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POPULATION PRESSURE AND FERTILITY CHANGES IN COSTA RICA, 1906-1970

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SOCIAL SCIENCE WORKING PAPER 109

January 1976

ABSTRACT

The demographic history of Costa Rica in the twentieth century is examined in the context of a model of dynamic adjustment to changing child survival probabilities and micro-level population pressure. Micro-level population pressure is viewed as resulting from a couple having children beyond its current optimal family size, given current prices and its income. Cantonal regression analyses for the time periods, 1927-1950, 1951-1953 to 1961-1963, and 1961-1963 to 1970 lend support to the hypothesis that the secular fertility decline in Costa Rica is a dynamic adjustment to high completed family size and increasing child survival probabilities.

INTRODUCTION

Recent work on the economic theory of fertility has focused almost exclusively on attempts to explain fertility fluctuations in countries which have already substantially completed the demographic transition. However, work by Easterlin (1968) on the United States fertility decline in the nineteenth century; by T. Paul Schultz (1973) on Taiwan, and by Nerlove and Schultz (1970) on Puerto Rico suggest that a theory of household decision making might be useful in explaining secular fertility declines in developing countries. This paper examines the demographic experience of Costa Rica in the context of a household decision making model which focuses on the responses of individual families to what we term micro-level population pressure and tests how well the hypotheses generated by that model explain both trends and fluctuations in Costa Rican fertility.

COSTA RICAN FERTILITY TRENDS, 1906-1970

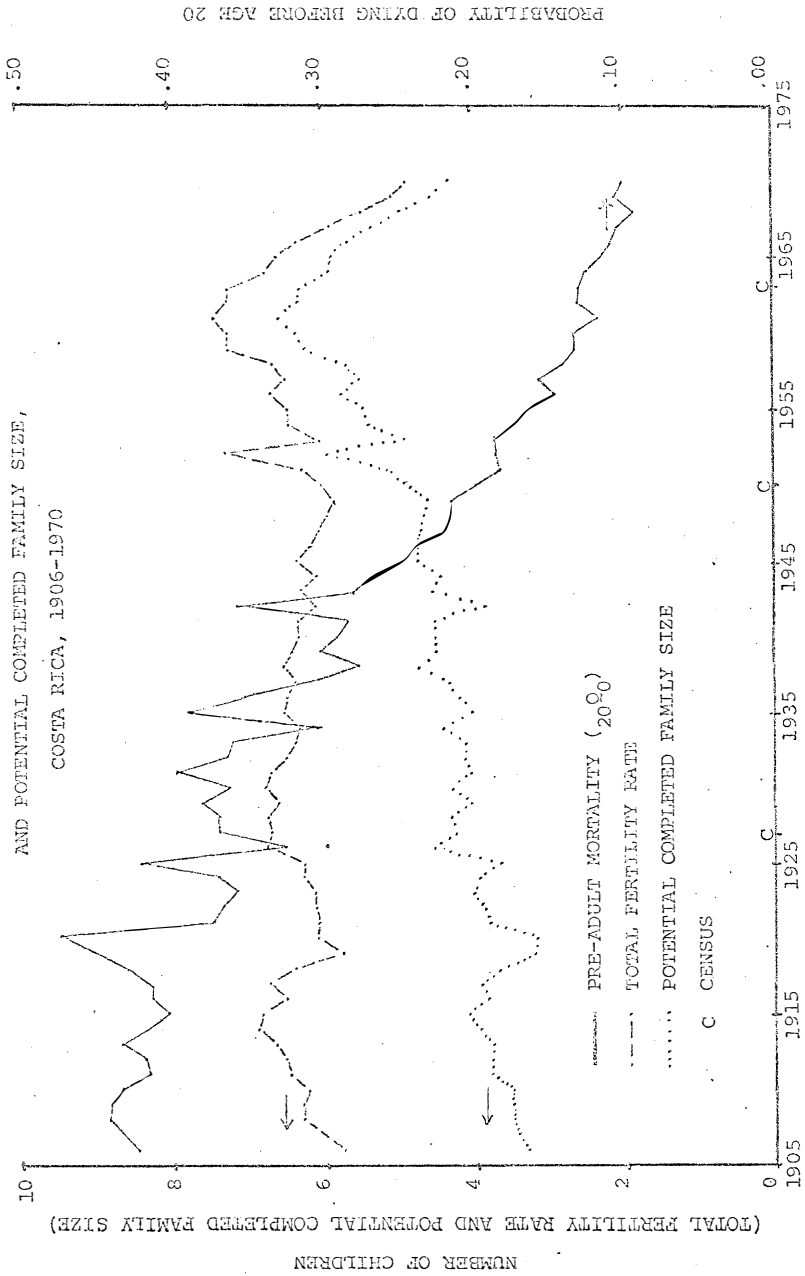
Many attempts to study secular fertility changes in less-developed countries cannot go beyond the purel descriptive stage because of lack of data, both cross-sectional and temporal. While the Costa Rican data are far from perfect and contain many important gaps, there are sufficient data to fairly accurately reconstruct the trends in fertility and mortality for Costa Rica and its provinces and cantons in the twentieth century. Age-specific population is available by canton from the 1892, 1950, and 1960 censuses and nationally from the 1927 census. (This research was completed prior to the publication of the 1973 census and therefore does not include those data.) Age-specific deaths and total births are available annually by canton from 1890 and births by age of mother by province from 1953.

Collver (1965) provides evidence that the national data from the 1892 and 1927 censuses were substantially correct and that, overall, the 1950 census was underenumerated by about five percent. Surviving the age-specific populations between censuses using age-specific deaths indicates that the national death registration is also substantially correct. Only the birth registration data present any problem. While the high degree of literacy and political stability in Costa Rica ensure that nearly all births are registered at some time, births are often registered many years after their occurrence, thus complicating an analysis of short-run fluctuations in fertility. Further, registration drives during election years tend to produce spurious fertility peaks as people reaching twenty-one years of age find they must have birth certificates to vote. Several attempts have been made to adjust fertility data to account for late birth registrations (Collver, 1965; United Nations, 1961). Collver's adjusted estimates are presented below for comparison with our demographic estimates, which are not adjusted for reporting bias.

Figure 1 sets out annual time series for Costa Rica since 1906 on the probability of dying before age 20, total fertility rate, and potential completed family size. Potential completed family size is the product of the total fertility rate and the probability at birth of surviving to age 20 and represents the average completed family size which would result if current

FIGURE 1

TOTAL FERTILITY RATES, PRE-ADULT MORTALITY, AND POTENTIAL COMPLETED FAMILY SIZE, COSTA RICA, 1906-1970



Source: estimated from censuses and annual vital statistics (Appendix A).

