies $p^* = p_a$. The proof of theorem 6 then applies, yielding $q^*(p^*) = q^*(p_a) > \bar{q}(p_a)$. To see that $q^*(p_a) < \bar{q}(p_a)$ assume the opposite. Then by definition

$$(30) \quad v_e \geq \frac{U(p_a, \bar{q}(p_a)) - c_r}{1 - \beta^s} = \frac{U(p_a, \bar{q}(p_a))}{1 - \beta^s}$$

But $v_e = B(p_a, v_e)$ implies

$$(31) \quad v_e = E[U(p_a, q) | p_a] + \beta^s v_e$$

when $q^*(p_a) = \bar{q}(p_a)$. (30) and (31) together imply

$$(32) \quad E[U(p_a, q) | p_a] \geq U(p_a, \bar{q}(p_a))$$

which is clearly a contradiction when $g(q|p_a)$ is nondegenerate.

Thus, $q^*(p_a) < \bar{q}(p_a)$.

If c_r remained positive as $c_s \rightarrow 0$, then it might pay to always resample since repurchase involves the cost c_r . But this is a silly assumption when $c_s = 0$. Corollary 7 describes the case analyzed by Nelson [1970]; the only relevant "search costs" are the opportunity costs of consuming at a low quality level.⁶ When $c_s = 0 = c_r$, then,

goods are truly experience goods in the sense that Nelson uses that term. But to get to this case it is not only necessary that the cost of direct sampling of quality be high (in this model it is infinite), but it is also necessary that the cost of observing price be zero.

To summarize, then, consider first $c_s = 0 = c_r$. In this case $p^* = p_a$ (the lowest price) and only quality matters. The consumer sets a reservation quality $q^* = q^*(p_a) = q^*(p^*)$ and samples from $g(q|p_a)$ until he finds a quality level which beats q^* . Now let $c_s \geq c_r > 0$. Assume $p_a < p^* < p_b$. In this case it may or may not be true that $q^*(p) > \bar{q}(p)$ for any given acceptable price ($p < p^*$); for some prices all quality levels may be acceptable (recall $\bar{q}(p)$ is the lower bound on $g(q|p)$). But at p^* , $q^*(p^*) > \bar{q}(p^*)$; at the margin it always pays to allow for the possibility of rejecting some quality levels. As c_s rises, so does p^* . Eventually $p^* = p_b$, the maximum price, and all prices are acceptable. Now in this case goods are experience goods in the sense that any price is acceptable, but acceptable quality levels are still sensitive to the observed price; that is, $q^*(p)$ is sensitive to the observed price, p .

Let c_s continue to rise. Here $B(p_b, v_e) > v_e$ and, by definition, $p^* = p_b$. In this case (22) can be written

$$(33) \quad c_s = E[B(p, v_e)] - v_e.$$

The question is, can c_s rise high enough that $q^*(p) = \bar{q}(p)$ for all p . Suppose this is the case. Then

$$(34) \quad B(p, v_e) = \frac{E[U(p, q) | p] - \beta^s c_r}{1 - \beta^s}$$

Thus, $p^* = p_b$ and $q^*(p) = \bar{q}(p)$ for all p implies

$$(35) \quad \frac{E[U(p_b, q) | p_b] - \beta^s c_r}{1 - \beta^s} > v_e = \int_{p_a}^{p_b} \frac{E[U(p, q) | p] f(p) dp}{1 - \beta^s} - \frac{\beta^s c_r}{1 - \beta^s} - c_s$$

or, after rearranging and letting $E^*[U(p, q)]$ be the expected value of $U(p, q)$ over $\phi(p, q)$,

$$(36) \quad E[U(p_b, q) | p_b] > E^*[U(p, q)] - (1 - \beta^s) c_s.$$

So, as c_s rises, eventually all prices become acceptable. But quality levels still matter. Only when c_s is large enough for (36) to be satisfied can quality become irrelevant also.

