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MONOPOLY AND THE RATE OF EXTRACTION OF EXHAUSTIBLE RESOURCES: COMMENT*

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ABSTRACT

The contention that a monopolist exhausts a natural resource at a slower than socially optimal rate is examined for two cases: (1) fixed operation costs; and (2) demand elasticity increasing with output. Under either or both of these assumptions monopoly extraction rates may be biased in the opposite direction towards excessive resource use. On balance, it is concluded, the effect of monopoly ownership on relative extraction rates must be determined empirically. Furthermore we suggest that the addition of fixed costs into the analysis will tend to destroy the Pareto optimal properties of resource extraction under competitive conditions.

MONOPOLY AND THE RATE OF EXTRACTION
OF EXHAUSTIBLE RESOURCES: COMMENT

In a recent paper appearing in this Review, Stiglitz (1976) demonstrates under a set of familiar conditions that a monopoly owned non-replenishable resource will tend to be exhausted at a slower rate than is socially optimal.[1] This supports earlier views on the subject by writers like Hotelling (1931) and Solow (1974). Stiglitz shows under the natural "first approximation" assumptions of constant demand elasticity and zero extraction costs, that monopolistic and socially optimal (competitive) extraction rates are identical. By assuming demand elasticity increasing with time or constant unit production costs, decreasing with time, he shows that competitive extraction rates exceed monopolistic rates, at least for an initial period of time.

In this note we present realistic alternative extensions to the iso-elastic, zero cost analysis which tend to bias monopolistic extraction rates in the opposite direction; i.e., towards excessive resource use. The first modification allows for costs that do not vary with the extraction rate. Occurring in the form of land rents, capital costs, and maintenance fees, these quasi fixed costs[2], F, are incurred during each period of production and often constitute a substantial portion of operating expenses[3]. The second extension considers demand elasticities which vary with consumption instead of time. In particular since the marginal units of inputs utilized in small amounts are often more essential than the marginal units of inputs utilized in large quantities, demand elasticities that increase with the extraction rate are of interest. We show that

[1]Similar analysis comparing monopoly and socially optimal extraction rates appear in Kay and Mirrlees (1975), Lewis (1976), Sweeny (1976), and Weinstein and Zechhauser (1975). Kamien and Schwartz (1976) compare relative extraction rates in a general equilibrium setting.

[2]Fixed costs of this variety which can be avoided by stopping production were first recognized and categorized as "avoidable fixed costs" by Smith (1961, pp. 257-259).

[3]E. g., see Hendry (1961)

if demand elasticity increases with quantity, or if all costs are fixed then the monopolist tends to deplete the resource too soon.

Heuristically, positive fixed costs provide an incentive to shorten the period during which they are incurred. Since monopoly returns represent only a fraction of society's net utility (i.e. the monopolist fails to capture all of consumer surplus), while period costs (F) are the same, there is greater incentive for the monopolist to accelerate depletion and encounter fixed costs over a smaller number of periods. Unfortunately, no such compelling yet straightforward argument exists for the purpose of explaining the mechanics of increasing demand elasticity in accelerating monopolistic depletion.

It is also important to note that competitive ownership of the resource, generally, will not result in socially optimal production when fixed operation costs exist. This is because it will be optimal to restrict the number of mines operating at any time to reduce total fixed costs. For example with zero variable costs, least cost production requires that one mine operate at a time to minimize fixed costs. Yet with discounting there will always be an incentive for individual competitively owned mines to operate in current time periods. While several alternative forms of market intervention might limit the number of operating mines to the social optimum, in general the behavior of an unregulated competitive industry under conditions of positive fixed costs is difficult to assess and well beyond the scope of this short note.[4] Consequently, our analysis will focus on socially optimal and monopolistic programs of resource extraction.

Letting $p(q)$ be the inverse demand function for the resource, Q_o be the initial resource supply, and $q_s(t)$ and $q_M(t)$ be the socially optimal and monopolistic rates of extraction, the respective maximization

[4]This topic is currently being pursued by the authors in a subsequent manuscript.

problems for the social maximizer[5] and the monopolist are[6]:

(A) $$\underset{q_s(t),\,T_s}{\text{maximize}} \quad \int_0^{T_s} \left[\int_0^{q_s(t)} p(q)dq - F\right] e^{-rt}\,dt$$

$$\text{subject to} \quad \int_0^{T_s} q_s(t)dt \leq Q_o; \quad q_s(t),\ T_s \geq 0$$

(B) $$\underset{q_M(t),\,T_M}{\text{maximize}} \quad \int_0^{T_M} [p(q_M(t))q_M(t) - F]\, e^{-rt}\,dt$$

$$\text{subject to} \quad \int_0^{T_M} q_M(t)dt \leq Q_o; \quad q_M(t),\ T_M \geq 0$$

where r is the discount rate and T_s and T_M are the terminal extraction dates . Note that these terminal dates are choice variables.

