DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

ENERGY VS. THE ENVIRONMENT

Tracy Lewis

SOCIAL SCIENCE WORKING PAPER 126

May 1976

ENERGY VS. THE ENVIRONMENT

Tracy Lewis

California Institute of Technology

ABSTRACT

Optimal development programs that explicitly account for the environmental impacts of extracting and consuming energy resources are analyzed. Following Lee and Orr (1975) we allow for the possibility of storing the resource above ground once it has been extracted. When environmental disruption results from resource extraction (as in the case of strip mining) or there are environmental costs associated with resource consumption (for example, the social costs of air pollution from fuel consumption) then the socially optimal rates of resource consumption and extraction depend on the severity of the environmental impact and on the prospects of storing the resource above ground.

ENERGY VS. THE ENVIRONMENT

Tracy Lewis
California Institute of Technology

INTRODUCTION

Energy development and environmental preservation are two important current national priorities.[1] Unfortunately, the two goals are sometimes incompatible with each other.[2] Certain lands are valued both for the natural resources and the environmental amenities they provide. Normally, productive inputs with multiple uses, like land, pose no real problems in a competitive free market economy. Use of the land goes to the highest bidder, the one for which the land is presumably most valuable. However, conservationists argue that with regard to the environment, the market is plagued with externality problems that preempt efficient resource allocation. Extraction activities like strip mining, deep hole mining, drilling

1. One has only to look at the proliferation of new federal agencies (like the EPA, FEA and ERDA) to deal with energy and environmental problems, at the quantity of new environmental and energy research projects and at the wealth of related literature to appreciate the public attention focused on these two goals. Of course public attention may soon shift to some other issues long before our energy and environmental problems are resolved. See the paper by Downs (1972) entitled: "The Issue-Attention Cycle and the Political Economy of Improving Our Environment" for an interesting discussion of this point.

2. Environmental and energy development interests have already collided on numerous issues including: the possible destruction of wildlife and the natural environs caused by the Alaskan pipeline, oil spills caused by offshore drilling, the destruction of the natural topography resulting from strip mining, possible underground water contamination in geothermal development, and so forth.

for oil and natural gas, and geothermal development may have adverse impacts on the natural environment during and even after the process has ceased. On the other hand, consumption activities like fuel burning contribute to air and water pollution. Consequently, government intervention is required to represent the interests of conservationists in decisions trading off environmental quality for energy development and consumption.

In what follows, we present an analysis of optimal resource allocation programs that explicitly account for environmental impacts. The rate of extraction and consumption for an exhaustible resource, like coal or oil, is chosen to maximize the discounted stream of the net economic returns. Primary attention in the paper is given to the case where environmental disruptions result from extraction. Under these circumstances, a fixed cost, independent of the extraction rate is incurred at each instant while the resource is being mined. This cost which captures the value of recreational and environmental services foregone because of mining, can be avoided only once extraction ceases and the environment is restored to its original state.[3] The effects on resource depletion of considering environmental impacts in extraction decisions is considered for two sets of circumstances. First for resources such as petroleum, we assume the resource must be consumed at the same rate it is pumped from the ground since the costs of storage are prohibitive. Second, following the analysis of Lee and Orr (1975) we assume above ground storage is feasible and that it is possible to

3. We assume environmental extraction costs, at least to a first degree approximation, are independent of the level of the extraction process. For example, the reduction in the recreational and environmental appeal of certain areas due to the construction of an oil well or the opening of a mine is independent of the rate of resource extraction. We also assume that environmental effects are not irreversible. Once the mine is closed or the oil well is removed the environment can be restored to its original state. Fisher et al (1972) and Arrow and Fisher (1975) analyze situations where economic development causes irreversible economic damages.

accumulate resource inventories. Lee and Orr show that if the unit extraction cost as a function of output is U-shaped, it may be economical to store resources as optimal extraction may exceed consumption in certain time periods. The U-shaped cost curve naturally occurs in our model as a result of the fixed environmental cost incurred each period.

In both the storage and no storage cases the inclusion of environmental extraction costs in the analysis serves to increase the rate of resource withdrawal and decrease the duration of the extraction process. If storage is impossible, consumption rates also increase and the resource is totally exhausted in a shorter time period. This suggests that in unregulated competitive markets, where environmental costs from extraction, are typically ignored, resources are being used at a slower than optimal rate.[4] If storage is feasible, we show that with environmental costs that resource consumption remains the same (increases) if marginal storage costs are constant (increase with larger inventories). Next, we demonstrate that increases in unit storage costs decrease the rate of extraction, but increase the rate of resource consumption.

