CALIFORNIA INSTITUTE OF TECHNOLOGY
Division of the Humanities and Social Sciences
Pasadena, California 91125

No not reorder
Send #183 in
place of this in
answer to requests

PRICE UNCERTAINTY AND THE HECKSCHER-OHLIN MODEL

Robert Forsythe

Social Science Working Paper
Number 118
June 1976

PRICE UNCERTAINTY AND THE HECKSCHER-OHLIN MODEL

Robert Forsythe

I. INTRODUCTION

There have been several recent attempts to incorporate uncertainty into international trade models.[1] All of these studies assume that markets exist only for currently available commodities and that no share trading is possible in order to provide for future consumption. Since without share trading there is no link between the behavior of the firm and the preferences of its stockholders, the reason that most of the standard theorems of trade theory under certainty fail to extend in these models is precisely due to this. Indeed, the general equilibrium trade models in nonstochastic environments assume (at least implicitly) that the profit of a firm is completely distributed among its stockholders. In this way, all wealth is distributed among consumers since nothing is taken out of the system by producers. It then follows immediately that producers in private-ownership economies should act so as to maximize their profits since that objective is in the best interests of each of their stockholders.

As we move to a general equilibrium model under uncertainty, Diamond [6] has shown that the proper objective of the firm becomes the maximization of its stock market value. If there is the equivalent of a complete set of contingent claim markets in the economy, value-maximizing decisions are agreed upon by all shareholders. In the absence of a sufficient number of markets, however, a difficulty frequently arises. Namely, stockholders may impute different sets of contingent claim prices and they will generally disagree about value-maximizing decisions. To circumvent this problem, it could be assumed that firms maximize the expected utility of profits.[2] We would argue that this is a departure from the perfectly competitive model since it is an expression of "the divorce of ownership from management."[3]

In this paper we will examine the effect of introducing price uncertainty into the Heckscher-Ohlin model while continuing to assume that we are dealing with private ownership economies in which consumers may also be stockholders. These economies will in general not possess the equivalent of a complete set of contingent claim markets, but only as many securities as there are firms. We will continue to parallel the complete market model in which firms seek to maximize their current stock market value by relying upon prices already implicit in the stock market. We have previously shown in [7] that there are no difficulties with this rule in the presence of price uncertainty if there are as many different firms as prices. This condition ensures that all individuals will impute the same certainty-equivalent to each price and will unanimously agree upon value-maximizing decisions. We will show that with a single random price, the standard two-section two-factor model satisfies this condition and we proceed to examine the traditional Heckscher-Ohlin model in this framework. Finally, we will show that by using the certainty-equivalent prices, the Rybczynski theorem, the Heckscher-Ohlin (HO) theorem, and the factor-price equalization theorem continue to hold.

II. THE GENERAL EQUILIBRIUM MODEL

We begin by using a state-preference version of a Fisherian model in which there is only a single commodity available for consumption now, c_0, or which may be invested in firms in order to provide for consumption later if state of the world θ occurs, c_θ, $\theta = 1, \ldots, S$. Also assume that there are I individuals in this economy, each possessing an endowment of the commodity, $\bar{c}^i$, and a portfolio consisting of fractions, $\bar{s}_j^i$, of each firm j. Each firm must choose the level of its decision variables, x_j, which determines the values of the firms, $V_j(x)$, and the returns each firm offers next period if state of the world θ occurs, $r_{j\theta}(x_j)$. Each individual then chooses a consumption-investment plan so as to maximize his expected utility,

$$EU^i(c_0^i, c_\theta^i)$$

subject to the budget constraints

$$c_0^i \leqq \bar{c}^i + \sum_{j=1}^{N} V_j(x)(\bar{s}_j^i - s_j^i) \tag{1}$$

$$c_\theta^i \leqq \sum_{j=1}^{N} r_{j\theta}(x_j)s_j^i, \qquad \theta = 1, \ldots, S \tag{2}$$

$$c_0^i \geqq 0, \qquad c_\theta^i \geqq 0, \qquad \theta = 1, \ldots, S.$$

Assuming sufficiently regular utility functions and letting short sales be unrestricted, the first-order conditions become