Performing the indicated maximitations, manipulation of the necessary conditions for (A) and (B) yield, respectively:

(1) $$\dot{q}_s(q) = \frac{rp(q)}{p'(q)} = -r\,e(q)q$$

(2) $$\dot{q}_M(q) = \frac{rR'(q)}{R''(q)} = -r\,e(q)q\left[1 - \frac{e'(q)q}{e(q)-1}\right]^{-1}$$

where $R(q) = p(q)q$ is the revenue function, which we presume is concave, and $e(q)$ is the demand elasticity. We assume $e > 1$ to ensure positive monopolistic output, so that (1),(2),

[5]Subject to the usual caveats, the social maximizer is assumed to maximize consumer surplus, the area beneath the demand curve.

[6]If the resource is contained in several mines or wells, least cost production with zero variable costs requires that one mine operate at a time to minimize total fixed costs. Consequently, F represents the fixed operating costs for one mine.

and $e'(q) \geq 0$ imply[7]

$$0 > \dot{q}_s(q) \geq \dot{q}_M(q) \tag{3}$$

The necessary terminal time conditions can be expressed as[8]

$$q_M(T_M)f'(q_M(T_M)) = f(q_s(T_s)) = F, \tag{4}$$

where the function $f(q)$ is defined by

$$f(q) = \int_0^q p(x)dx - qp(q). \tag{5}$$

Note that $f''(q) = p'(q)\ e(q)^{-1} - p(q)e'\ e(q)^{-2} < 0$ since $e'(q) \geq 0$. The concavity of f together with $f(0) = 0$ and (4) imply[9]

$$q_M(T_M) \geq q_s(T_s) \tag{6}$$

Changing variables of integration from t to q in the resource constraint equations yields

$$Q_o = \int_{q_M(T_M)}^{q_M(0)} -[q/\dot{q}_M(q)]dq = \int_{q_s(T_s)}^{q_s(0)} -[q/\dot{q}_s(q)]dq \tag{7}$$

Consistency between (3), (6), and (7) requires that $q_M(0) \geq q_s(0)$; i.e., the monopoly initially extracts at a rate no slower than is

[7]Lest the point of this section be made vacuously we hasten to assert that demand functions satisfying these requirements exist. In particular if the social welfare function is $U(q) = \ln q + 2q^{1/2}$, then $dU/dq = q^{-1} + q^{-1/2} = p(q)$. From this one easily obtains $e > 1$ and $e' > 0$. Moreover $R''(q) < 0$ everywhere.

[8]Terminal time conditions are obtained by maximizing the Lagrange expression for this problem with respect to T_s and T_M.

[9]If $F = 0$ we have $T_M = T_S = \infty$ and $q_M(\infty) = q_S(\infty) = 0$. This follows because $e'(q) \geq 0$ implies $\lim_{q \to 0} p(q) = \infty$

socially optimal. Since inequality (3) is strict if $e'(q) > 0$, and inequality (6) is strict if $F > 0$, the initial monopoly extraction rate will be excessive in <u>either</u> case. From equation (3) the time path $q_M(T)$ crosses $q_s(T)$ at most once, and only from above. Thus the monopolist either extracts too fast for the entire extraction period before exhaustion (Figure 1), or too fast initially and too slow thereafter (Figure 2). In both cases, the resource remaining, $Q_M(t)$, is always less than socially optimal and depletion occurs too soon so that $T_M \leq T_s$.[10] This is clear for the case in Figure 1, and the fact that $q_M(0) > q_s(0)$, $q_M(T_M) = q_s(T_S) = 0$ and that the paths $q_M(T)$ and $q_s(T)$ intersect only once establish this result for the case in Figure 2.

Summarizing our argument, we have established the following: <u>Proposition</u>: In comparing monopolistic and socially optimal rates of extraction for the case of elastic demand and zero variable extraction costs, if either:

(a) $F > 0$, $e'(q) \geq 0$, or

(b) $F \geq 0$, $e'(q) > 0$

then monopolistic exploitation of the resource will be excessive in that:

(i) $T_M \leq T_s$ (with strict inequality for case (a))

(ii) $q_M(t) > q_s(t)$ initially, (for $0 < t \leq T_M$ when $F > 0$, $e'(q) = 0$)

(iii) $Q_M(t) < Q_s(t)$, $0 < t < T_s$

[10]For $F = 0$, we obtain the general result, $Q_M(t) \lesseqgtr Q_s(t)$ as $e'(q) \gtreqless 0$. The analysis for $e'(q) < 0$ is in Lewis (1976).