For situations where there are adverse environmental impacts from resource consumption a per unit tax on resource use is levied to reflect external damages. The effect of the tax is to decrease the rate of resource consumption in both the storage and no storage cases. This is in contrast to the situation where the environmental costs from extraction cause a more rapid rate of resource use.

4. For interesting discussions of resource use under competitive conditions see Peterson (1972), Stiglitz (1974), and Weinstein and Zeckhauser (1975).

THE GENERAL MODEL WITH ENVIRONMENTAL EXTRACTION COSTS

Resource Demand

There exists a fixed and presumably known quantity Q_0 of the resource. We are interested in socially desirable programs of resource exploitation, and the optimality of the consumption stream is measured by the discounted sum of consumer plus producer surplus. In each period the demand function for resource consumption is

$$p(t) = \phi(q(t)) \; ; \; \phi' < 0 \tag{1}$$

where q(t) is the rate of resource consumption in period t and p(t) is the price. For now we will assume there are no environmental costs from using the resource. Consequently the value of resource consumption or consumer surplus is represented by

$$S(q(t)) = \int_0^{q(t)} \phi(\xi)d\xi \tag{2}$$

(subject to the usual proviso of constant marginal utility of income). Assuming storage is possible let e(t) and $\dot{I}(t)$ be the rate of extraction and the rate of change in resource inventories, I(t), in period t. Then

$$q(t) = e(t) - \dot{I}(t) \tag{3}$$

or $\dot{I}(t) = e(t) - q(t)$; inventories decrease (increase) as consumption rates exceed (fall short of) extraction rates.

Extraction and Storage Costs

The variable costs of extraction are represented by C(e(t)) with $C', C'' > 0$.[5] Fixed costs are incurred at a positive rate, F,

5. We are abstracting from "depletion effects" that cause extraction costs to rise as the resource stock diminishes.

as compensation for the recreational and environmental services preempted during the extraction process. Storage costs are represented by $W(I(t)$ with $W' > 0$ and $W'' \geq 0$.

Objective Function

Following Lee and Orr (1975) we can represent the present value of returns for an extraction and consumption program by

$$\pi = \int_0^{T_1} \{S(e(t) - \dot{I}(t)) - C(e(t)) - F - W(I(t))\}e^{-rt}dt + \int_{T_1}^{T_2} \{S(- \dot{I}(t)) - W(I(t))\}e^{-rt}dt \tag{4}$$

where r is the constant social rate of discount, T_1 and T_2 are the extraction and consumption time horizons, respectively, $T_1 \leq T_2$ and $e(t)$, $\dot{I}(t)$ and $I(t) \geq 0$. Note that environmental fixed costs, F, are incurred during the entire time horizon $[0, T_1]$.

OPTIMAL PROGRAMS WITH NO STORAGE AND ENVIRONMENTAL EXTRACTION COSTS

If above ground storage is not possible, the resource is extracted at the desired consumption rate. Once the extraction process begins environmental costs occur. Implicitly these costs represent a charge for leaving the resource in the ground until the time it is to be consumed. Naturally as environmental or underground storage costs increase, there is an incentive to hasten the extraction process to avoid costs.

To establish this formally, first we derive the conditions for an optimal extraction program choosing e(t) and T_1 to maximize π subject to the resource availability constraint

$$\int_0^{T_1} e(t)dt = Q_0 \tag{5}$$

Note that with no storage $I(t) = 0$ and $e(t) = q(t)$ for all t. The first order conditions for a maximum include the Euler equation,

$$\{S'(e(t)) - C'(e(t))\}e^{-rt} = \lambda \ ; \ t \in [0, T_1] \tag{6}$$

(where λ is the LaGrange multiplier attached to eq. (5)), the resource availability constraint, the nonnegativity of e(t), and the terminal time condition[6]

$$\{S(e(T_1)) - C(e(T_1)) - F\}e^{-rT_1} = \lambda \tag{7}$$

The multiplier λ is the value of having an additional unit of the resource available. Equation (6) indicates that in each period the present value of net returns are balanced off against the foregone value of future returns resulting from current extraction.