$$V_j(x)EU_0^i = EU_\theta^i r_{j\theta}(x_j), \quad j = 1, \ldots, N, \tag{3}$$

where

$$EU_0^i = \frac{\partial EU^i(c_0^i, c_\theta^i)}{\partial c_0^i} \quad \text{and} \quad U_\theta^i = \frac{\partial U^i(c_0^i, c_\theta^i)}{\partial c_\theta^i}, \quad \theta = 1, \ldots, S,$$

denote individual i's marginal utility for consumption now and for consumption later if state of the world θ occurs, respectively. The conditions, (3), when taken together with the market clearing conditions

$$\sum_{i=1}^{I} s_j^i = 1, \qquad j = 1, \ldots, N,$$

determine an equilibrium in the capital market at the prices V_j, $j = 1, \ldots, N$, for given decisions $x = (x_1, \ldots, x_N)$. We may rewrite (3) as

$$V_j(x) = \sum_{\theta=1}^{S} \rho_\theta^i r_{j\theta}(x_j) \tag{4}$$

where ρ_θ^i is individual i's contingent commodity price for consumption in state of the world θ, i.e.

$$\rho_\theta^i = \frac{U_\theta^i \pi_\theta^i}{EU_0}$$

and π_θ^i is individual i's subjective probability that state of the world θ may occur.

In this paper, we wish to examine this model where the returns of the firm in each state are simply the profit of the firm in that state. Further, we will assume that the only uncertainty that is present is price uncertainty. Thus, each firm's return function is of the form

$$r_{j\theta}(x_j) = w(\theta) f^j(x_j), \qquad \theta = 1, \ldots, S, \quad j = 1, \ldots, N, \tag{5}$$

where $w(\theta)$ is a 1 x K vector of prices which occur in state θ and $f^j(x_j)$ is a K x 1 vector of state-independent decision functions of firm j. We will further assume that no random price is perfectly correlated with the set of other random prices. To see that this assumption is not particularly restrictive, suppose that $w_K(\theta)$ is perfectly correlated with $\{w_1(\theta), \ldots, w_{K-1}(\theta)\}$. Then there exist constants $a_1, \ldots, a_{K-1}$, such that

$$w_K(\theta) = \sum_{j=1}^{K-1} a_j w_j(\theta), \qquad \theta = 1, \ldots, S,$$

and $w_K(\theta)$ may be replaced by this sum in (5).

In this model, firms seek to maximize their value, which is given by substituting (5) into (4) as

$$V_j = \sum_{\theta=1}^{S} \rho_\theta^i \sum_{k=1}^{K} w_k(\theta) f_k^j(x_j)$$

$$= \sum_{k=1}^{K} f_k^j(x_j) \sum_{\theta=1}^{S} \rho_\theta^i w_k(\theta).$$

Although individuals have imputed different sets of contingent claim prices, if they agree upon each certainty-equivalent price given by

$$\tilde{w}_k = \sum_{\theta=1}^{S} \rho_\theta^i w_k(\theta), \qquad k = 1, \ldots, K, \quad i = 1, \ldots, I, \tag{6}$$

the value of a firm is given by

$$V_j = \sum_{k=1}^{K} \tilde{w}_k f_k^j(x_j)$$

and value-maximizing decisions may be determined from

$$\sum_{k=1}^{K} \tilde{w}_k \frac{\partial f_k^j(x_j)}{\partial x_j} = 0. \quad (7)$$

The solution to (7) will be the same independently of which stockholder's contingent claim prices are used.

We have seen in an earlier paper, [7], that (6) will hold if there are as many independent firms[4] as random prices.[5] In this framework, the certainty-equivalent price, $\tilde{w}_k$, may be interpreted as the value each individual would pay in the current period for an asset which returned the random price, $w_k(\theta)$, in the next period. We should also note at this point that if one of the prices is nonstochastic, then all individuals in this model will also agree upon $\sum_{\theta=1}^{S} \rho_\theta^i$, which is the price they would pay for a risk-free asset which would yield one dollar in every state of the world next period.[6]

III. THE HECKSCHER-OHLIN MODEL

In this section, we will alter the traditional Heckscher-Ohlin model to allow for one of the output prices to be random. As usual, we will begin by examining those factors which determine the relative commodity prices in an economy. To do this, we shall need to maintain the usual assumptions that are made in this type of analysis.