We have shown that the inclusion of either fixed costs or demand elasticity increasing with quantity causes the monopoly to deplete the resource faster than is optimal, while the Stiglitz extensions lead to the opposite conclusion. The net effect of all these presumably realistic extensions is analytically indeterminate and must be ascertained empirically.

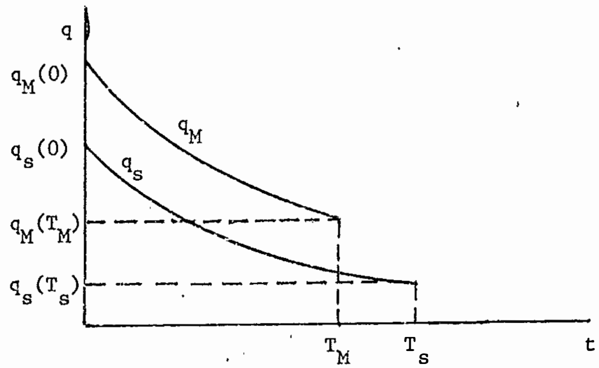

Figure 1. Non intersecting extraction paths (ex. iso-elastic demand and positive fixed costs).

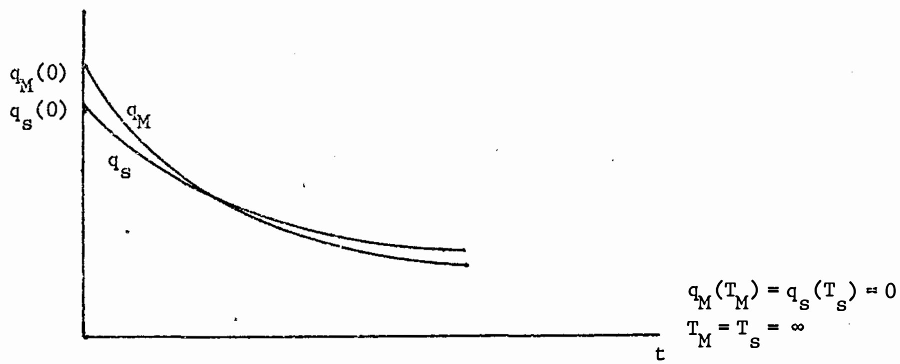

Figure 2. Intersecting extraction paths (ex. zero fixed costs and demand elasticity increasing with quantity).

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The social maximizer's problem is to maximize the discounted present value of consumers' surplus, subject to a resource constraint and nonegativity constraints; i.e.,

(A) $$\underset{q_s(t),\, T_s}{\text{maximize}} \quad \int_0^{T_s} \left[\int_0^{q_s(t)} p(q)dq - F\right] e^{-rt}\, dt$$

$$\text{subject to} \quad \int_0^{T_s} q_s(t)dt \leq Q_o;$$

$$q_s(t) \geq 0, \; T_s \geq 0$$

Applying the maximum principle to the Hamiltonian

$$H'_s \equiv H_s e^{rt} = \int_0^{q_s} p(q)\, dq - F - \lambda_s q_s,$$

we obtain the following necessary relations for an interior solution:

(1) $$\frac{\partial H'_s}{\partial q_s} = p(q_s) - \lambda_s = 0$$

(2) $$-\frac{\partial H'_s}{\partial Q_s} = 0 = \dot{\lambda}_s - r\lambda_s$$

(3) $$\lim_{t \to T_s} \left[\int_0^{q_s} p(q)\, dq - F - \lambda_s q_s\right] e^{-rt} = 0$$

Differentiation of (1) and the substitution of (2) into the result produces

(4) $$\dot{q}_s = r \frac{p(q_s)}{p'(q_s)}$$

The problem facing the monopolist is similar, with revenue, $R(q) = p(q)q$, replacing consumer's surplus, i.e.,

(B) $$\underset{q_M(t),\, T_M}{\text{maximize}} \quad \int_0^{T_M} [R(q) - F]\, e^{-rt}\, dt$$

$$\text{subject to} \quad \int_0^{T_M} q_M(t) \leq Q_o;$$

$$q_s(t) \geq 0, \; T_M \geq 0$$

Analogous to (1)-(3) above, the necessary conditions for an interior solution to this problem are

(5) $$R'(q_M) - \lambda_M = 0$$

(6) $$\dot{\lambda}_M - r\lambda_M = 0$$

(7) $$\lim_{t \to T_M} [R(q_M) - F - \lambda_M q_M]\, e^{-rt} = 0$$

Expressions (5) and (6) yield

(8) $$\dot{q}_M = r \frac{R'(q_M)}{R''(q_M)}$$

Throughout we shall make the usual assumptions that $p'(q) < 0$, $R'(q) > 0$, $R''(0) < 0$, and $\lim_{q \to 0} R(q) = 0$. If $e(q) = -d\ln q/d\ln p$ defines demand elasticity as a function of q, then $e(q) > 1$ if $R'(q) = p(q)[1 - e(q)^{-1}] > 0$.