Limon excluded, 1906-1927

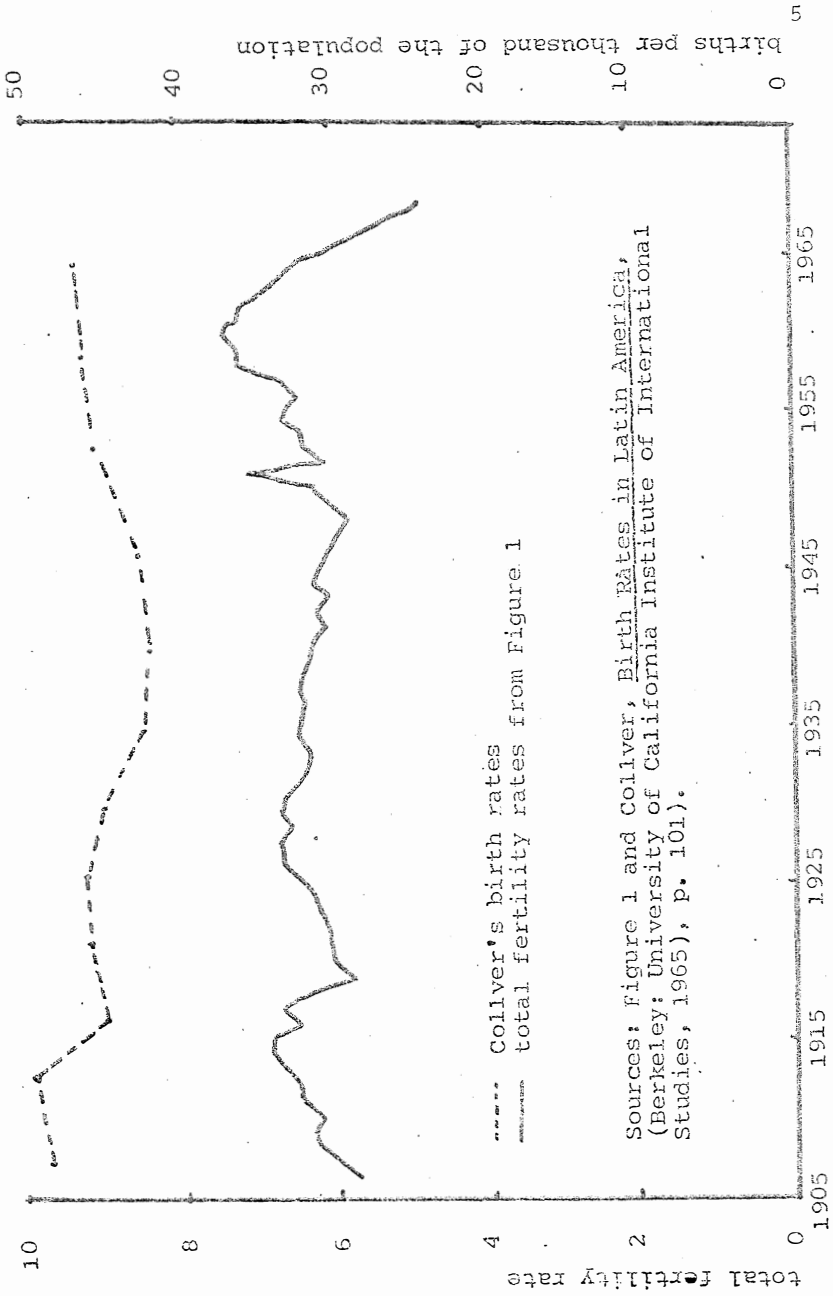
mortality and fertility persisted. (Appendix A provides a discussion of the estimation procedure.)

Pre-adult mortality declines unevenly to the 1940s and more rapidly and uniformly thereafter, leveling off at the end of the period. Fertility declines slowly and steadily from a peak in the 1920s to 1950, then rises rapidly to 1961. After 1961, it declines precipitously to levels lower than at any previous time in this century. Potential completed family size rises slowly between the mid-1920s and 1950. It increases more rapidly in the 1950s as fertility rises and the mortality decline accelerates. Between 1961 and 1970 potential completed family size declines rapidly, to near the level prevailing between 1906 and 1925. A comparison of these fertility and mortality estimates with Collver's (1965) adjusted figures for 1905-1909 to 1955-1959 (see figure 2) indicates that the timings of the adjusted and unadjusted fertility swings correspond quite closely. On that basis we accept our unadjusted national figures as usable for our analysis. Where we use cantonal data for hypothesis testing we will treat the problems caused by data quality (see pp. 17-24).

The demographic trends since 1927 can be separated into three distinct periods. 1927 to 1950 is a period of declining pre-adult mortality, slower declining fertility, and rising potential completed family size. Between 1950 and 1961 fertility and potential completed family size rise dramatically, while pre-adult mortality declines rapidly. 1961 to 1970 is characterized by a slower decline in pre-adult mortality, and rapidly declining fertility and potential completed family size. In the remainder of this paper we develop a model to explain the observed demographic behavior of Costa Rica and test this model with canton-level demographic data.

Figure 2

COMPARISON OF TOTAL FERTILITY RATES
FROM FIGURE 1 WITH COLLVER'S ADJUSTED
BIRTH RATES, 1906-1970



Sources: Figure 1 and Collver, Birth Rates in Latin America, (Berkeley: University of California Institute of International Studies, 1965), p. 101).

A MODEL OF ADJUSTMENT TO FAMILY LEVEL POPULATION PRESSURE AND CHANGING CHILD SURVIVAL PROBABILITIES

A model which is to explain the Costa Rican experience must account for both a secular fertility decline and rather extreme short-run variations in fertility. The following model builds upon the household model of Becker (1960); and the work of T. Paul Schultz (1969, 1973) on economic models of family planning, Easterlin (1968, 1973) on the American baby boom and responses to economic stress, Davis (1963) on the theory of multi-phasic response, and Namboodiri (1972) and Lindert (1973) on decision making with regard to the use of contraceptives.

The standard household decision making model is a one-period model in which a couple enters family formation with perfect knowledge regarding prices, future income, and tastes for children and other goods, with child mortality assumed constant or absent. While this kind of model has yielded significant empirical results using recent United States data, it is probably unsatisfactory to explain the experience of a developing country undergoing sustained declines in pre-adult mortality and fertility. While the lack of cross-sectional economic data for Costa Rica does not allow the testing of a comprehensive model, we offer the following somewhat simplified model in keeping with the available data.

