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ATTITUDES TOWARDS RISK AND THE OPTIMAL EXPLOITATION OF AN EXHAUSTIBLE RESOURCE

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*The author wishes to thank
Robert Forsythe and R. Schmalensee
for helpful comments on an earlier
draft.

SOCIAL SCIENCE WORKING PAPER 103

Revised March 1976

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ABSTRACT

The exploitation of a non renewable natural resource, such as petroleum or mineral ores, is analyzed in a stochastic framework with price uncertainty. The market setting may be either monopolistic or competitive. We demonstrate that the rate of extraction varies directly with the resource owner's willingness to accept risk. Risk preferring owners use the resource more rapidly than risk neutral owners, who in turn deplete the resource more rapidly than risk averse owners. It is also seen that the usual practice of increasing the discount rate to account for risk induces a more rapid rate of resource use, when in fact a slower rate of depletion is desired.

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I. INTRODUCTION

Recent concern over the adequacy of future supplies of certain energy and mineral resources has stimulated much interest about factors affecting the rate at which non-renewable resources are used up. A number of authors including Heal et al. (1974), Lewis (1974), Long (1975, 1974), Schmalensee (1975), Smith (1974), Stiglitz (1975), Vousden (1973), and Weinstein and Zeckhauser (1975) have described the patterns of resource depletion occurring under different assumptions about resource demand, market structure, the rate of time discount, the possibility of backup resources, changes in technology and so forth.

One factor, yet to be analyzed, which is the subject of this paper, is the effect on depletion rates of random variations in the market price of resources. For example, owners of petroluem reserves such as the OPEC[1] block are faced with price uncertainties resulting from changes in the demand for oil, in the availability of substitute supplies, and in national energy conservation programs.[2] In what follows, we present a theoretical analysis comparing extraction rates of an exhaustible resource for owners who are risk preferring,

risk neutral, or risk averse regarding variations in the returns from extraction. The resource whether it be oil, gas, minerals, etc., is owned by a single firm or cartel in the case of OPEC.[3] The analysis allows for situations where the firm operates in a competitive market with price perceived as given, or in a monopolistic market where price is sensitive to the quantity of the resource sold by the firm. In either case price uncertainty exists in the form of random additive shifts in the demand schedule facing each owner. The firm presumes to know the total resource stock available for extraction. However, in each period it must choose the amount of oil to pump or the quantity of ore to extract knowing only the probability distribution for prices at which the resource can be sold.[4] We assume the firm always sells all of its output at the prevailing market price. This is because opportunities for storage or arbitrage, in which distributors buy and store resources when prices are depressed and sell their inventories when prices rise, are presumed to be already reflected in current prices.[5]

Complete contingency markets are presumed not to exist, so that the owner is necessarily subject to some risk in the returns from the resource. We assume the rate of extraction is chosen in each period to maximize the present value of expected utility, with preferences for risk bearing being reflected in the form of the owner's utility function.

In the analysis to follow, we demonstrate that the rate of pumping or extraction varies directly with the owner's willingness to accept risk. Risk preferring owners use the resource more rapidly than risk neutral owners, who in turn deplete the resource more rapidly than risk averse owners. This is analogous to the static results derived by Leland (1972) and Sandmo (1971) who find that risk averse firms produce less than risk neutral firms. Also, increasing the discount rate to account for risk, a common practice in the public investment literature, is shown to induce a more rapid rate of resource use when in fact a slower rate of depletion is desirable.

This section is concluded with a specification and description of our model. In section II optimal extraction programs are derived and compared for owners with different risk bearing attitudes. The method of risk adjusted discounting is shown to differ markedly from the procedure of capturing risk in the form of the utility function. A short summary and discussion of our results is presented in section III.