Differentiating equation (6) with respect to time we obtain

$$\dot{e}(t) = r\{\frac{S'(e(t)) - C'(e(t))}{S''(e(t)) - C''(e(t))}\} < 0. \tag{8}$$

Combining equations (6) and (7) we obtain

$$(S(e(T_1) - C(e(T_1)) - F)/e(T_1) = S'(e(T_1)) - C'(e(T_1)) \tag{9}$$

Thus according to equations (8) and (9) over the extraction horizon e(t) falls steadily to $e(T_1)$, the rate at which average and marginal returns are equated. Consider the optimal programs corresponding to different levels of fixed costs F_1 and F_2 with $F_1 < F_2$. Let $\{e(t, F_1), T_1(F_1)\}$ and $\{e(t, F_2), T_1(F_2)\}$ be the corresponding controls for these programs. Since both programs must satisfy the resource constraint

6. Equation (7) is derived by maximizing π subject to eq. (5) with respect to T_1.

$$\int_0^{T_1(F_1)} e(t, F_1)dt = \int_0^{T_1(F_2)} e(t, F_2)dt \tag{10}$$

By a change of variable we have

$$\int_{e(T_1(F_1))}^{e(0, F_1)} \frac{e}{-\dot{e}}de = \int_{e(T_1(F_2))}^{e(0, F_2)} \frac{e}{-\dot{e}}de \tag{11}$$

noting that from equation (8) $-\dot{e}$ is a positive function of e. Implicitly differentiating equation (9) we obtain $de(T_1(F))/dF > 0$ establishing that $e(T_1(F_2)) > e(T_1(F_1))$. But this implies that $e(0, F_2) > e(0, F_1)$ since the integrand on both sides of (11) is the same. Consequently equation (8) implies $e(t, F_2) > e(t, F_1)$ for $t \in [0, T_1(F_2)]$ and equation (10) therefore implies that $T_1(F_2) < T_1(F_1)$.[7, 8]

The effect of accounting for environmental impacts in extraction decisions is to increase the rate of resource depletion. This conclusion is bound to be questioned by those arguing that nonreplenishable resources are already being consumed too rapidly. The possibility of above ground storage, however, allows us to extend the consumption horizon for resources, as demonstrated in the next section.

---

7. To see that $e(0, F_1) < e(0, F_2)$ implies $e(t, F_1) < e(t, F_2)$ rewrite eq. (6) as

$$\{S'(e(t)) - C'(e(t))\} - e^{rt}\{S'(c(0)) - C'(e(0))\} = 0$$

and differentiate implicitly with respect to e(0) to obtain

$$\frac{de(t)}{de(0)} = -\frac{-e^{rt}\{S''(e(0)) - C''(e(0))\}}{\{S''(e(t)) - C''(e(t))\}} > 0.$$

8. Burness (1976) and Schmalensee (1976) has derived similar results in different contexts.

OPTIMAL PROGRAMS WITH STORAGE AND ENVIRONMENTAL EXTRACTION COSTS

For most resources above ground storage is possible at some positive cost. This allows for an additional degree of freedom in extracting and distributing the resource for consumption over time. Naturally, with increases in environmental or under ground storage costs there is still an incentive to speed up the extraction process. However, now consumption and extraction rates need not coincide. Resource consumption may be deferred to a more opportune time through storage.

The conditions for an optimal program with storage are derived by choosing e(t), q(t), T_1 and T_2 to maximize π subject to the resource availability constrain equation (5). Assuming an interior solution the first order conditions for a maximum include the Euler equations,[9]

$$\{S'(q(t)) - C'(e(t))\}e^{-rt} = \lambda \; ; t \in [0, T_1] \tag{12}$$

$$\frac{d}{dt}\{S'(q(t))e^{-rt}\} = W'(I(t))e^{-rt} \; ; t \in [0, T_2] \tag{13}$$

the resource constraint, equation (5), the nonnegativy of e(t) q(t) and I(t), the boundary conditions $I(0) = I(T_2) = 0$ and the terminal time conditions[10]

$$(C(e(T_1)) - F)/e(T_1) = C'(e(T_1)) \tag{14}$$

$$S(q(T_2)) = 0. \tag{15}$$

---

9. We are assuming that extraction proceeds fast enough relative to consumption so that the nonnegativity constraint on I(t) is not binding.