Building upon Lindert's (1973) and Namboodiri's (1972) sequential analyses, we might think of a couple deciding sequentially for each time period, whether or not they wish to have a child in that time period. While our discussion refers to a couple, we recognize that an individual can also decide to have or not to have a child. We assume that a couple's preferences for goods, child quality, numbers of children, and contraceptive practices can be represented by a quasi-concave utility function

and that the couple behaves as though it is attempting to maximize that function. The decision at the beginning of a time period whether to have an additional child at that time will involve weighing the utility of an additional child of a desired quality against the utility from the next best uses of the income which would go to that additional child. These alternatives would include both goods for parents and inputs to increase the quality of already surviving children.

The assumption of quasi-concavity implies that at each utility level, as parity increases, a family will wish to give up fewer and fewer units of goods for parents and child quality in order to get an additional child. Thus, each child that dies lowers the marginal rate of substitution of an additional child relative to child quality and goods for parents, and each child that lives raises it, ceteris paribus. A family which was indifferent between having and not having the last child, therefore, should not wish to have an additional child unless another child should die.

As Lindert (1973) and Namboodiri (1972) point out, however, parents cannot decide whether or not to have another child in the next time period, but must actually choose some contraceptive strategy. A couple seeking to maximize utility will trade the psychic and monetary costs associated with using different contraceptive strategies against the probability of having an additional child while using each strategy, and will choose that strategy which yields the highest expected utility. The utility maximizing strategy, therefore, may not be the most efficient form of contraception. For example, total abstinence, which is a perfectly efficient form of contraception carrying no monetary costs, may have little appeal to the couple because of its high psychic cost.

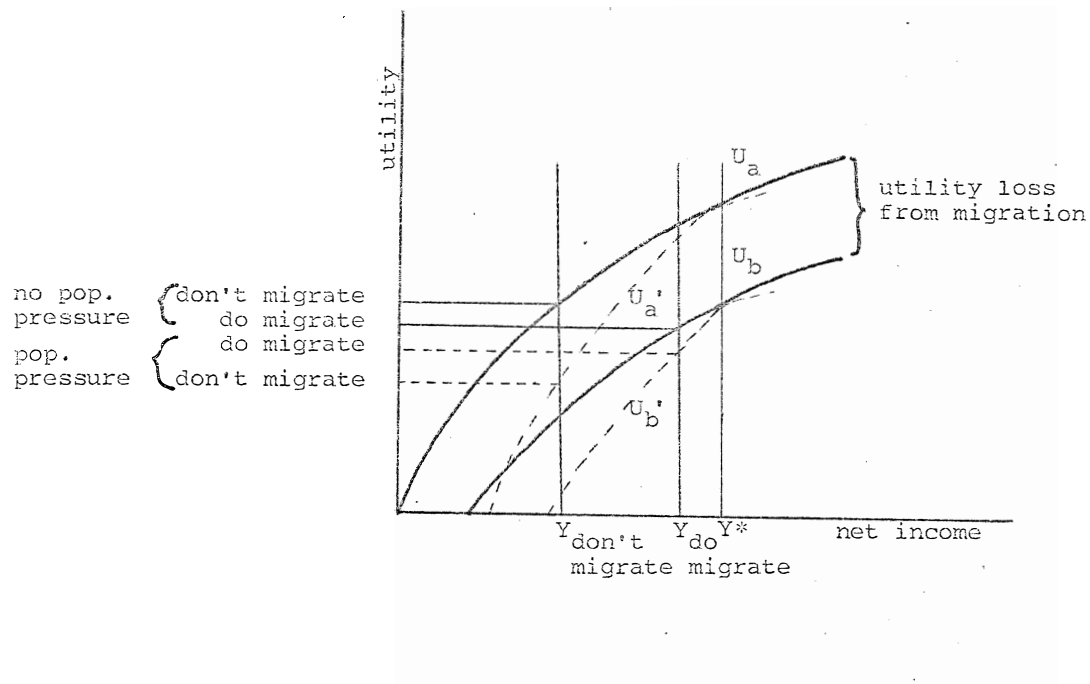
With the exceptions of total abstinence and sterilization, and the after-the-fact methods of abortion and infanticide, all methods of fertility regulation involve some probability of conception, live birth, and child survival. Short of infanticide, a family having an additional surviving child is forced to allocate resources to that child and away from goods for parents and quality of previous surviving children, given constant income and prices. Additionally the family must allocate time resources to that child, which may involve the reallocation of time among leisure, home production, and income-earning activities. If a family that was at the margin finds itself with an additional surviving child, either through contraceptive failure, non-use of contraceptives, or inaccurate expectations regarding income, prices, or child survival, we say that this family suffers micro-level or family-level population pressure. If the pressure on the individual family is perceived to be sufficiently great, we might expect the recognition of this pressure from excess fertility to result in the adoption of more efficient contraception.

The addition of a constraint on consumption decisions in the form of suboptimal children may imply that a family under population pressure would have a higher marginal utility of income than it would have if it were not experiencing population pressure. (See Appendix B for an outline of a proof of this statement.) A family whose marginal utility of income has increased due to population pressure might be moved to adopt potentially income-increasing behavior in the face of population pressure. In particular, if a course of action yields a higher expected income, but also involves psychic costs, a family might adopt it only under population pressure.

For example, a couple may have the opportunity to increase its future income, net of discounted monetary costs of migration, by moving to a new location. Figure 3 shows the net lifetime incomes for the couple if it chooses to migrate and if it does not. The solid lines show hypothetical utilities from income without population pressure, i.e., when the number of children is allowed to vary optimally. The dotted lines show hypothetical utilities from income for some fixed number of children, where that fixed number of children is optimal at Y^* and for which population pressure is experienced at incomes lower than Y^*. The dotted lines are steeper than the solid lines when children are greater than the optimum number, reflecting a higher marginal utility of income under population pressure. The upper two utility functions (U_a and U_a') apply if the couple does not migrate. The lower two utility functions (U_b and U_b') apply if it does migrate. The vertical distance between the two sets of utility functions represents the discounted psychic costs or utility loss from leaving friends, family, and familiar surroundings when migration is undertaken. We assume this cost or utility loss is invariant with the level of income. When no population pressure exists (U_a and U_b), the couple does not migrate because the anticipated loss in utility from migration is greater than the expected gain in utility from the increase in income. On the other hand, if the couple represented in figure 3 is experiencing population pressure, (U_a' and U_b'), the couple can raise its net utility by migrating and earning a higher income.

We will not deal with this issue in any greater detail because the Costa Rican data do not allow a test of its usefulness in explaining demographic trends in Costa Rica. However, we feel it has sufficient potential explanatory value that it should be investigated where data permit.