Our analysis will be done in discrete time under stationary conditions. Let q_t be the quantity of the resource extracted or pumped from the ground (in the case of oil) during time period t, and let S_t be the physical size of the available resource stock at the beginning of the t th time period. (Time arguments will be omitted where no confusion exists). For a given amount of extraction the per unit market price for the resource, $p(q_t)$ is random with

$$\tilde{p}(q_t) = p(q_t) + \tilde{\eta}_t \qquad (A.1)$$

where $\tilde{\eta}_t$ is a random variable which is identically and independently distributed over time with density function $\phi(\tilde{\eta}_t)$.[6] For the case where the owner is the only local supplier of the resource we assume $p'(q) < 0$, otherwise $p(q) = \bar{p}$ in the competitive situation.[7] Consistent with other analysis assuming additive stochastic demand by Mills (1959) and Zabel (1972), we restrict $\tilde{\eta}$ to be non negative to avoid the possibility of negative prices. Expected demand equals $p(q) + \hat{\eta}$ where $E(\tilde{\eta}) = \hat{\eta}$. Instead of (A.1) we might have specified a multiplicative stochastic demand, although the additive and multiplicative assumptions are equivalent in the competitive case where $p(q) = \bar{p}$. As Zabel (1972) points out, models with multiplicative demand are intrinsically more difficult to work with. This is also

true in our analysis. For example, in the monopoly case with stochastic multiplicative demand, we have been unable to unambiguously determine the effect of preferences for risk bearing on resource use.

In each period, the owner incurs a cost of extraction, $C(q_t)$ which is non random with

$$C_q > 0, \; C_{qq} \geq 0 \qquad (A.2)$$

This is a simplified specification in that we are abstracting from possible effects of resource depletion on extraction costs.[8]

In choosing the optimal extraction program, we assume the owner selects q_t in each period to maximize the present value of expected utility from extraction profits given by

$$\underset{q_t}{\text{maximize}} \sum_{t=0}^{\infty} B^t EU(\tilde{\pi}(q_t)) \; ; \; B = \frac{1}{1+\rho} \qquad (1)$$

subject to the constraint on resource availability

$$S_0 - \sum_{t=0}^{\infty} q_t \geq 0 \; ; \text{ and } q_t, S_t \geq 0 \; ; \; \rho > 0 \qquad (2)$$

where profits $\tilde{\pi}(q) = q\tilde{p}(q) - C(q)$ and ρ is the discount rate. We assume the utility function, U, has a positive first derivative with $U''(> 0, = 0, < 0)$ according to whether the owner is risk preferring, risk neutral, or risk averse. To make the analysis relevant we make one additional assumption that the resource is exploitable, that is

$$EU'[0]\tilde{\pi}_q(0) > 0 \qquad (A.3)$$

II. OPTIMAL EXTRACTION PROGRAMS

The necessary condition for the maximization of (1) subject to (2) is $$B^t E U' \tilde{\pi}_q - \lambda \leq 0 \text{ and } \left[B^t E U' \tilde{\pi}_q - \lambda\right] q_t = 0 \tag{3}$$

where λ is the Lagrange multiplier associated with the resource availability constraint in (2). Clearly λ is positive by (A. 3), and (A. 3) and (3) together imply $q_t > 0$ whenever $S_t > 0$. λ has the usual interpretation of being the marginal value of an addition to the resource stock. Equation (3) simply states that along the optimal path increases in expected current returns are exactly offset by decreases in future returns resulting from current resource use.

Condition (3) is also sufficient for a maximum of (1) if

$$E\left[U'' \tilde{\pi}_q^{\,2} + U' \tilde{\pi}_{qq}\right] < 0 \tag{4}$$

Sufficient conditions for (4) to be negative are that $\tilde{\pi}$ be strictly concave in q, and that firms be risk neutral or risk averse ($U'' \leq 0$) or that $\tilde{\pi}$ be linear in q and the firm be risk averse. We shall assume $\tilde{\pi}$ is strictly concave in q and that (4) is negative for all q (even with $U'' > 0$). In Appendix A we also obtain the same results as presented here when (4) is not negative for all q, which may result when $U'' > 0$, or when $U'' \geq 0$ and $\tilde{\pi}$ is linear.