10. Equations (14) and (15) are derived (after some substitution and rearrangement) by maximizing π subject to equation (5) with respect to T_1 and T_2.

Effects of Environmental Costs on Optimal Programs

Along with Lee and Orr (1975) let us assume (for now) that total storage costs are proportional to inventory size and thus $W(I(t)) = \alpha I(t)$; $\alpha > 0$. Then, proceeding as in the no storage case we can demonstrate the "speeding-up" effect of increasing environmental costs on extraction programs.[11] The proof is omitted here because of its similarity with the no storage case. However, a somewhat surprising result (at least at first blush) awaits us when we examine the effect of environmental costs on the optimal consumption path and horizon.

From equation (13) and substituting for $W'(I(t))$ we obtain

$$\frac{d}{dt}\{S'(q(t))e^{-rt}\} = \alpha e^{-rt} \tag{16}$$

which implies

$$\dot{q} = \frac{\alpha + rS'(q(t))}{S''(q(t))} < 0. \tag{17}$$

According to the terminal time condition in equation (15)

$$q(T_2) = 0 \tag{18}$$

and the boundary conditions $I(0) = I(T_2) = 0$ imply

$$\int_0^{T_2} q(t)dt = Q^0. \tag{19}$$

11. This is true as long as the inventory constraint $I(t) \geq 0$ is not binding.

Taken together equations (17)-(19) are sufficient to determine T_2 and the entire time path of $q(t)$. Note that none of these equations depends explicitly on F and therefore the path $q(t)$ and T_2 are independent of changes in environmental costs.

Thus with a change in F, resource consumption proceeds at the same pace, despite increases in total above ground storage cost caused by the more rapid extraction of the resource. The reason for this is that inter-period consumption decisions are based on marginal quantities, and the marginal cost of storage is independent of the level of inventories. To see this, integrate equation (16) between 0 and t to obtain

$$S'(q(t)) = S'(q(0))e^{rt} + \frac{\alpha}{r}(e^{rt} - 1). \tag{20}$$

With some rearranging we obtain

$$e^{-rt}\{S'(q(t)) + \frac{\alpha}{r}\} = S'(q(0)) + \frac{\alpha}{r} . \tag{21}$$

The term $\frac{\alpha}{r}$ on both sides of equation (21) is the present value marginal cost of storing an additional unit of $I(t)$ for an infinitely long period. Equation (21) implies that resource consumption is allocated across time periods so as to equate the present value of marginal returns, including $\frac{\alpha}{r}$ the marginal storage cost avoided by current consumption.

If marginal storage costs increase with inventory size then both the extraction and consumption rates increase with increases in F. As greater quantities of the resource are extracted, inventories accumulate causing an increase in marginal storage costs. Consequently current consumption increases due to the higher costs of keeping resources for future consumption. Since the mathematics of the analysis are quite messy with variable marginal storage costs these results are formally established in the Appendix.

Effects of Increasing Storage Costs on Optimal Programs

Retaining the assumption of constant marginal storage costs we now examine the effect of an increase in α on optimal resource programs.

Extraction Programs

Consider the optimal program corresponding to different marginal storage costs α_1 and α_2 with $\alpha_1 < \alpha_2$. Let $\{e(t,\alpha_1), T_1(\alpha_1)\}$ and $\{e(t,\alpha_2), T_1(\alpha_2)\}$ be the corresponding controls for these programs. From equations (12) and (13) we know that $\frac{d}{dt}\{C'(e(t))e^{-rt}\} = \alpha e^{-rt}$ for $t \in [0, T_1]$ which implies

$$\dot{e}(\alpha_1) = \frac{\alpha_1 + rC'(e(t))}{C''(e(t))} > 0 \tag{22}$$

$$\dot{e}(\alpha_2) = \frac{\alpha_2 + rC'(e(t))}{C''(e(t))} > 0 \tag{23}$$

and from equation (14), $e(T_1(\alpha_1)) = e(T_1(\alpha_2))$. Since both programs must satisfy the resource availability constraint

$$\int_0^{T_1(\alpha_1)} e(t,\alpha_1)dt = \int_0^{T_1(\alpha_2)} e(t,\alpha_2)dt. \tag{24}$$

By a change of variable we have

$$\int_{e(0,\alpha_1)}^{e(T_1(\alpha_1))} \frac{e}{\dot{e}(\alpha_1)}de = \int_{e(0,\alpha_2)}^{e(T_1(\alpha_2))} \frac{e}{\dot{e}(\alpha_2)}de \tag{25}$$

noting that from equations (22) and (23), $\dot{e}(\alpha_1)$ and $\dot{e}(\alpha_2)$ are both positive functions of e. Since $\alpha_1 < \alpha_2$, for a given e, $\dot{e}(\alpha_1) < \dot{e}(\alpha_2)$ which implies $e(0_1\alpha_1) > e(0_1\alpha_2)$ in equation (25).