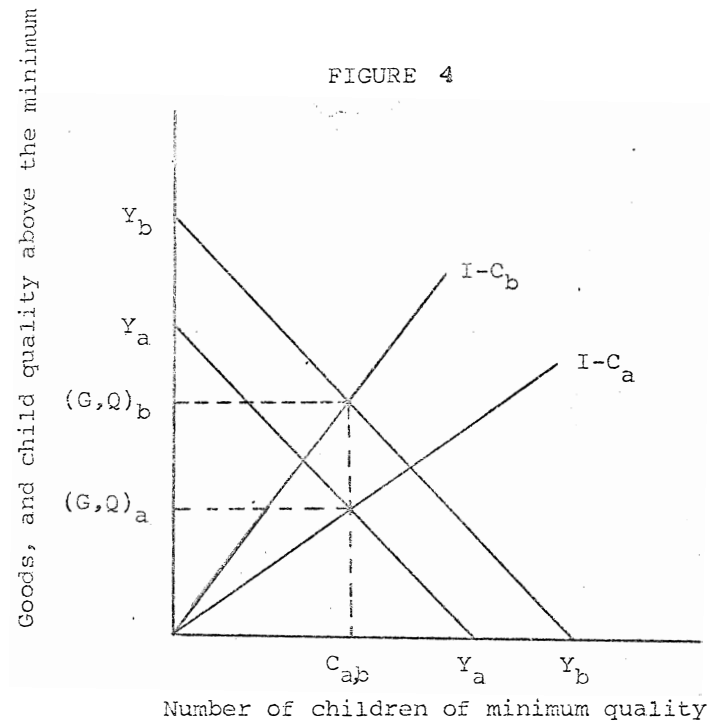
FIGURE 3



A final issue which needs to be considered in developing a dynamic model of fertility behavior is taste formation and change. Many taxonomies of factors accounting for differences in tastes have been developed, but there have been only a few attempts to develop a systematic theory of taste formation and change. One such attempt is by Richard Easterlin, which we wish to examine for its usefulness in explaining the swings in fertility in Costa Rica.

Easterlin (1970) argues that couples' consumption aspirations were formed when they were teenagers in their parents' households and reflect their parents' consumption expenditures at those times. During prolonged steady income growth, each successive couple that enters the child-bearing years will have a greater expected income, but to the extent that their consumption standards were formed in their parents' households, they will have higher preferences for goods and child quality relative to numbers of children, as well. The income-consumption lines in figure 4 represent the combinations of numbers of children of minimum quality and a composite good, consisting of goods for parents and child quality, that each of two couples would choose at each level of income in order to maximize utility. The higher consumption line ($I - C_b$), reflecting stronger preferences for goods and child quality relative to numbers of children, belongs to a couple that grew up and is entering child-bearing at a later time than the couple with the lower line ($I - C_a$). The higher budget line represents the income constraint of the later couple, and the lower one applies to the earlier couple. In this example, desired completed family size for the later couple is the same as for the earlier couple. Depending on the relative shifts in income

FIGURE 4



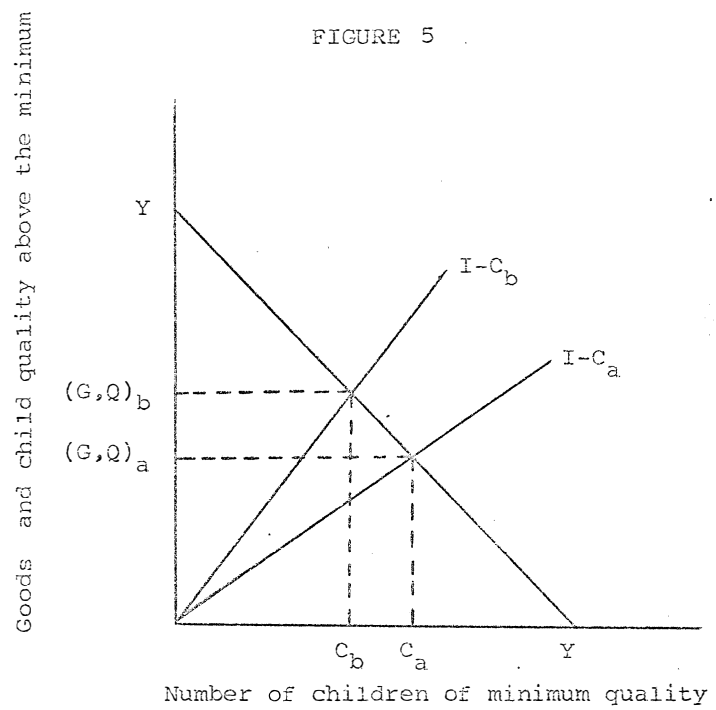
and preferences over time, and assuming constant relative prices, desired completed family size may remain constant, as in figure 4, or increase or decrease.

For a given level of child survival, Easterlin argues that swings in income will produce swings in fertility, with the pattern of fertility changes dependent on both the pattern of income changes and the age at which consumption aspirations are formed. For example, rising and then constant income can produce a downward swing in fertility, as is indicated by the change in desired family size shown in figure 5. The couple entering family formation at the beginning of a period of constant incomes has consumption aspirations ($I - C_a$) based on the lower income levels that prevailed in its parents' households. It is followed by a couple that has higher consumption aspirations ($I - C_b$), because it was raised during the period of constant, higher incomes. The first couple has a lower income-consumption line, and thus higher desired completed family size, than does the second, leading to a lower desired completed family size.

HYPOTHESES

While the data for Costa Rica are not comprehensive, some testable hypotheses emerge from the model discussed above. To begin, 1927 to 1950 is a period of declining pre-adult mortality, more slowly declining fertility, and rising potential completed family size. We might assume that rising potential completed family size either reflects an increase in average desired family size, or, for some proportion of Costa Rican families, population pressure. If the rise in potential completed family size represents population pressure, the decline in fertility might be a response to that pressure. At

FIGURE 5



the same time, the fertility decline might be a dynamic adjustment to the decline in pre-adult mortality.

Easterlin's hypothesis suggests that the dramatic rise in fertility between 1950 and 1960 might be due to an increase in desired family size, perhaps reflecting low previous consumption experience for cohorts entering the peak childbearing age groups, or sufficient increases in present income. The continued decline in pre-adult mortality suggests that there might be a continuing adjustment to improving child survival. Thus, we might expect fertility to have risen even further if child survival probabilities had remained constant.

Finally, 1961 to 1970 is characterized by a slower decline in pre-adult mortality, and rapidly declining fertility and potential completed family size. The model suggests the hypothesis that this decline in fertility to pre-1950 levels may reflect high previous consumption experience for cohorts entering the peak childbearing age groups, or sufficient decreases in present income. The decline in pre-adult mortality may contribute to the decline in fertility by reducing the number of births necessary to achieve the hypothesized lower desired completed family size.

