Radiation reaction for spinning bodies in effective field theory. I. Spin-orbit effects

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We compute the leading post-Newtonian (PN) contributions at linear order in the spin to the radiation-reaction acceleration and spin evolution for binary systems, which enter at fourth PN order. The calculation is carried out, from first principles, using the effective field theory framework for spinning compact objects, in both the Newton-Wigner and covariant spin supplementary conditions. A nontrivial consistency check is performed on our results by showing that the energy loss induced by the resulting radiation-reaction force is equivalent to the total emitted power in the far zone, up to so-called “Schott terms.” We also find that, at this order, the radiation reaction has no net effect on the evolution of the spins. The spin-spin contributions to radiation reaction are reported in a companion paper.

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I. INTRODUCTION

The potentially large amount of scientific information on strong gravitational fields that can be extracted from the observations of gravitational waves detected by ground-based detectors [1–3] and future space-based antennas motivates the computation of highly accurate theoretical models for the dynamics and gravitational wave emission from binary systems in general relativity. The gravitational dynamics and emitted power from compact binaries has been computed to seventh order in the relative speed \( v \) (also called 3.5 post-Newtonian or 3.5PN order) for nonrotating bodies (see [4,5] for extensive reviews). In addition, the gravitational potential has recently been rederived with EFT methods in [71] employing the for-...
calculated in [75] (and the radiation-reaction Hamiltonian in [76]) using different methodologies and spin supplementarity conditions (SSCs). For completeness, here we carry out the calculation in both the Newton-Wigner and covariant SSCs. We also perform a nontrivial consistency check by showing the equivalence between the energy loss induced by the resulting radiation-reaction acceleration and the total emitted power in the far zone which follows from the well-known multipole expansion, up to total time derivatives. (The latter are called “Schott terms” in electrodynamics [80], which account for energy stored in near-zone fields. See also [81].) We find that, in the spin-orbit sector, the radiation reaction has no net effect on the evolution of the spins. Our findings are compatible with the derivations in [75]. Spin-spin backreaction effects, which first enter at 4.5PN order, are studied in a companion paper [82].

As it is customary in the literature, we define the following useful quantities:

\[
S \equiv S_1 + S_2, \tag{1.1}
\]

\[
\Sigma \equiv \frac{m_1}{m_2} S_2 - \frac{m_2}{m_1} S_1, \tag{1.2}
\]

\[
S_{\text{eff}}^{\text{rad}} = -\int dt \sqrt{g_{00}} \left[ M(t) - \sum_{\ell=2} \left( \frac{1}{\ell!} I^\ell(t) \nabla_{L-2} E_{i-1i} + \frac{2\ell'}{(2\ell' + 1)!} I^{\ell'}(t) \nabla_{L-2} B_{i-1i} \right) \right], \tag{2.1}
\]

with \( L = (i_1 \cdots i_\ell) \) being a multi-index. The quantity \( M(t) \) is the (Bondi) mass associated with the binary while \( I^\ell(t) \) and \( J^\ell(t) \) are the mass- and current-type source multipole moments, respectively. The electric and magnetic components of the Weyl tensor are denoted by \( E_{ij} \) and \( B_{ij} \), respectively, which depend only on the metric in the radiation region, \( \hat{h}_{\mu\nu} \). See [60] for more details.

To incorporate the nonconservative effects of radiation reaction at the level of the action, we need to formally double the number of degrees of freedom [72,73] so that \( x_a \rightarrow (x_{a(1)}, x_{a(2)}) \) and \( \hat{h}_{\mu\nu} \rightarrow (\hat{h}_{\mu\nu}^{(1)}, \hat{h}_{\mu\nu}^{(2)}) \) in (2.1). Here, \( a = \{1, 2\} \) labels the particles. Solving for both potential and radiation fields generates an effective action of the form

\[
S_{\text{eff}}[x_{\pm}] = \int dt (L_{\text{eff}}[x_{a(1)}] - L_{\text{eff}}[x_{a(2)}] + R_{\text{eff}}[x_{a(1)}, x_{a(2)}]), \tag{2.2}
\]

where \( L_{\text{eff}} = \int dt (K - V) \) is the Lagrangian for the conservative sector and \( R_{\text{eff}} \) accounts for all nonconservative effects. It is convenient to translate the expressions into the \( \pm \) variables,

\[
x_{a+} \equiv (x_{a(1)} + x_{a(2)})/2, \quad x_{a-} \equiv x_{a(1)} - x_{a(2)}, \tag{2.3}
\]

such that the equations of motion follow from the variational principle,

\[
\frac{\delta S_{\text{eff}}[x_{\pm}]}{\delta x_{a-}} \bigg|_{\text{PL}} = 0. \tag{2.4}
\]