Following Nelson, it would be desirable to get comparative statics with respect to s . But as it stands there is no way to let s change without changing the net utility per period obtained from a given purchase. One way around this is to let $U(p, q)$ be linear in p and adjust prices to keep per period utility constant when s changes. This is taken up in the next section.

5. DURABILITY: CHANGES IN s

In order to make comparisons between different durabilities, it is necessary to control for the flow of utility obtained by paying particular prices. Let the net utility of purchase be linear in price and assume

$$(37) \quad U(p, q) = \sum_{i=1}^s \beta^{i-1} [u(q) - p] = b(s) [u(q) - p],$$

where $b(s) = \sum_{i=1}^s \beta^{i-1}$. At any given price-quality pair, the difference in a purchase at duration s and duration $s + 1$ is $\beta^s [u(q) - p]$. This is desirable since in the absence of repurchase costs (c_r) the total discounted "cost" of maintaining a given per-period flow of utility $u(q_0)$ when the pair (p_0, q_0) has been drawn from $\phi(p, q)$ is independent of the durability of the good. Assuming $U(p, q)$ is defined as in (37), an increase in s has two effects when $c_r > 0$; since repurchase is postponed into the future, the cost of observing a low value of q is increased, but at the same time the repurchase cost, c_r , is paid less often.

A second major change in the basic model is to assume that price and quality are distributed independently. The necessary condition $\partial \Psi(w|p) / \partial p > 0$ is maintained but independence makes some computation easier.

Recall the definition of $B(p, V_e)$:

$$(38) \quad B(p, V_e) = E[U(p, q) | p] + \beta^s \{V_e G(q^*(p, V_e) | p) + \frac{1}{1-\beta^s} \int_{q^*(p, V_e)}^{\bar{q}(p)} [U(p, q) - c_r] g(q | p) dq\}$$

Integrating by parts and substituting from (37),

$$(39) \quad B(p, V_e) = b(s) \int_{q_a}^{q_b} [u(q) - p] g(q) dq + \frac{\beta^s}{1-\beta^s} \{b(s) [u(q_b) - p] - c_r - b(s) \int_{q^*(p, V_e)}^{q_b} u'(q) G(q) dq\}$$

where $g(q)$ is the p.d.f of quality defined on $[q_a, q_b]$. When $p^* \in (p_a, p_b)$, it still holds that $V_e = B(p^*, V_e)$. Thus, using the definition of $q^*(p, V_e)$ and letting u_e be the expected value of $u(q)$ with respect to $g(q)$,

$$(40) \quad \frac{b(s) [u(q^*(p^*)) - p^*] - c_r}{1-\beta^s} = V_e = B(p^*, V_e) \\ = b(s) [u_e - p] + \frac{\beta^s}{1-\beta^s} \{b(s) [u(q_b) - p^*] - c_r - b(s) \int_{q^*(p^*)}^{q_b} u'(q) G(q) dq\} \\ = b(s) u_e - \frac{1}{1-\beta} p^* - \frac{\beta^s}{1-\beta^s} c_r + \frac{\beta^s}{1-\beta} \{u(q_b) - \int_{q^*(p^*)}^{q_b} u'(q) G(q) dq\}.$$

$q^*(p^*)$ must satisfy (40) for any value p^* . Therefore, define

$$(41) \quad \theta(z) = (1-\beta^s) u_e - u(z) + (1-\beta) c_r + \beta^s \{u(q_b) - \int_z^{q_b} u'(q) G(q) dq\}.$$

Then $\theta(q^*(p^*)) = 0$ is equivalent to (40).

Lemma 8: There exists a unique solution z^* to $\theta(z) = 0$. Furthermore, $p^* \in (p_a, p_b)$ implies $z^* > q_a$, and for c_r small, $q_a < z^* < q_b$.