TESTS OF HYPOTHESES

While the data for Costa Rica do not allow unambiguous tests of the hypotheses developed above, preliminary tests with the available data suggest some rather striking conclusions and raise interesting questions regarding the experience of other Central American countries. To adequately test the above hypotheses one should have a stratified, longitudinal sample of Costa Rican households, since the model develops a strictly micro

rather than a macro concept. Unfortunately, however, only estimated canton data of varying quality are available longitudinally. (The estimation procedures are discussed in Appendix A.) Recognizing that household data would be more desirable, we present regression results using the canton data as indicators of significant relationships, discussing the quality of the data underlying each regression.

First, if the decline in fertility from 1927 to 1950 represents an adjustment to improving child survival and a response to population pressure, cantons with higher potential completed family size and greater increases in child survival should have larger declines in total fertility rates. If literacy or urban residence, two variables often cited as explanations of fertility differentials, reflect the differences among cantons in desired family size, differences in potential completed family size should reflect differences in excess family size when they are held constant. In the absence of data on literacy and urbanization, a dummy variable representing Central Highlands Residence (Central Highlands = 1; Non-Central Highlands = 0) is used in their place, where Central Highlands is a proxy for urban areas. This regression is run for 52 cantons of Costa Rica. Six cantons were eliminated from the analysis in equation (1) due to suspected data errors. (See Appendix A.)

$$DTFR_{27-49} = -4.59 + 0.91\ PFS_{27} + 0.97\ CHR + 6.49\ ISP_{27-49} \quad R^2 = .49 \qquad (1)$$

$$t = 5.76 \qquad t = 3.62 \qquad t = 3.04$$

All coefficients have the predicted sign, and are significant at greater than the .99 level. The decline in the total fertility rate ($DTFR_{27-49}$) is strongly associated with high initial potential

family size (PFS_{27}), Central Highland residence (CHR)(urbanization), and increases in child survival probabilities (ISP_{27-49}). If the six cantons eliminated from the estimation of equation (1) are retained in the sample, the results are little changed:

$$DTFR_{27-49} = -5.02 + 1.01\ PFS_{27} + 1.08\ CHR + 6.34\ ISP_{27-49} \quad R^2 = .58 \qquad (2)$$

$$t = 7.40 \qquad t = 4.03 \qquad t = 3.23$$

The evidence presented above provides support for the hypothesis that the decline in fertility between 1927 and 1950 was a response to both population pressure existing at the beginning of the period and increasing probabilities of child survival.

The data underlying equations (1) and (2) were estimated by surviving the age-specific populations by canton from 1950 to 1927, as described in Appendix A. The estimated age groups 0 - 4 plus 4/5 (5 - 9) were then compared with each enumerated cantonal population 0 - 8 from the 1927 census. The distribution of percentage deviations of the survived from the enumerated populations is given in Table I.

Table I indicates that the estimation procedure in general introduced little error into estimates of 0 - 8 population. What error there is is fairly randomly distributed and is not likely to consistently bias the results if the errors described above are indicative of errors in the total survived population. To check for possible bias, equation (1) was estimated using only those cantons whose error was between -15percent and +15 percent and equation (2) was estimated using all the cantons. The results were little changed. Thus, it is assumed that the estimation procedure does not seriously bias the results.

Table I

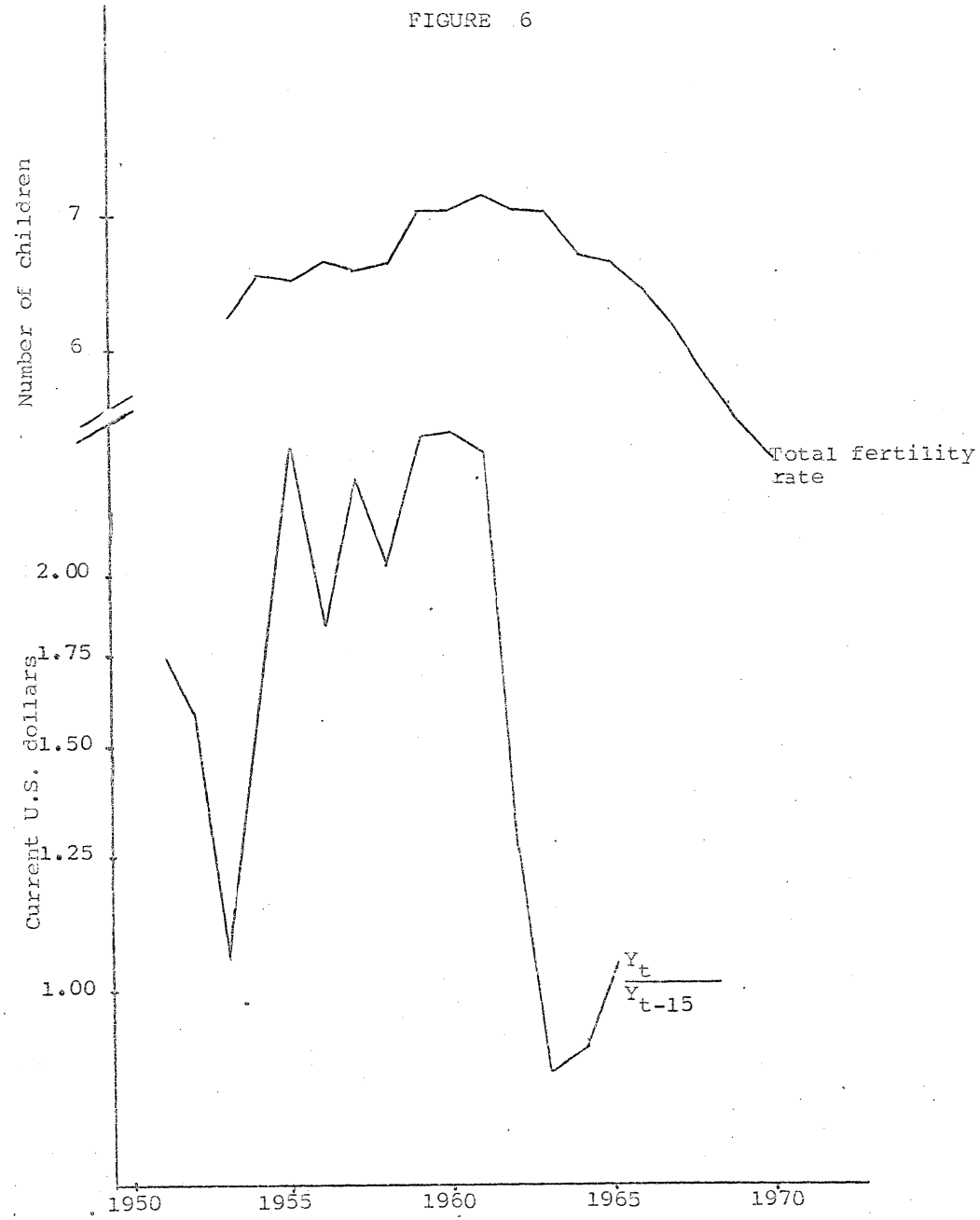
Percentage Deviations of Survived Populations, 0-8 from Enumerated Populations, 0-8, by Canton in 1927