To simplify the analysis, we introduce the following notation. Let $\left[q^p_t, q^n_t, q^a_t\right]$, $\left[S^p_t, S^n_t, S^a_t\right]$, and $\left[T^p, T^n, T^a\right]$ represent the amount of extraction at time t, the amount of the resource remaining at time t, and the time at which the resource is finally exhausted according to optimal extraction programs for a risk preferring, risk neutral, and risk averse owner respectively. We wish to establish that:

Proposition 1:

If $T^a \geq 2$ then

$$q^n_t > q^a_t \qquad t < t'$$

$$q^n_t \leq q^a_t \quad \text{for} \quad t = t' \qquad t' \in (0, T^a) \tag{5a}$$

$$q^n_t < q^a_t \qquad t > t'$$

$$S^n_t < S^a_t \quad \text{for} \quad t \in (0, T^a) \tag{5b}$$

$$T^n \leq T^a \tag{5c}$$

otherwise $q^n_0 = q^a_0 = S_0$

and

Proposition 2:

if $T^n \geq 2$ then

$$q^p_t > q^n_t \qquad t < t'$$

$$q^p_t \leq q^n_t \quad \text{for} \quad t = t' \qquad t' \in (0, T^n) \tag{6a}$$

$$q^p_t < q^n_t \qquad t > t'$$

$$S^p_t < S^n_t \quad \text{for} \quad t \in (0, T^n) \tag{6b}$$

$$T^p \leq T^n \tag{6c}$$

otherwise $q^p_0 = q^n_0 = S_0$

or in other words that risk preferring owners deplete more rapidly than risk neutral owners, who in turn deplete more rapidly than risk averse owners.[9]

For the discussion here, we will assume that extraction horizons are at least two periods long. The results for single period horizons are demonstrated in Appendix B. To establish Propositions 1 and 2 we need to derive some additional properties about optimal extraction programs. First,

$$\text{sign}\ \frac{d}{dq}\frac{E\tilde{\pi}_q}{EU'\pi_q} = \text{sign}\left\{\frac{EU'\tilde{\pi}_q E\tilde{\pi}_{qq} - E\tilde{\pi}_q\left[EU''\tilde{\pi}_q^2 + EU'\tilde{\pi}_{qq}\right]}{(EU'\pi_q)^2}\right\} \qquad (7a)$$

$$= \text{sign}\left\{\frac{\tilde{\pi}_{qq}\left[EU'\tilde{\pi}_q - EU'E\tilde{\pi}_q\right] - EU''\tilde{\pi}_q^{\,2}E\tilde{\pi}_q}{(EU'\tilde{\pi}_q)^2}\right\} \qquad (7b)$$

$$= -\ \text{sign}\left\{\left[EU'\tilde{\pi}_q - EU'E\tilde{\pi}_q\right] + EU''\tilde{\pi}_q^{\,2}E\tilde{\pi}_q\right\} \qquad (7c)$$

$$= -\ \text{sign}\left\{E\left[U' - U'(\tilde{\pi}(\hat{\eta}))\right]\left[\tilde{\pi}_q - E\tilde{\pi}_q\right] + EU''\tilde{\pi}_q^{\,2}E\tilde{\pi}_q\right\} \qquad (7d)$$

$$\lesseqgtr 0 \ \text{ as } \ U'' \gtreqless 0. \qquad (7e)$$

Noting that $E\tilde{\pi}_{qq} = \tilde{\pi}_{qq} = p''(q)_q + 2p'(q) - C_{qq}$ and factoring we obtain (7b) from (7a). Eq. (7c) follows since $\pi_{qq} < 0$ and $(EU'\tilde{\pi}_q)^2 > 0$. We obtain (7d) by noting that $E\tilde{\pi}_q = \tilde{\pi}_q(\hat{\eta})$ and by adding and subtracting $U'(\tilde{\pi}(\hat{\eta}))E\tilde{\pi}_q$ from the r.h.s. of (7c). Eq. (7e) then follows immediately since $[U' - U'(\tilde{\pi}(\hat{\eta}))][\tilde{\pi}_q - E\tilde{\pi}_q] \lesseqgtr 0$ for all q as $U'' \lesseqgtr 0$.