Proof: Differentiating (41) with respect to z ,

$$\theta'(z) = -u'(z) [1 - \beta^s G(z)].$$

$u'(z) > 0$ for all z implies $\theta'(z) < 0$ since $G(z)$ is the c.d.f. of quality. Moreover,

$$\theta(q_a) = [u_e - u(q_a)] + (1-\beta) c_r$$

$$\theta(q_b) = -(1-\beta^s) [u(q_b) - u_e] + (1-\beta) c_r$$

Clearly $\theta(q_a) > 0$. This combined with $\theta'(z) < 0$ for all $z > 0$ and

$\lim_{z \rightarrow \infty} \theta(z) < 0$ implies z^* such that $\theta(z^*) = 0$ exists and is unique.

$z^* < q_b$ if and only if $\theta(q_b) < 0$. This is the case when c_r is small.

Lemma 8 establishes that $q^*(p^*)$ is independent of p^* -- at the margin, acceptable quality is constant. Moreover, at the margin it always pays to allow for rejecting some quality levels. But the important result is that, other things constant, as s rises, $q^*(p^*)$ falls.

Let $q^*(p^*; s)$ be the reservation quality associated with the reservation price when durability is s periods.

$$(43) \quad \frac{dq^*(p;t)}{dp} = \frac{1}{u'(q^*(p;t))}$$

Now, $u''(q) < 0$ implies $u'[q^*(p^*(s);s)] < u'[q^*(p^*(s+1);s+1)]$

since $q^*(p^*(s);s) > q^*(p^*(s+1);s+1)$. Thus,

$$(44) \quad \frac{1}{u'[q^*(p^*(s);s)]} = \frac{dq^*(p^*(s);s)}{dp} < \frac{dq^*(p^*(s+1);s+1)}{dp} = \frac{1}{u'[q^*(p^*(s+1);s+1)]}$$

which implies $u'[q^*(p^*(s);s)] > u'[q^*(p^*(s+1);s+1)]$, contradicting $u''(q) < 0$. Therefore it must be that $p^*(s) > p^*(s+1)$. $\quad |$

Theorem 10 establishes that regardless of the size of c_r and regardless of whether $V_e(s)$ rises or falls with an increase in s , the optimal reservation price, $p^*(s)$, is decreasing in s . The reason for this is that as the durability of the good increases, opportunities to correct for poor quality observations are postponed into the future. Thus, the consumer needs to be more selective prior to evaluating the good -- he invests relatively more in search and $p^*(s+1) < p^*(s)$ for all $s \geq 1$.

The question of whether $V_e(s)$ rises or falls as s increases is more difficult to answer. If $c_r = 0$ then $V_e(s) > V_e(s+1)$; as s increases there are no benefits to offset the cost of postponing repurchase decisions. As c_r rises, benefits began to weigh against these costs as total repurchase costs fall when s increases.

Conceivably, $V_e(s) < V_e(s+1)$ for some c_r .

A final question concerns the expected number of purchases before a satisfactory good is found. For a given s this is defined by the inverse of the probability that a satisfactory good is found

on a given purchase. This value is

$$(45) \quad E(N;s) = \{F(p^*(s))^{-1} \int_{p_a}^{p^*(s)} f(p)[1-G(q^*(p;s))]dp\}^{-1}.$$

As s increases there are two effects; p^* falls but $q^*(p;s)$ may rise or fall. This leaves $\Delta E(N;s)/\Delta s$ ambiguous. The precise expression is complicated, but it appears that the normal case is $\Delta E(N;s)/\Delta s < 0$; as durability increases, the expected number of purchases prior to finding a satisfactory good falls. At the same time, $p^*(s) > p^*(s+1)$ implies the expected number of searches or inspections prior to making an initial purchase increases when s rises; there is a shift toward inspection and away from evaluation when durability increases.