We can show that $e(t,\alpha_1) > e(t,\alpha_2)$ for $t \in [0, T_1(\alpha_1)]$ thus implying $T_1(\alpha_2) > T_1(\alpha_1)$ by equation (24). For if there exists some $t' \in [0, T_1(\alpha_1)]$ such that $e(t,\alpha_1) > e(t,\alpha_2)$ for $t \in [0,t']$ and $e(t',\alpha_1) = e(t',\alpha_2)$ we obtain

$$Q^0 - \int_0^{t'} e(t,\alpha_1)dt < Q^0 - \int_0^{t'} e(t,\alpha_2)dt$$

or

$$\int_{t'}^{T_1(\alpha_1)} e(t,\alpha_1)dt < \int_{t'}^{T_1(\alpha_2)} e(t,\alpha_2)dt \tag{26}$$

By a change of variable equation (26) implies

$$\int_{e(t',\alpha_1)}^{e(T_1(\alpha_1))} \frac{e}{\dot{e}(\alpha_1)}de < \int_{e(t',\alpha_2)}^{e(T_1(\alpha_2))} \frac{e}{\dot{e}(\alpha_2)} \tag{27}$$

which is impossible since $\dot{e}(\alpha_1) < \dot{e}(\alpha_2)$ as a function of e.

The intuition behind this result is that as above ground storage costs increase it is economical to store more of the resource under ground by slowing the rate of extraction.

Consumption Programs

Let $\{q(t,\alpha_1), T_2(\alpha_1)\}$ and $\{q(t,\alpha_2), T_2(\alpha_2)\}$ be the controls for the optimal consumption programs corresponding to the marginal storage costs α_1 and α_2 with $\alpha_1 < \alpha_2$. From equations (17) and (18) we obtain

$$\dot{q}(\alpha_1) = \frac{\alpha_1 + rS'(q(t))}{S''(q(t))} < 0 \tag{28}$$

$$\dot{q}(\alpha_2) = \frac{\alpha_2 + rS'(q(t))}{S''(q(t))} < 0 \tag{29}$$

and $q(T_2(\alpha_1)) \quad q(T_2(\alpha_2)) = 0$. From the resource availability constraint we obtain

$$\int_0^{q(0,\alpha_1)} \frac{q}{-\dot{q}(\alpha_1)}\,dq = \int_0^{q(0,\alpha_2)} \frac{q}{-\dot{q}(\alpha_2)}\,dq \tag{30}$$

which implies $q(0,\alpha_2) > q(0,\alpha_1)$ since $-\dot{q}(\alpha_1) < -\dot{q}(\alpha_2)$ as a function of q. Furthermore there exists some $t' \in (0, T_2(\alpha_2))$ such that

$$q(t,\alpha_1) \begin{matrix} > \\ = \\ < \end{matrix} q(t,\alpha_2) \text{ as } t \begin{matrix} < \\ = \\ > \end{matrix} t' \qquad \begin{matrix} (31a) \\ (31b) \\ (31c) \end{matrix}$$

and

$$T_2(\alpha_2) < T_2(\alpha_1) \tag{31d}$$

Since $q(0,\alpha_1) < q(0,\alpha_2)$ and $q(t)$ is continuous some t' exists for which $q(t',\alpha_1) = q(t',\alpha_2)$ otherwise $q(t,\alpha_1) < q(t,\alpha_2)$ for all $t < T_2(\alpha_2)$ which is impossible, thus establishing 31a and 31b.[12] Looking at figure 1 which plots the time paths for $q(t,\alpha_1)$ and $q(t,\alpha_2)$ it is apparent that conditions (31c) and (31d) then follow from the fact that $|\dot{q}(\alpha_1)| > |\dot{q}(\alpha_2)|$ for a given q.