Second, from equation (3) we obtain,

$$\frac{d}{dt}q_t < 0;\ \text{for } q_t > 0 \qquad (8)$$

$$\frac{EU'\tilde{\pi}_q(q_{t+\tau})}{EU'\tilde{\pi}_q(q_t)} = \frac{1}{B^{\tau}}\ \text{ for } q_t, q_{t+\tau}, \tau > 0 \qquad (9)$$

independent of attitudes for risk bearing.

Using (7)-(9) we can establish

Lemma 1:

$$\text{if}\quad q^n_{t'} \le q^a_{t'}\ \text{ for some } t',\ \text{then}$$
$$q^n_t < q^a_t\ \text{ for all } t > t' \text{ unless } q^n_t = q^a_t = 0 \qquad (10)$$

and

Lemma 2:

$$\text{if}\quad q^p_{t'} \le q^n_{t'}\ \text{ for some } t',\ \text{then}$$
$$q^p_t < q^n_t\ \text{ for all } t > t' \text{ unless } q^p_t = q^n_t = 0. \qquad (11)$$

To prove Lemma 1 assume contrary to (10) that $q^n_{t'} \le q^a_{t'}$ and $q^n_t \ge q^a_t$ for some $t > t'$ where all q's are positive. Then since $\tilde{\pi}$ is strictly concave in q,

$$E\tilde{\pi}_q\left[q^n_{t'}\right] \ge E\tilde{\pi}_q\left[q^a_{t'}\right] = \alpha EU^{a\prime}\tilde{\pi}_q\left[q^a_{t'}\right] \qquad (12a)$$

where α is some positive constant, and U^a is the risk averse owner's cardinal utility function.[10] By (7), (8) and (12a)

$$\frac{E\tilde{\pi}_q(q^a_{t'})}{EU^{a'}\tilde{\pi}_q(q^a_{t'})} = \alpha > \frac{E\tilde{\pi}_q(q^a_t)}{EU^{a'}\tilde{\pi}_q(q^a_t)}$$

and therefore with a little rearranging we obtain

$$\alpha EU^{a'}\tilde{\pi}_q(q^a_t) > E\tilde{\pi}_q(q^a_t) \geq E\tilde{\pi}_q(q^n_t) \tag{12b}$$

It follows from (12a)-(12b) that

$$\frac{E\tilde{\pi}_q\left[q^n_t\right]}{E\tilde{\pi}_q\left[q^n_{t'}\right]} - 1 = \frac{E\tilde{\pi}_q\left[q^n_t\right] - E\tilde{\pi}_q\left[q^n_{t'}\right]}{E\tilde{\pi}_q\left[q^n_{t'}\right]}$$

$$< \frac{\alpha\left[EU^{a'}\tilde{\pi}_q\left[q^a_t\right] - EU^{a'}\tilde{\pi}_q\left[q^a_{t'}\right]\right]}{\alpha EU^{a'}\tilde{\pi}_q\left[q^a_{t'}\right]} = \frac{EU^{a'}\tilde{\pi}_q\left[q^a_t\right]}{EU^{a'}\tilde{\pi}_q\left[q^a_{t'}\right]} - 1 \tag{13}$$

which implies a contradiction of (9) if we let the risk neutral utility function, $U^n(\tilde{\pi}) = \tilde{\pi}$. Thus $q^n_t < q^a_t$. The result in (11) for q^p_t and q^n_t can also be established using the same proof as above.

From (10) and (11), the results in Propositions 1 and 2 follow readily. First with respect to (5), we note that $q^a_0 < q^n_0$. Otherwise if $q^a_0 \geq q^n_0$, (10) implies $\sum_{t=0}^{\infty} q^a_t > \sum_{t=0}^{\infty} q^n_t$ which can't occur if both the risk neutral and risk averse programs satisfy the resource availability constraint with equality. With $q^a_0 < q^n_0$ (5a) is an immediate consequence of (10). Condition (5b) follows from (5a) and the fact that $\sum_{t=0}^{\infty} q^a_t = \sum_{t=0}^{\infty} q^n_t$. Finally, (5c) follows directly from (5b). The same proof as above can be applied to Proposition 2 as well.