1. Given endowments of two factors, K and L (say, capital and labor) may be utilized to produce two commodities, y_1 and y_2. These goods are produced using linear homogeneous and concave production functions, $F_1(K_1, L_1)$ and $F_2(K_2, L_2)$, under conditions of perfect competition, full employment, inelastic factor supplies, and irreversible factor intensities.

2. The price of the first commodity, X_1, is random and given by $p_1(\theta)$ and the price of the second commodity, p_2, is nonstochastic. Decisions on the amount of K and L to be employed must be made prior to the realization of $p_1(\theta)$.

3. As discussed in the previous section, both producers wish to maximize the value of their shares. We shall assume that after the realization of the state of the world, θ, the profits of each firm are completely distributed among their shareholders. Letting $\pi_1(\theta)$ be the profits of the first industry if state of the world θ occurs, then

$$\pi_1(\theta) = p_1(\theta)F_1(K_1, L_1) - wL_1 - rK_1,$$

where w is the wage rate and r the rental rate of capital. The profits of the second industry, π_2, are nonrandom and given by

$$\pi_2 = p_2F_2(K_2, L_2) - wL_2 - rK_2.$$

Taking prices as given, firms in the first industry act to maximize their value

$$V_1 = \sum_{\theta=1}^{S} \rho_\theta^i \pi_1(\theta) = \sum_{\theta=1}^{S} \rho_\theta^i [p_1(\theta)F_1(K_1, L_1) - wL_1 - rK_1]$$

$$= F_1(K_1, L_1) \sum_{\theta=1}^{S} \rho_\theta^i p_1(\theta) - (wL_1 - rK_1) \sum_{\theta=1}^{S} \rho_\theta^i.$$

Similarly, producers in the second industry wish to maximize

$$V_2 = \sum_{\theta=1}^{S} \rho_\theta^i (p_2F_2(K_2, L_2) - wL_2 - rK_2).$$

Since there are two independent firms, it should be clear that this model satisfies the conditions given in the preceding section which ensure unanimity regarding

$$\sum_{\theta=1}^{S} \rho_\theta^i p_1(\theta) \text{ and } \sum_{\theta=1}^{S} \rho_\theta^i .$$

Let

$$\tilde{p} = \sum_{\theta=1}^{S} \rho_\theta^i p_1(\theta)$$

denote the certainty-equivalent random price, and

$$\tilde{q} = \frac{1}{r} = \sum_{\theta=1}^{S} \rho_\theta^i$$

be the certainty equivalent risk-free price, since r is the riskless rental rate per unit of capital and thus must equal the risk-free rate of return.

We will use as the numeraire the quantity $\tilde{q}p_2$, which is the value in the present period of an asset which returns p_2 next period in every state of the world. This choice of numeraire has the effect of discounting all prices in the future period into current period terms. Thus, the values of firms in these two industries may be alternatively written as

$$V_1 = \tilde{p}^* F_1(K_1, L_1) - w^* L_1 - r^* K_1$$

$$= \tilde{p}^* L_1 f_1(k_1) - w^* L_1 - r^* K_1$$

and

$$V_2 = F_2(K_2, L_2) - w^* L_2 - r^* K_2$$

$$= L_2 f_2(k_2) - w^* L_2 - r^* K_2$$

respectively, where the starred prices denote real prices (in current period terms) and k_i is the capital-labor ratio in the ith industry.

The two first-order conditions for value maximization by firms in the first industry are

$$\tilde{p}^* f_1'(k_1) - r^* = 0 \qquad (8)$$

and

$$\tilde{p}^*[f_1(k_1) - k_1 f_1'(k_1)] - w^* = 0. \qquad (9)$$

Of immediate interest here should be the fact that, just as in the certainty model, the value of the firm in equilibrium is zero.[7] To see this, substitute (8) into (9), giving

$$0 = \tilde{p}^* f_1(k_1) - k_1 r^* - w^*$$

$$= \tilde{p}^* L_1 f_1(k_1) - w^* L_1 - r^* K_1$$

$$= V_1.$$

Likewise, for firms in the second industry the value-maximizing conditions are given by

$$f_2'(k_2) - r^* = 0 \qquad (10)$$

and

$$f_2(k_2) - k_2 f_2'(k_2) - w^* = 0. \qquad (11)$$

Again we see that the values of firms in this industry are also zero in equilibrium.