Percentage Deviations	Number of Cantons
more than +15	6
+11 to +15	1
+6 to +10	2
0 to +5	14
0 to -5	20
-6 to -10	11
-11 to -15	2
less than -15	0

In order to test the hypothesis that the rise in fertility between 1950 and 1961 reflects either relatively low consumption expectations or relatively high current income, it would be necessary to have cantonal income estimates for the period 1935 to 1960. Unfortunately, however, even crude estimates of national income can only be constructed from 1950 on. Voertman's (unpublished) estimates of real per capita agricultural production for 1950 to 1965 constitute the best figures available for the period. Before 1950, the only income data available are figures for the value of exports. Since coffee and bananas account for nearly 85 percent of the value of Costa Rican exports in 1950, the export figures were deflated using an index of coffee and banana prices to construct a very crude indicator of the trend in real agricultural output from 1936 to 1950. The indicators of current income underlying figure 6 were constructed by splicing the export series with Voertman's estimates.

Since we hypothesize that consumption expectations are formed in a cohort's parents' households and reflect parents' incomes at that time, firgure 6 uses income lagged fifteen years as a proxy for the consumption expectations of couples in the peak childbearing years. As in Easterlin's analysis of the American baby boom (1970), the trend in the ratio, current income to lagged income, is a proxy for the relative movements of income and consumption expectations. Thus, if the relative income proxy increases, current income becomes larger relative to consumption expectations, and fertility should rise. Similarly, as relative income falls, fertility should fall. In figure 6, relative income rises sharply in the early 1950s and stays high until 1961, the year the total fertility rate peaks, and then falls as fertility falls. While the fertility trend is not exactly

FIGURE 6



replicated by the relative income trend, the rise in actual income relative to consumption expectations might well be sufficient to move families from population pressure to increasing desired family size and induce a rise in fertility. And, the equally precipitous fall in relative income after 1961 might be sufficient to bring desired completed family size well below actual completed family size and account for the timing of the fertility decline after 1961. While these data are suggestive at best and in no way provide a test of the hypothesis, they do raise interesting questions about the relationship between relative income and fertility changes in other countries which benefitted from the inflation of banana export prices during World War II.

The hypothesis that increases in child survival rates kept the fertility rise in the 1950s from being even greater suggests that the fertility increase should be inversely associated with the increase in child survival rates. When the increase in the total fertility rate is regressed on the increase in child survival probability between 1951 and 1961-3 for 65 cantons of Costa Rica, we obtain the following result:

$$ITFR_{51-63} = 2.00 - 8.91\ ISP_{51-63} \quad R^2 = .15 \qquad (3)$$
$$t = 3.33$$

While the increase in the child survival probability (ISP_{61-63}) only explains 15 percent of the variance in the increase in the total fertility rate ($ITFR_{51-63}$), the hypothesis is supported at greater than the .99 level that the increasing probability of child survival kept fertility from rising even higher during the 1950s.

The data underlying regression (3) were calculated from complete annual vital statistics and intercensal population interpolations. While the five percent underenumeration of the 1950 census may bias the vital rates by reducing the estimated fertility increase and increasing the estimated mortality decline, five percent is probably not sufficient to create a spurious relationship if the errors are random. If, however, they are not random, but concentrated in those cantons with the greater fertility increases the error might lead to a false correlation between small fertility increases and large mortality declines. For that reason, we regard the results of equation (3) as tentative.

The decline in fertility in the 1960s, like the fertility decline from 1927 to 1950, is hypothesized to be both a response to population pressure from excess completed family size and an adjustment to rising child survival probability. For this period, calculations can be made from the 1963 census of the proportion of the population of childbearing age which is literate, and the proportion of the total population living in urban areas in each canton. These provide direct measures of literacy and urbanization. Central Highlands residence is retained as a dummy variable. Cantons experiencing larger family size in 1961-63 and greater increases in child survival probabilities between 1961-63 and 1970 should have greater declines in fertility between 1961-63 and 1970 when literacy, urban residence, and Central Highlands residence are held constant. The following equation shows the regression results for 65 cantons:

$$DTFR_{61-70} = 6.78 + \underset{t=5.35}{1.07\ PFS_{61-63}} + \underset{t=3.00}{13.33\ ISP_{61-70}} - \underset{t=0.26}{.10\ CHR}$$

$$+ \underset{t=0.96}{.03\ PLIT} \quad R^2 = .55 \qquad (4)$$

(The coefficient for percent rural is 0.) The improvement in child survival probability (ISP_{61-70}) and initial potential completed family size (PFS_{61-63}) both have the predicted sign and are significant at better than the .99 level, while the other independent variables are not significant at the .90 level.

Even though potential family size and total fertility levels for 1961-63 have a correlation of 0.98, when fertility level in 1961-63 is held constant, potential family size retains its significant relationship with fertility decline, as shown in the following equation:

$$DTFR_{61-70} = 1.57 + \underset{t=5.03}{3.57\ PFS_{61-63}} + \underset{t=4.82}{24.25\ ISP_{61-70}} - \underset{t=3.65}{2.23\ TFR_{61-63}}$$

$$\underset{t=1.05}{-.03\ PLIT} + \underset{t=0.70}{.01\ PRUR} + \underset{t=0.51}{.18\ CHR} \quad R^2 = .63 \qquad (5)$$

The data underlying equations (4) and (5) were calculated by surviving population from the 1960 census. While this procedure introduces error where internal migration is widespread, the effect of this error is either to slow or to accelerate both the mortality and the fertility declines at the same time. This error may bias the results by increasing the coefficient for child survival, but we assume that it will not seriously alter the significance of the relationships described in equations (4) and (5).

Although various lags in survival probabilities were tried in the regression equations (1)-(5), current survival

probabilities added the most to the explanation of the variance in fertility change. The results of the regression analysis provide support for the hypothesis that the Costa Rican fertility decline reflects an adjustment to population pressure and increasing child survival rates.