12. Because of the time stationarity of our problem, the controls q(t) are continuous throught time.

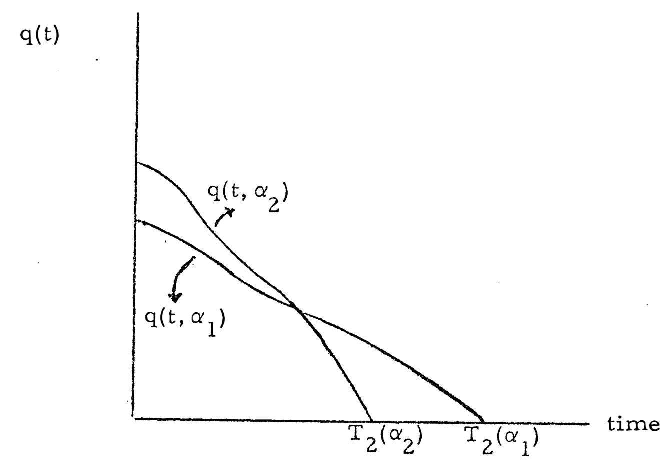

Figure 1

Conditions (31a) - (31d) imply that the total amount of the resource consumed as of a certain date increases with larger storage costs.

Consequently, it is economical to consume the resource more rapidly, but extract it from the ground at a slower pace when the above ground storage costs increase. The best of all worlds from both an environmental and energy conservation point of view occurs when above ground storage costs are small. Then the environmentalist is happy since the extraction horizons decrease as it becomes cheaper to store resources above ground.[13] Energy conservationists are pleased since lower storage costs promote longer consumption times.

13. One might argue that environmentalist should be indifferent to the length of the extraction horizon as long as the environmental costs are being paid. However, in practice the total compensation for environmental damages may not be forthcoming. Also, it is unlikely that those individuals who are actually harmed by the environmental disruption can be identified and fully compensated.

OPTIMAL PROGRAMS WITH ENVIRONMENTAL COSTS FROM RESOURCE CONSUMPTION

Now consider the case where a degrading of the environment occurs as energy resources are used. An example of this is air pollution resulting from fuel consumption. Suppose that the environmental damage from consumption is reflected by a constant per unit tax on energy use. Then the net value of resource consumption measured in consumer surplus terms is $S(q(t)) - \tau q(t)$ where the tax τ represents the environmental damage from consumption.

The inclusion of environmental damages in our model is formally equivalent to placing an ad valorem tax on energy sales. In the case where above ground storage is not possible, it is easy to show that the effect of such a tax is to decrease the rate of resource consumption.[14] This is also true for the storage case as can be formally demonstrated with a proof similar to that used in the previous section to show that consumption rates increase with larger storage costs.[15] Consequently, the interests of environmental and energy conservationists appear to coincide when the environmental impacts of resource consumption are included in allocation decisions.

---

14. See Burness (1976).

15. We are reminded that it will be economical to store resources only if there are fixed costs, F, to extraction.

CONCLUSION

In considering the environmental impacts of energy programs our results suggest that the interests of energy and environmental conservationists conflict or coincide depending on whether environmental costs result from extracting or consuming the resource. Our primary concern has been with the impacts of environmental extraction costs. The effect of including environmental extraction costs in allocations decisions is to cause an increase in resource extraction. Consumption of the resource need not increase if above ground storage is possible at constant marginal costs. If however, marginal costs increase with inventory size, increases in the environmental costs of extraction will ultimately lead to shorter consumption times. This result is bound to upset those energy conservationist who argue that current energy consumption is already excessive.

Of course no such problem exists if environmental impacts result from consuming rather than extracting the resource. The value of environmental amenities is hard to quantify, yet the tradeoffs between the environment and other goods like energy are apparent in our discussion. It would not be too surprising to see environmental preservation regarded (as it conflicts with resource extraction) more as a "luxury good" if the so-called energy crunch persists.

An interesting question we have not considered is the intergenerational conflict that may occur if environmental costs are included in extraction decisions. Imagine a situation in which energy is to be supplied in a quasi competitive market subject to government regulated assessments for environmental costs. All else being equal, those resources having the smallest environmental impacts from extraction will be taped first. In this regard future generations will be inheriting a stock of resources for which the environmental cost of extraction are high. To account for environmental impacts in the present, we may inadvertently leave future generations with a lower quality environment, one subject to more severe disruptions from energy development.