6. SUMMARY AND CONCLUSION

The key results of this paper are in sections 4 and 5. Section 5 analyzes the general model and bears out the contention in the introduction that Nelson's theory of search and experience is very restrictive. In particular, when goods are characterized by multiple characteristics and there is a natural ordering of observation of these characteristics (here enforced by assuming that quality cannot be observed prior to purchase), then there is no such thing as a search good; if it pays consumers to distinguish goods by price, then quality is never completely irrelevant. Similarly, price is never irrelevant unless all price and quality pairs are

acceptable. Suppose $p^* = p_b$. Then all prices are acceptable. But unless $q^*(p) = \bar{q}(p)$ for all $p \in [p_a, p_b]$, then price still matters in the sense that acceptable quality is sensitive to observed price. The only case in which acceptable quality is independent of the distribution of price is when $c_s = 0$ and $p^* = p_a$. In this case it is true that only quality is relevant to the decision as to which goods are acceptable, but the driving force in that conclusion is perfect information with respect to price.

Section 5 considers the important question of how the optimal policy responds to changes in the lifetime of goods. The main result there is that p^* declines as s rises, regardless of the relationship of $q^*(p;s)$ to $q^*(p;s+1)$. The "normal" case, however, is that in which $V_e(s) > V_e(s+1)$ and, as a consequence, $q^*(p;s) > q^*(p;s+1)$. An increase in durability in this case causes a shift away from investment in evaluation and toward investment in inspection.

Of course, this model suffers the same shortcoming of all one-sided search models, it doesn't explain what generates $\phi(p,q)$. The existence theorem (given in the appendix) shows that an optimal policy exists regardless of the specific form of $\phi(p,q)$. While uniqueness of p^* as well as some other properties of the optimal policy are sensitive to the form of $\phi(p,q)$, most of the results of section 4 are not affected. The same cannot be said for the results of section 5 (except perhaps in the case where $\partial B(p, V_e)/\partial p > 0$; p^* is unique, and high prices are acceptable).

As far as extensions of the present analysis are concerned, the natural direction is to allow for observing quality prior to

purchase at some finite cost. While this assumption is not valid for all dimensions of quality, there are certainly some trade-offs. As noted in the introduction, this is essentially the kind of problem analyzed by MacQueen [1962]. The major difference is that MacQueen did not allow for the possibility of new search at the end of the good's lifetime if quality turns out to be unsatisfactory. That is, after observing price at cost c_s the consumer could either (1) reject the good; (2) buy the good without observing quality; or (3) pay c_T and observe quality prior to purchase. In the last case, if the observed quality is low the good is rejected. If it is high, the good is purchased and search terminates forever. In case (2), where the good is purchased without observing quality, there is no opportunity for new search when quality turns out to be poor. Allowing for this possibility is an interesting extension of the present analysis. However, if some qualitative features involve very high inspection costs (i.e., if it is very expensive to observe these features prior to purchase) then the basic results of this paper will be preserved.

APPENDIX

This appendix gives the proof of theorem 1 on page 12.

Theorem 1: Given (A1)→(A6), there exists a unique continuous, bounded solution to the functional equation (9).

Proof: The proof of this theorem is a straightforward application of Denardo [1967]. Let $h(\cdot)$ be a continuous, bounded function on $[p_a, p_b]$. Consider the operator

$$(Th)(x) = -c_s + \max\{h_e, B(x, h_e)\}$$

where $B(x, h_e)$ is defined in (7) and (8) and h_e is the expected value of $h(x)$ with respect to the marginal p.d.f. $f(p)$. $(Th)(x)$ is clearly bounded and continuous since $U(p, q)$ is bounded and continuous. Therefore, if T is monotone and a contraction, then by Denardo [1967], there exists a unique $h^*(x)$ such that $(Th^*)(x) = h^*(x)$ for all $x \in [p_a, p_b]$.

i) T is monotone: Let $h^1(x)$ and $h^2(x)$ be continuous and bounded on $[p_a, p_b]$ such that $h^1(x) \geq h^2(x)$ for all $x \in [p_a, p_b]$. Then $h_e^1(x) \geq h_e^2(x)$ where h_e^1 is the expected value of $h^1(x)$ with respect to $f(p)$. Furthermore, $B(x, h_e^1) \geq B(x, h_e^2)$ for all $x \in [p_a, p_b]$. Then, $(Th^1)(x) \geq (Th^2)(x)$ for all $x \in [p_a, p_b]$.