CONCLUSION

The model set out in this paper and the evidence presented for a developing country, Costa Rica, suggest that fertility decisions are made in the context of prevailing child survival probabilities. In addition, current income and the family's consumption preferences may affect fertility decisions. If actual or expected family size is greater than desired, families may respond to real or threatened (micro-level) population pressure by reducing fertility. On the other hand, measured fertility increases in developing countries may reflect increases in desired family size and not simply improved nutrition or registration. However, inaccurate mortality expectations or imperfect contraception may lead to continued excess fertility and population pressure for individual families. Measures which help families to reduce contraceptive failure and improve the accuracy of child survival expectations may not only lower fertility, but also may help to improve the well-being of individual families.

APPENIX A

Estimation of Cantonal Components of Completed Family Size, 1927-1970

Annual age-specific population by canton were estimated between the 1950 and 1963 censuses (Costa Rica, Direccion General de Estadistico y Censos, 1953; 1966) by a simple geometric interpolation. Estimates of age-specific populations from 1964 to 1970 were made by surviving from the 1963 census figures, employing annual age-specific deaths and annual total births (Costa Rica, Direccion General de Estadistico y Censos, 1906-70).

Due to the lack of age-specific population figures by canton for 1927, the following estimation procedure was employed. First, annual total populations by canton were estimated between 1927 and 1950, using equation A-1.

$$P_t = P_o e^{\left[\frac{t}{T} \ln \frac{P_T}{P_o} - \frac{(\Delta RNI)(t)}{2} - \frac{(\Delta RNI)(t^2)}{2T}\right]}, \qquad \text{(A-1)}$$

where:

P_t = total population estimated at time t;

P_o = initial census year total population (1927);

P_T = ending census year total population (1950);

T = number of years between censuses;

t = number of years from initial census to time t; and

ΔRNI = change in rate of natural increase between censuses.

This procedure has the effect of accelerating the annual rate of population growth between censuses to account for a substantial increase in the rate of natural increase.

Next, starting with the age distribution of the population by canton in 1950 (Costa Rica, Direccion General de Estadistico y Censos, 1953), each age group was survived backwards to 1949, using age-specific deaths (Costa Rica, Direccion General de Estadistico y Censos, 1906-70). The sum of the 1949 survived populations was then subtracted from the interpolated population total. This residual was attributed to migration and distributed among the age groups according to a standardized age distribution of migration, which was adapted from Thomas (1938, pp. 42-44). This distribution procedure is based on the assumption that the relative rates of migration across age groups are constant. That is, that

$$\left(\frac{\frac{M_1}{P_1}}{R_1}\right) = \left(\frac{\frac{M_2}{P_2}}{R_2}\right) = \ldots = \left(\frac{\frac{M_n}{P_n}}{R_n}\right), \text{ where} \qquad \text{(A-2)}$$

1, n = age groups;

M = number migrating in the ith age group;

P = population in the ith age group; and

R = percentage contribution of the ith age group to the sum of the age-specific migration rates for the base period.

The M_i's can be found from the following matrix equation:

$$\begin{bmatrix} M_1 \\ M_2 \\ \cdot \\ \cdot \\ \cdot \\ M_n \end{bmatrix} = \begin{bmatrix} P_1 & O & \ldots & O \\ O & P_2 & \ldots & O \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ O & O & \ldots & P_n \end{bmatrix} \begin{bmatrix} P_1/R_1 & (P_2-P_1) & \ldots & (P_n-P_1) \\ (P_1-P_2) & P_2/R_2 & \ldots & (P_n-P_2) \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ (P_1-P_n) & (P_2-P_n) & \ldots & P_n/R_n \end{bmatrix}^{-1} \begin{bmatrix} MT \\ MT \\ \cdot \\ \cdot \\ \cdot \\ MT \end{bmatrix}, \qquad \text{(A-3)}$$

where MT = total migration.

For each age group, the sum of the survived population and the migration residual was then used as the basis for surviving back to 1948 and this procedure was repeated for each year back to 1927. While age-specific populations by canton are not available for 1927 for all age groups, the census does give the number of children in each canton eight years and younger (Costa Rica, Direccion General de Estadistico y Censos, 1960, pp. 52-53). A comparison was made between this number and the sum of the survived age groups 0-1, 1-4 and 80% of 5-9 for 1927. For those cantons in which the survived age groups deviated by more than 15% from the enumerated figures, the observation was eliminated. Cantons San Jose and Tibas in San Jose Province, Puntarenas and Osa in Puntarenas Province, and Poccoci and Siquirres in Limon Province were eliminated.

The probability of dying before age 20 (${}_{20}Q_0$) was calculated annually from 1927 to 1970 using annual age-specific deaths (Costa Rica, Direccion General de Estadistico y Censos, 1906-70). The annual total fertility rates for the same time period were estimated for each canton by distributing the cantonal total births (Costa Rica, Direccion General de Estadistico y Censos, 1906-70) among the female age groups according to the distribution of births by age of mother in 1953 in the province containing that canton.

Estimation of National Components of Completed Family Size, 1906-1970

Annual age-specific populations between the 1927 and 1950 censuses (Costa Rica, Direccion General de Estadistico y Censos, 1960; 1953) were estimated by estimating provincial populations in the same manner as the cantonal populations and

then summing. Estimations of age-specific populations between the 1892 and 1927 censuses, and between the 1950 and 1963 censuses (Costa Rica, Direccion General de Estadistico y Censos, 1893; 1960; 1953; 1966) were estimated by simple geometric interpolations. For 1963 to 1970, the cantonal estimation procedure was also employed at the national level.

The probability of dying before age 20 between 1906 and 1970 was calculated annually in the same manner as the cantonal rates were calculated. The annual total fertility rates for 1906 to 1952 were estimated by distributing annual total births (Costa Rica, Direccion General de Estadistico y Censos, 1906-70) according to the distribution of births by age of mother in 1953. Annual total fertility rates for 1953 to 1970 were calculated from annual births classified by age of mother (Costa Rica, Direccion General de Estadistico y Censos, 1906-70).

APPENDIX B

In this appendix we outline a proof of the claim that there exists a neighborhood in which the marginal utility of income is higher when the actual number of children is constrained to exceed the optimal number, assuming positive income elasticity for numbers of children. Consider a utility function of the form:

$$U = U(N, Q, X), \text{ where} \tag{B-1}$$

N = numbers of children of minimum quality;

Q = total quality inputs to children above the minimum; and

X = goods for parents.

In the unconstrained case we wish to maximize the Lagrangian:

$$L = U(N, Q, X) + \lambda(Y - p_N N - p_Q Q - p_X X), \text{ where} \tag{B-2}$$

Y = income; and

p_N, p_Q, p_X = prices of N, Q, and X, respectively.