APPENDIX

Consider the storage model where marginal storage costs are sensitive to inventory levels and W', $W'' > 0$. We shall prove that as F increases both the extraction and consumption horizons decrease.

Let $\{e(t, F_i), q(t, F_i), T_1(F_i), T_2(F_i)\}$ be the optimal controls corresponding to F_i for $i = 1, 2$ with $F_1 < F_2$. First, it is clear that the optimal controls for each value of F must differ if for no other reason that the terminal time condition (14) implies $e(T_1(F_1)) < e(T_1(F_2))$ and that therefore the controls are not identical.

To establish $T_1(F_1) > T_1(F_2)$ define

$$\pi(F_i) = \int_0^{T_1(F_i)} \{S(e(t, F_i) - \dot{I}(t, F_i)) - C(e(t, F_i) - W(I(t, F_i))\}e^{-rt}dt$$

$$+ \int_{T_1(F_i)}^{T_2(F_i)} \{S(-\dot{I}(t, F_i)) - W(I(t, F_i))\}e^{-rt}dt \qquad (A.1)$$

$$- F_i \int_0^{T_1(F_i)} e^{-rt}dt$$

$$= V(F_i) + F_i\, \theta(F_i) \qquad (A.2)$$

where $V(F_i)$, equal to the first two terms on the right hand side of (A1), are the variable returns for an optimal program corresponding to F_i, and $F_i\theta(F_i) = F_i \int_0^{T_1(F_i)} e^{-rt}dt$ is the total discounted fixed costs corresponding to F_i.

Since the programs differ with fixed costs we have

$$\pi(F_1) > V(F_2) - F_1\theta(F_2) \qquad \text{(A. 3)}$$

$$\pi(F_2) > V(F_1) - F_2\theta(F_1). \qquad \text{(A. 4)}$$

From (A. 3) we have

$$V(F_1) - V(F_2) > F_1(\theta(F_1) - \theta(F_2)). \qquad \text{(A. 5)}$$

Assume, contrary to our assertion that $T_1(F_1) < T_2(F_2)$. Therefore $\theta(F_1) < \theta(F_2)$. Since $F_2 > F_1$, multiplying the right hand side of (A. 5) by F_2/F_1 still yields the same inequality and we have

$$V(F_1) - V(F_2) > F_2(\theta(F_1) - \theta(F_2)) \qquad \text{(A. 6)}$$

but it can be shown that this contradicts (A. 4) thus $T_1(F_1) > T_1(F_2)$.

Now we will show $T_2(F_2) < T_1(F_2)$. Suppose to the contrary that $T_2(F_2) \ge T_1(F_2)$. From equation (13) and (15) we can establish

$$\dot q(F_i) = \frac{W'(I(t, F_i) + rS'(q(t, F_i))}{S''(q(t, F_i))} < 0 \qquad i = 1, 2 \qquad \text{(A. 7)}$$

$$q(T_2(F_i)) = 0 \qquad i = 1, 2 \qquad \text{(A. 8)}$$

We have traced some possible paths for $q(t, F_i)$ in figure 2, assuming $T_2(F_2) > T_2(F_1)$.