Since factor rewards will be identical with perfectly competitive markets, (8), (9), (10), and (11) yield the factor market equilibrium conditions

$$\tilde{p}^* f_1' = f_2' \qquad (12)$$

and

$$\tilde{p}^*(f_1 - k_1 f_1') = f_2 - k_2 f_2' \qquad (13)$$

Finally, the model may be closed using the full employment assumption which requires that

$$L_1 + L_2 = L \qquad (14)$$

FOOTNOTES

1. Most notable are the contributions by Batra [1, 2], Brainard and Cooper [3], and Turnovsky [13].

2. As in Batra [1, 2].

3. This has previously been pointed out by Radner [12].

4. In this framework, independence of firms requires that the state-distribution of returns offered by a firm cannot be written as some linear combination of returns of all other firms.

5. To see this, substitute (5) into the first-order conditions, (4), which gives

$$V_j(x) = \sum_{\theta=1}^{S} \rho_\theta^i \sum_{k=1}^{K} w_k(\theta) f_k^j(x_j), \qquad j = 1, \ldots, N \qquad \theta = 1, \ldots, S.$$

If there are K independent firms, then this provides us with a s K equations in K unknowns, $\sum_{\theta=1}^{S} \rho_\theta^i w_k(\theta)$. Since each individual i confronted with the same set of equations, they must necessaril agree upon each certainty-equivalent random price, i.e.

$$\sum_{\theta=1}^{S} \rho_\theta^i w_k(\theta) = \tilde{w}_k, \qquad k = 1, \ldots, K \qquad i = 1, \ldots, I.$$

6. At this point, it should be noted that this result could also have been derived by using the "unanimity" framework of Ekern [5], Ekern and Wilson [6], Leland [8, 9], and Radner [11]. This was previously pointed out in [7].

7. This result may at first seem paradoxical but it is the exact analogy of the certainty model in which firms possessing linear homogeneous production functions earn zero profits in equilibrium. A stronger version of this is proved in [7], where we show that, independent of the functional form chosen, as long as a firm's return function is linear homogeneous, the value of that firm in equilibrium must be zero.

8. See, for instance, the proof given by McKenzie [10].

REFERENCES

1. Batra, R. N. "Production Uncertainty and the Heckscher-Ohlin Theorem," Review of Economic Studies, 42 (April 1975), 259-267.

2. Batra, R. N. The Pure Theory of International Trade under Uncertainty (Halstead Press, New York, 1975).

3. Brainard, W. C. and Cooper, R. N. "Uncertainty and Diversification in International Trade," Food Research Institute Studies in Agricultural Economics, Trade and Development, 8 (1968), 257-285.

4. Diamond, P. A. "A Stock Market in a General Equilibrium Model," American Economic Review, 52 (September 1967), 759-776.

5. Ekern, S. "Some Aspects of Firms' Decision Making in an Economy with Incomplete Capital Markets," Swedish Journal of Economics, 76 (March 1974), 117-130.

6. Ekern, S. and Wilson, R. "On the Theory of the Firm in an Economy with Incomplete Markets," Bell Journal of Economics and Management Science, 5 (Spring 1974), 171-180.

7. Forsythe, R. "Unanimity and the Theory of the Firm under Multiplicative Uncertainty," Social Science Working Paper No. 90, California Institute of Technology, 1975.

8. Leland, H. E. "Capital Asset Markets, Production and Optimality: A Synthesis," Technical Report No. 115, Institute for Mathematical Studies in the Social Sciences, Stanford University, 1973.

9. Leland, H. E. "Production Theory and the Stock Market," Bell Journal of Economics and Management Science, 5 (Spring 1974), 125-144.

10. McKenzie, L. W. "Equality of Factor Prices in World Trade," Econometrica, 23 (July 1955), 239-257.

11. Radner, R. "A Note on Unanimity of Stockholders' Preferences Among Alternative Production Plans: A Reformulation of the Ekern-Wilson Model," Bell Journal of Economics and Management Science, 5 (Spring 1974), 181-184.

12. Radner, R. "Problems in the Theory of Markets under Uncertainty," American Economic Review, 60 (May 1970), 454-460.

13. Turnovsky, S. J. "Technological and Price Uncertainty in a Ricardian Model of International Trade," Review of Economic Studies, 41 (April 1974), 201-217.