<u>Lemma 1</u>: If $e'(q) \gtreqless 0$ for some $q > 0$,

$$\frac{p(q)}{p'(q)} \gtreqless \frac{R'(q)}{R''(q)}$$

Proof: Since $R''(q) = p'(1-e^{-1}) + pe'e^{-2} = (p'/p)R' + pe'e^{-2}$, we have

$$\frac{R''}{R'} = \frac{p'}{p} + \frac{pe'}{R'e^2}$$

The result now follows from our assumptions $R''<0$, $R'>0$, and $p'<0$.

From (4), (8), and Lemma 1, we have established

Lemma 2: Considering $\dot{q}_s(q)$ and $\dot{q}_M(q)$ as functions of q, we have $\dot{q}_s<0$, $\dot{q}_M<0$, and

(9) $$\dot{q}_s(q) \lesseqgtr \dot{q}_M(q) \quad \text{as} \quad e'(q) \lesseqgtr 0 \quad \text{for } q>0$$

For use in the next lemma, define

(10) $$f(q) = \int_0^q p(x)dx - R(q)$$

Observe that $f(0) = 0$.

Lemma 3: If $e'(q) \geq 0$ for $q>0$, then

$$q_M(T_M) > q_s(T_s) \text{ if } F>0$$

Furthermore, regardless of the sign of $e'(q)$, $q_M(T_M) = q_s(T_s) = 0$ if $F=0$

Proof: We first consider the case $F>0$. Inspection of the problems (A) and (B) reveals that never will the benefit flows $\int_0^{q_s} p(q)dq - F$ and $R(q)-F$ be negative. Hence if $F>0$, the continuity of $R(q)$ and $\int_0^{q_s} p(q)dq$ imply that $q_M(T_M)>0$ and $q_s(T_s)>0$. The resource constraint then implies T_M, $T_s<\infty$. Now (1), (3), (5), (7), and (10) yield

(11) $$q_M(T_M)\, f'(q_M(T_M)) = f(q_s(T_s)) = F,$$

since $f'(q) = p(q) - R'(q)$. Differentiation of $f(q)$ results in $f''(q) = p' - R'' = p'e^{-1} - pe'e^{-2}$, since we assume $e' \geq 0$, we have $f''(q)<0$. Since f is concave and $f(0)=0$,

(12) $$qf'(q) < f(q) \quad \text{for } q>0$$

Now (11), (12) and $F>0$ imply $q_M(T_M) > q_s(T_s)$.

For the case $F=0$, even if $T_s=\infty$ the resources constraint forces $\lim_{t\to\infty} q_s(t) = q_s(T_s) = 0$. Similarly, even if $T_M=\infty$ we have $q_M(T_M)=0$ and so $q_M(T_M)f'(q_M(T_M))=0$. Hence (11) holds for the case $F=0$ too. (10) and (11) imply $q_M(T_M)R'(q_M(T_M)) = R(q_M(T_M))$ when $F=0$, and hence the concavity of $R(q)$ yields $q_M(T_M)=0$. Because $p'(q)<0$, only at $q=0$ is $f(q)$ not positive, so (11) also implies that $q_s(T_s)=0$ when $F=0$.

Proposition 1: For $F=0$, the following is true:

(13) $$q_M(0) \gtreqless q_s(0) \quad \text{as} \quad e'(q) \gtreqless 0 \quad \text{for } q>0$$

For the case $F>0$ and $e'(q) \geq 0$, we still have $q_M(0) > q_s(0)$.

Proof: Clearly, in both (A) and (B) the resource constraint will be binding if extraction ever occurs at all. A change of integration variables in that constraint from t to q yields

(14) $$Q_o = \int_{q_M(T_M)}^{q_M(0)} -[q/\dot{q}_M(q)]dq = \int_{q_s(T_s)}^{q_s(0)} -[q/\dot{q}_M(q)]dq$$

If $F=0$, then $q_M(T_M) = q_s(T_s) = 0$ by Lemma 3.