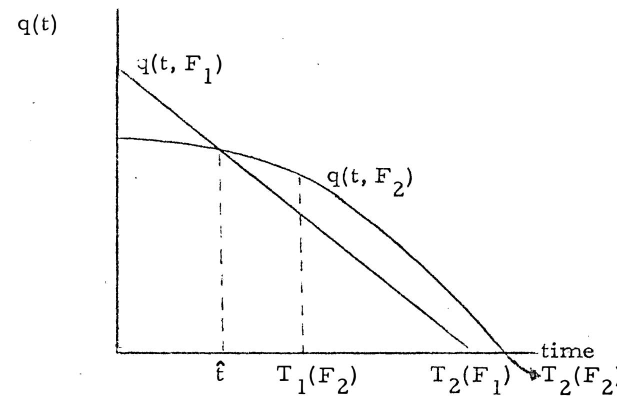

Figure 2

If $T_2(F_2) > T_2(F_1)$ then clearly there exists some $\hat{t} \in [0, T_2(F_2)]$ such that $q(t, F_2) > q(t, F_1)$ for $t \in (\hat{t}, T_2(F_2))$. It is also true that $q(t, F_2) < q(t, F_1)$ for some $t \notin (\hat{t}, T_2(F_2))$ otherwise $q(t, F_2) > q(t, F_1)$ for all t which is impossible. By continuity of q, we see that $q(\hat{t}, F_1) = q(\hat{t}, F_2)$ and therefore $\dot q(\hat{t}, F_2) \ge \dot q(\hat{t}, F_1)$, ($\dot q$ is represented by the slope of q(t) in figure 2). By (A. 1) this implies $I(\hat{t}, F_2) \le I(\hat{t}, F_1)$. Also $\hat{t} < T_1(F_2)$ otherwise if $\hat{t} \ge T_1(F_2)$ and $I(\hat{t}, F_2) \le I(\hat{t}, F_1)$ we have

$$I(\hat{t}, F_2) - \int_{\hat{t}}^{T_2(F_2)} q(t, F_2)dt < I(\hat{t}, F_1) + \int_{\hat{t}}^{T_1(F_1)} e(t, F_1)dt - \int_{\hat{t}}^{T_2(F_1)} q(t, F_1)dt = 0$$

which violates the $I(T_2(F_2)) = 0$ constraint,(note that $e(t, F_1) = 0$ for $t > T_1(F_1)$). Consequently, to satisfy the inventory constraint $I(t, F_2) > I(t, F_1)$ for $t \ge T_1(F_2)$.

Thus far we have established that:

$$I(\hat{t}, F_2) < I(\hat{t}, F_1) \tag{A.9}$$

$$I(T_1(F_2), F_2) > I(T_1(F_2), F_1) \tag{A.10}$$

$$\hat{t} < T_1(F_2). \tag{A.11}$$

To simplify the analyses (the proof still holds without this assumption) suppose $q(t, F_2)$ and $q(t, F_1)$ intersect only once at $\hat{t}$. Then figure 2, (A.9) and (A.10) imply

$$q(t', F_1) > q(t', F_2) \text{ for some } t' <$$

$$e(t', F_1) > e(t', F_2)$$

and

$$q(t'', F_1) < q(t'', F_2)$$

$$e(t'', F_1) < e(t'', F_2) \text{ for some } t'' e(\hat{t}, T_1(F_2))$$

but this can be shown to violate the first order condition in equation (12). Consequently $T_2(F_1) < T_2(F_2)$ is impossible. Note that the same proof applies if $T_2(F_1) = T_2(F_2)$ and $q(t, F_2) > q(t, F_1)$ for $t \in [\hat{t}, T_2(F_2)]$. Using the same type of proof we can also show that $T_2(F_1) = T_2(F_2)$ with $q(t, F_1) \leq q(t, F_2)$ for all t in some neighborhood of $T_2(F_2)$ is also impossible. Thus we have eliminated the possibility of $T_2(F_2) \geq T_2(F_1)$ and our proof is complete.

Arrow, K. J. and Fisher A. "Environmental Preservation Uncertainty and Irreversibility," Quarterly Journal of Economics 88 (May 1974): 312-19.

Burness, Stuart. "On the Taxation of Non-Replenishable Natural Resources," unpublished manuscript, University of Kentucky, 1976.

Downs, A. "The Issue-Attention Cycle and the Political Economy of Improving Our Environment," in The Political Economy of Environmental Control by Downs et al, 1972.

Fisher, A., et al. "The Economics of Environmental Preservation: A Theoretical and Empirical Analysis," American Economic Review 62 (September 1972): 605-19.

Lee, Dwight and Orr, Daniel. "The Private Discount Rate and Resource Convervation," Canadian Journal of Economics 8 (1975): 351-363.

Peterson, F. "Analysis of a Variational Model of Natural Resources Extraction," paper presented at the Toronto meetings of the Econometric Society, 1972.

Schmalensee, Richard. "Resource Exploitation Theory and the Behavior of the Oil Cartel," European Economic Review 7 (1976): 1-23.

Stiglitz, J. "Growth with Exhaustible Natural Resources: The Competitive Economy," Review of Economic Studies Symposium pp. 139-152, 1974.

Weinstein, M. and Zeckhauser, R. "The Optimal Consumption of Depletable Natural Resources," Quarterly Journal of Economics (1975): 371-392.