Totally differentiating the first order conditions for a regular relative maximum yields a matrix equation of the form:

$$\begin{bmatrix} U_{NN} & U_{NQ} & U_{NX} & -p_N \\ U_{QN} & U_{QQ} & U_{QX} & -p_Q \\ U_{XN} & U_{XQ} & U_{XX} & -p_X \\ -p_N & -p_Q & -p_X & 0 \end{bmatrix} \begin{bmatrix} dN \\ dQ \\ dX \\ d\lambda \end{bmatrix} = \begin{bmatrix} \lambda dp_N \\ \lambda dp_Q \\ \lambda dp_X \\ -dY + Ndp_N + Qdp_Q + Xdp_X \end{bmatrix} \tag{B-3}$$

Interpreting λ as the marginal utility of income, and holding all parameters except income constant,

$$d\lambda = \frac{-dYA_{44}}{|A|}, \text{ where} \tag{B-4}$$

A_{44} is the 44 cofactor of the bordered Hessian matrix A and $|A|$ is its determinant.

Now we constrain N to be at $\bar{N}$, maximizing the Lagrangian:

$$L = U(N, Q, X) + \lambda(Y - p_N N - p_Q Q - p_X X) - \varkappa(\bar{N} - N) \tag{B-5}$$

Totally differentiating the first order conditions adds an additional border to matrix A:

$$\begin{bmatrix} U_{NN} & U_{NQ} & U_{NX} & -p_N & 1 \\ U_{QN} & U_{QQ} & U_{QX} & -p_Q & 0 \\ U_{XN} & U_{XQ} & U_{XX} & -p_X & 0 \\ -p_N & -p_Q & -p_X & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} dN \\ dQ \\ dX \\ d\lambda \\ d\varkappa \end{bmatrix} = \begin{bmatrix} \lambda dp_N \\ \lambda dp_Q \\ \lambda dp_X \\ -dY + Ndp_N + Qdp_Q + Xdp_X \\ d\bar{N} \end{bmatrix} \tag{B-6}$$

We designate the second bordered matrix B. Then

$$d\lambda = \frac{-dYB_{44}}{|B|} \tag{B-7}$$

Thus,

$$\left.\frac{\partial\lambda}{\partial Y}\right|_{N \text{ constrained}} = \frac{-B_{44}}{|B|} \tag{B-8}$$

Expressing the unconstrained case (B-4) in terms of matrix B we have:

$$\left.\frac{\partial\lambda}{\partial Y}\right|_{N \text{ unconstrained}} = \frac{-B_{55,44}}{B_{55}}, \text{ where} \tag{B-9}$$

$B_{55,44}$ is the cofactor B_{44} with the 5th row and 5th column deleted.

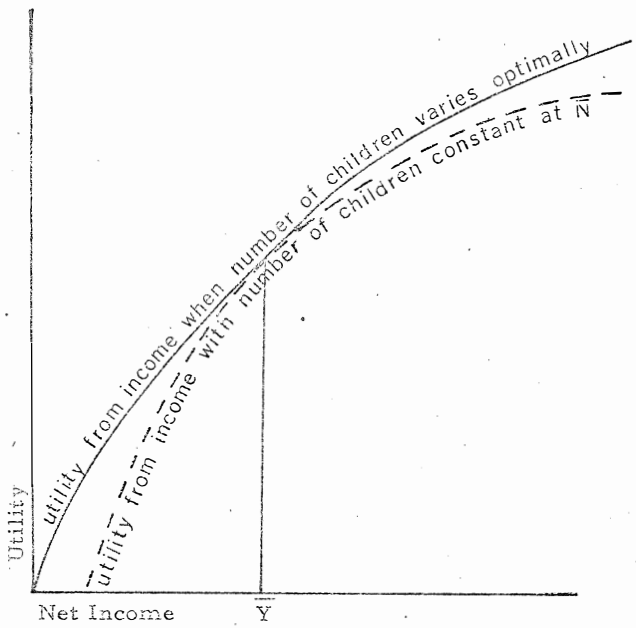
We now wish to sign the expression:

$$\left[\left.\frac{\partial\lambda}{\partial Y}\right|_{N \text{ unconstrained}} - \left.\frac{\partial\lambda}{\partial Y}\right|_{N \text{ constrained}}\right].$$ If

$\left.\frac{\partial\lambda}{\partial Y}\right|_{N \text{ unconstrained}}$ is greater than $\left.\frac{\partial\lambda}{\partial Y}\right|_{N \text{ constrained}}$ the utility function in the constrained case lies below the envelope curve of the utility function in the unconstrained case, except at tangency points, where the constrained number of children is optimal ($\bar{N}$ = N). If the constrained case lies below the unconstrained case, then the marginal utility of income is greater in the constrained case in neighborhoods to the left of points of tangency. For example, in figure B-1 the second derivative of the envelope curve, N unconstrained, is greater than the second derivative of the curve, N constrained, at the point of tangency. In this example, the constrained curve lies below the unconstrained curve, giving a higher marginal utility of income for incomes less than $\bar{Y}$, when children are constrained to be greater than optimal.

Figure B-1

HYPOTHETICAL UTILITY FUNCTIONS WITH CONSTRAINED AND UNCONSTRAINED NUMBERS OF CHILDREN



$$\left.\frac{\partial\lambda}{\partial Y}\right|_{N \text{ unconstrained}} - \frac{\partial\lambda}{\partial Y}_{N \text{ constrained}} =$$

$$\frac{-B_{55,44}}{B_{55}} - \left(\frac{-B_{44}}{|B|}\right) = \frac{B_{44}B_{55} - |B|B_{55,44}}{|B|B_{55}} \qquad \text{(B-10)}$$

Applying Jacobi's theorem on determinants,

$$\frac{B_{44}B_{55} - |B|B_{55,44}}{|B|B_{55}} = \frac{B_{44}B_{55} - (B_{55}B_{44} - B_{54}B_{45})}{|B|B_{55}} = \frac{B_{54}B_{45}}{|B|B_{55}} \qquad \text{(B-11)}$$

Since B is a symmetric matrix $B_{54} = B_{45}$ and the numerator is positive. The denominator is also positive because bordered principal minors of such negative definite matrices must be of the same sign regardless of the number of bordering rows. Thus the difference is positive and the marginal utility of income is higher under population pressure in neighborhoods to the left of points of tangency.

ACKNOWLEDGMENTS

An earlier version of this paper was presented at the Population Association of America annual meetings in New York, April 19, 1974. The authors wish to thank E. David Emery, David Grether, Barbara Janowitz, T. Paul Schultz and anonymous reviewers for their comments on earlier drafts; Miguel Gomez, Ricardo Jimenez, Director of C. E. S. P. O., and Fernando Naranjo for their valuable advice and assistance in Costa Rica; and Robert Voertman for allowing the authors to use his unpublished materials.

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