When risk preferences are captured in the form of the utility function, we see that the rate of resource depletion decreases as we move from risk preferring to risk neutral to risk averse owners. On the other hand, if we maximize the stream of expected profits discounted by a risk adjusted interest rate (higher discount rates corresponding to greater risk aversion) we find that resource depletion is more rapid for risk averse than risk neutral owners and so on. This is the message of

Proposition 3:

$$\begin{aligned}
&T^{\rho} \geq 2 \\
&q^{\rho'}_t > q^{\rho}_t && t < t' \\
&q^{\rho'}_t \leq q^{\rho}_t \quad \text{for} && t = t' \quad t' \in (0, T^{\rho}) \\
&q^{\rho'}_t < q^{\rho}_t && t > t' \\
&S^{\rho'}_t < S^{\rho}_t \quad \text{for} && t \in (0, T^{\rho}) \\
&T^{\rho'} \leq T^{\rho}
\end{aligned} \tag{14}$$

otherwise $q^{\rho'}_0 = q^{\rho}_0 = S_0$.

The quantities $[S^{\rho'}_t, S^{\rho}_t]$, $[q^{\rho'}_t, q^{\rho}_t]$, and $T^{\rho'}, T^{\rho}]$ represent the stock remaining at time t, the extraction rate at time t, and the time when the resource is exhausted according to the optimal extraction programs corresponding to discount rate ρ' and ρ respectively, with $\rho' > \rho$. The proof of Proposition 3 follows directly from (3) and (4) and the resource availability constraint.

In accounting for risk preferences through the discount rate, we assume uncertainty enters as a time compounding phenomenon. However, in our model, variations in returns are proportional to the extraction rate, q, and independent of time. In this context it is not surprising that altering the discount rate to account for risk preferences is inadequate.[11]

III. CONCLUSION

According to our analysis, the rate of resource use is affected in rather predictable ways by the risk bearing attitudes of the resource owner. Whether these results hold in a more general context where extraction costs depend on the amount of the resource available, and where possibilities for storing the resource are explicitly considered requires further investigation.

Besides this several other topics for further research come to mind. One is the effect on extraction rates of increased price uncertainty. There are many factors tending to distablize the world oil market. These include uncertainties about existing and future supplies of energy, the development of energy saving technology, and even the social and political climate of the energy consuming countries. Such uncertainties are bound to add to existing variations in oil prices. In the case of domestic resource suppliers it would be helpful for policy purposes to examine the effect of resource use of various kinds of extraction taxes. The level of total income including extraction profits and other "outside" income is likely to affect the risk preferences of owners. Therefore, lump sum taxes and profit taxes on resource extraction which are normally perceived as being neutral with respect to extraction rates will undoubtedly affect resource use under conditions of uncertainty.[12] In that income is

reduced, we would expect the imposition of these taxes to cause a decrease in the rate of extraction assuming that owners are risk averse and display decreasing absolute risk aversion as defined by Arrow (1965) and Pratt (1964). However, we suspect that the impact of these taxes must be examined on a specific case by case basis. We have found that the simple conditions imposed on our model are too general to derive unambiguous results on the effect of taxation.

APPENDIX A

We wish to show that the results in (6) hold even if equation (4) is not negative for all q, a sufficient condition for maximization in equation (1). Equation (4) will be non negative if (1) π is linear in q and $U'' \geq 0$, or possibly if (2) $U'' > 0$. For the first case, with π linear if follows that $q^P_0 = q^n_0 = S_0$ and the resource is depleted as rapidly as possible.