By Lemma 2, we have

(15) $$\frac{1}{-\dot{q}_M} \lesseqgtr \frac{1}{-\dot{q}_s} \quad \text{as} \quad e'(q) \gtreqless 0 \quad \text{for } q \geq 0$$

Hence (14) and (15) imply (13) when $F=0$. If $F>0$, then $q_M(T_M) > q_s(T_s)$ by Lemma 3, and so (14) and (15) will also imply that $q_M(0) > q_s(0)$ if $e'(q) \geq 0$.

Proposition 2 (Stiglitz): If $F = 0$ and $e'(q) = 0$ for $q > 0$, then (i) $q_s(t) = q_M(t)$ for $t \geq 0$, and (ii) $T_M = T_s = \infty$

Proof: (i) is immediate from Lemma 2 ($\dot{q}_s(q) = \dot{q}_M(q)$)and Proposition 1 ($q_M(0) = q_s(0)$). From Proposition 1, $q_s(T_s) = q_M(T_M) = 0$, and for $e' = 0$ (4) and (8) yield $\dot{q}_s(q) = \dot{q}_M(q) = -rqe(q)$. Hence (ii), since only at ∞ can $q_s(t) = q_M(t) = 0$

Proposition 3: If $F > 0$ and $e'(q) = 0$ for $q > 0$, then

(i) $T_M < T_s$

(ii) $q_M(t) > q_s(t), \quad 0 \leq t \leq T_M$

(iii) $Q_M(t) > Q_s(t), \quad 0 < t < T_s$

If $F > 0$ and $e'(q) \geq 0$, then (i) and (iii) hold, and there exists a time T, $0 < T \leq T_M$, such that

(ii') $q_M(t) \gtreqless q_s(t)$ as $t \lesseqgtr T$ and $t < T_s$

If $F = 0$ and $e'(q) > 0$, then (ii') and (iii) are true, and (i) becomes

(i') $T_M = T_s = \infty$

Proof: For $F > 0$ and $e'(q) = 0$, (9) becomes $\dot{q}_s(q) = \dot{q}_M(q)$, so the two parts $q_s(t)$ and $q_M(t)$ are either identical or one lies entirely above the other while it is positive. But by proposition (13), we have $q_M(0) > q_s(0)$. Hence $q_M(t)$ is always greater than $q_s(t)$ while it is positive. (i), (ii), and (iii) now follow because of the constraint equations

$$Q_o = \int_0^{T_M} q_M(t)\, dt = \int_0^{T_s} q_s(t) dt.$$

For $F > 0$ and $e'(q) > 0$, (13) becomes $q_M(0) > q_s(0)$ and (9) becomes $0 < \dot{q}_s(t) < \dot{q}_M(t)$.

Thus $q_M(t)$ can cross $q_s(t)$ at most once and only from above; i.e, (ii'). Hence $Q_M(t) < Q_s(t)$ for the initial period when $q_M(t) > q_s(t)$. If (iii)were not true, then there must be a time $t_o < T_s$ such that $Q_M(t_o) = Q_s(t_o) > 0$ and $q_M(t_o) < q_s(t_o)$. This implies that either the monopolist or the social maximizer is not optimizing, since a recomputation of the problems (A) and (B) with Q_o replaced by $Q_M(t_o) = Q_s(t_o)$ would yield a contradiction to Proposition 1; i.e., $q_M(t_o) > q_s(t_o)$. Hence (iii) is true, and $T_M \leq T_s$. But since $q_M(T_M) > q_s(T_s)$ by Lemma 3, (ii') now implies (i) $T_M < T_s$.

If $F = 0$ and $e'(q) > 0$, the same argument shows (ii') and (iii). By Lemma 3, $q_M(T_M) = q_s(T_s) = 0$. Since $e'(q) > 0$, $R'(q) > 0$, and $R''(q) < 0$ imply

$$\lim_{q \to 0} p(q) = \lim_{q \to 0} R'(q) = \infty$$

(1) and (5) conjoined imply

$$\lim_{t \to T_s} \lambda_s(t) = \lim_{t \to T_M} \lambda_M(t) = \infty$$

Since (2) and (6) imply $\lambda_s(t) = \lambda_s(0)e^{rt}$ and $\lambda_M(t) = \lambda_M(0)\, e^{rt}$, respectively, (i') follows.

Proposition 4: If $F = 0$, then

$$Q_M(t) \lesseqgtr Q_s(t) \quad \text{as } e'(q) \lesseqgtr 0 \quad \text{for } q > 0$$

Proof: The proof follows that of Proposition 2 and, hence, is omitted.