ii) T is a contraction: Let $\rho(h^1; h^2) = \sup_{x \in [p_a, p_b]} |h^1(x) - h^2(x)|$.

then showing T is a contraction reduces to showing that $|h_e^1 - h_e^2| \leq \rho(h^1, h^2)$ and $|B(x, h_e^1) - B(x, h_e^2)| \leq k\rho(h^1, h^2)$ for some $0 < k < 1$ and any continuous, bounded functions on $[p_a, p_b]$, h^1 and h^2 . Clearly $f(p)$ non-degenerate implies $|h_e^1 - h_e^2| \leq \rho(h^1, h^2)$.

Now consider $|B(x, h_e^1) - B(x, h_e^2)|$. Integrating by parts and defining $\Psi(w|p) = G(\hat{q}(w, p)|p)$ where $\hat{q}(w, p)$ solves $w = U(p, \hat{q})$, this reduces to

$$|B(x, h_e^1) - B(x, h_e^2)| = \left(\frac{\beta^s}{1-\beta^s} \right) \left[\int_{h_e^1(1-\beta^s)+c_r}^{\bar{w}(x)} \Psi(w|x) dw - \int_{h_e^2(1-\beta^s)+c_r}^{\bar{w}(x)} \Psi(w|x) dw \right]$$

where $\bar{w}(x) = U(x, \bar{q}(x))$. Thus,

$$|B(x, h_e^1) - B(x, h_e^2)| \leq \frac{\beta^s}{1-\beta^s} \left| \int_{h_e^1(1-\beta^s)+c_r}^{h_e^2(1-\beta^s)+c_r} \Psi(w|x) dw \right|$$

But $\Psi(w|x) < 1$ over the above limits of integration. Thus,

$$|B(x, h_e^1) - B(x, h_e^2)| < \beta^s |h_e^1 - h_e^2|.$$

FOOTNOTES

1. These definitions of general and specific information obviously draw from Becker's [1975] work on human capital. The crucial feature of the analogy between training and information is that specific training involves an interaction between the worker and firm which is unique to each match-up; a worker cannot "buy" specific human capital short of actually taking a job in the firm associated with the training. On the other hand, general training, like general information, can be acquired without any special interaction with firms.
2. Prices and quality will be restricted to some compact subset of R^2 . $U(p,q)$ is defined over all positive values of p and q . Thus it will be possible to talk of the stream of utility obtained for any conceivable price/quality pair, even if the pair will never be observed.
3. There are two elements of "search" in this model. Inspection consists of drawing observations from $\phi(p,q)$ and observing p . Evaluation is the subsequent purchase of a good and the observation of q . Inspection is assumed to be a timeless activity, the only cost is the direct sampling cost, c_s . Evaluation is assumed to take time. This stems from the necessary consumption of the good and induces two types of cost onto evaluation activity; the opportunity cost of consuming at a low quality level and the postponement into the future of the possibility of rectifying the purchase of a poor quality good. Inspection is assumed to be a timeless activity in order to avoid confusing the latter effect.
4. It is implicit that price and quality combinations are such that it never pays to throw away a poor quality good and initiate new inspection prior to the end of the s -period lifetime of the good. Also, since inspection is timeless, it never pays to draw new samples from $\phi(p,q)$ (in the event that (p_0, q_0) is a poor price/quality pair) until the end of s periods.
5. These definitions take account of the fact that while $U(p,q)$ is bounded on $[p_a, p_b] \times [q_a, q_b]$, it is well defined for all $p \geq 0, q \geq 0$. See footnote 2.
6. Kohn and Shavell [1974] have included this case in their study of the theory of search. But in their model, as in Nelson's, there is no opportunity for the consumer to investigate only some aspects of the good; either the cost of search is defined to include the foregone opportunity cost of consuming at a low quality level, or it is not -- there is no possibility for the consumer to decide.

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