In the second case clearly two possibilities exist; either $T^P = 1$, or $T^P > 1$. If $T^P = 1$ the results in (6) follow immediately. If $T^P > 1$ then q^P_t must satisfy equation (3), otherwise there would exist some t and t' such that $B^{t'} EU'\tilde{\pi}_q(q^P_{t'}) < B^t EU'\tilde{\pi}_q(q^P_t)$ with $q^P_{t'}$ and $q^P_t > 0$, which is non optimal since we can decrease $q^P_{t'}$ and increase q^P_t on the margin to increase the total value of the program. However, in the body of the paper we have already derived the results in (6) for the case where condition (4) yields a true maximum in (1).

APPENDIX B

In (5) we wish to show that if $T^a = 1$ and $q_0^{\ a} = S_0$ then $T^n = 1$ and $q_0^{\ n} = S_0$. Let us assume to the contrary that $T^a = 1$ but $T^n > 1$. Then (3) implies:

$$E\tilde{\pi}_q(q_0^{\ n}) = BE\tilde{\pi}_q(q_1^{\ n}) \tag{B1}$$

and

$$EU^{a\prime}\tilde{\pi}_q(S_0) \geq BEU^{a\prime}\tilde{\pi}_q(0). \tag{B2}$$

Since $S_0 > q_0^{\ n}$ (4) implies

$$EU^{a\prime}\tilde{\pi}_q(S^0) < EU^{a\prime}\tilde{\pi}_q(q_0^{\ n}). \tag{B3}$$

Because U^a is a cardinal utility index, we can choose some constant α such that

$$EU'\tilde{\pi}_q(q_0^{\ n}) = \alpha E\tilde{\pi}_q(q_0^{\ n}) = \alpha BE\tilde{\pi}_q(q_1^{\ n}). \tag{B4}$$

Condition (7) implies

$$BEU^{a\prime}\tilde{\pi}_q(q_1^{\ n}) > \alpha BE\tilde{\pi}_q(q_1^{\ n}). \tag{B5}$$

However, combining equations (B3)--(B5) we obtain

$$BEU^{\alpha\prime}\tilde{\pi}_q(0) > BEU^{a\prime}\tilde{\pi}_q(q_1^{\ n}) > EU'\tilde{\pi}_q(S_0) \tag{B6}$$

which contradicts equation (B2). Thus it follows that $T^n = 1$ whenever $T^a = 1$. The same type of proof can also be employed in (6) to show that $T^P = 1$ whenever $T^n = 1$.

FOOTNOTES

1. OPEC refers to the "Organization of Petroleum Exporting Countries." It is composed primarily of oil rich Middle East countries and was established in 1960 as a cartel to influence world petroleum prices.

2. For example President Ford's "energy independence" program is likely to effect OPEC's oil revenues substantially. However, the exact impact of the program will in the long run depend on the social, political and economic climate in the United States.

3. For a comprehensive discussion of OPEC and the world petroleum market, see Adelman (1974).

4. We are assuming a free market in natural resources, one without direct government price regulation.

5. The extent to which resource storing can dampen oscillations in price will depend on the cost of maintaining inventories. When these costs are high, as in the case of petroleum, storage may not be feasible. For an explicit treatment of resource exploitation in which storage is possible, see Lee and Orr (forthcoming).

6. For notation purposes a variable with a tilde over it is a random variable, and $f_x = df/dx$.
   E(*) represents the expectation operator.

7. In the competitive case, an alternative assumption is that firms perceive prices as given in a particular time period, but that expected price may rise or fall throughout time. For example, unless substitute materials exist, competitive firms may expect prices to rise over time as resource supplies are diminished. This possibility is to be examined in a forthcoming paper by the author.

8. We might have also assumed there are positive fixed costs to extraction although this assumption would not alter the nature of our results. Schmalensee (forthcoming) provides an interesting analysis of the effect of fixed costs on resource use.

9. T^a, T^n, and T^p may be infinite if $\tilde{\pi}_q(0) = 00$. T^a will also be infinite if $U'(0) = 00$.

10. Because we are dealing in a von Neumann-Morgenstern expected utility framework, all utility functions are necessarily cardinal. The value of α is arbitrary and will depend on the scaling of U^a.

11. See Prest and Turvey (1965) on this point.

12. See Peterson (1972), pp 20-22.

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