The “PL” subscript indicates the “physical limit” to be taken wherein all “\( - \)” variables vanish and the “\( + \)” variables are set to their physical values. In other words, in terms of the relative coordinates, \( r_-, v_- \rightarrow 0 \), \( r_+ \rightarrow r \), \( v_+ \rightarrow v \), etc. From here we obtain the (relative) acceleration due to radiation reaction, given by

\[
a_{\text{RR}}^i = \frac{1}{m_v} \left[ \frac{\partial R_{\text{eff}}(r_{\pm}, v_{\pm})}{\partial r_{\pm}(t)} - \frac{d}{dt} \left( \frac{\partial R_{\text{eff}}(r_{\pm}, v_{\pm})}{\partial v_{\pm}(t)} \right) \right]_{\text{PL}}. \tag{2.5}
\]

See [72,73] for a more complete exposition of classical mechanics and field theories for generic, nonconservative systems.
At leading order in Newton’s constant, the effective action is expressed in terms of Feynman diagrams as

\[ iS_{\text{eff}}[x_a^{(\pm)}] = \sum_{\ell \geq 2} \int dt J_{\ell}^{\pm} + \int dt J_{\ell}^{\pm} \]

and yields the following nonconservative piece [19]:

\[ \int R_{\text{eff}} dt = \sum_{\ell \geq 2} \left( -\frac{1}{\ell^2} \ell^2 + 2 \frac{2^{\ell+1} \ell + 2}{\ell (2 \ell + 1)!} \right) dt \left( \frac{2\ell + 1}{2\ell (2 \ell + 1)!} I_{\ell}^{(-)}(t) I_{2\ell+1}^{(+)}(t) + \frac{2^{\ell+3} \ell}{2 \ell (2 \ell + 1)!} I_{\ell}^{(-)}(t) I_{2\ell-1}^{(+)}(t) \right). \]

The superscript \((n)\) in the multipole moments represents the number of time derivatives. For this paper, the lowest mass- and current-type multipole terms suffice to capture the contributions from the leading order spin effects to radiation reaction,

\[ R_{\text{eff}}[p_\pm] = -\frac{1}{5} \frac{I_{ij}^{(5)}(t) I_{ij}^{(5)}(t) - 16}{45} \frac{I_{ij}^{(5)}(t) I_{ij}^{(5)}(t)}{t}. \]

For nonspinning bodies, we have at leading PN order the expressions for the mass quadrupole,

\[ I_{ij}^{(0)-}(t) \equiv I_{ij}^{(0)}(t, x_a^{(1)}) - I_{ij}^{(0)}(t, x_a^{(2)}) = \sum_a m_a \left( x_a^i - x_a^i + x_a^i - x_a^i - \frac{2}{3} \delta^i_j x_a^j \cdot x_a^j \right) + \mathcal{O}(x_a^3), \]

\[ I_{ij}^{(0)+}(t) \equiv \frac{1}{2} \left( I_{ij}^{(1)}(t, x_a^{(1)}) + I_{ij}^{(1)}(t, x_a^{(2)}) \right) = \sum_a m_a \left( x_a^i + x_a^i - \frac{1}{3} \delta^i_j x_a^j \cdot x_a^j \right) + \mathcal{O}(x_a^3). \]

Using the first term of (2.7) in (2.5), we obtain the leading order radiation-reaction piece of the full acceleration,

\[ (a_{\text{LO}}^{\text{RR}})^i = -\frac{2}{5} x_a^i I_{ij}^{(5)}(t). \]

This result is the well-known Burke-Thorne acceleration [61,62] and was derived in the EFT approach in [64,71].

### B. Spinning bodies

#### 1. Basics

The spin of an object is described by a 3-vector. Therefore, a theory that is manifestly Lorentz invariant, which often represents spin by a tensor \( S^\mu \), must unavoidably introduce redundancies. To reduce the system to the required 3 degrees of freedom, a spin supplementary condition must be enforced. The two most popular SSCs are

\[ S^\mu p_\nu = 0 \quad (\text{Covariant}), \]

\[ (p_i^j \frac{p_i^j}{p_0 + m} - S^{ij}) = 0 \quad (\text{Newton–Wigner}), \]

where

\[ a_{\text{SW}} = \kappa S^{ij} n^j, \]

with \( n^\mu \equiv dx^\mu / dt \), and \( \kappa = (1, 1/2) \) in covariant and Newton-Wigner cases, respectively.

The SSCs are second class constraints, implying that their straightforward implementation in the effective action results in a modification of the symplectic structure, which introduces the so-called Dirac brackets; see, e.g., [85]. Nevertheless, we can retain the spin tensor until the end of the calculations, using instead a set of Lagrange multipliers. Because the spin degrees of freedom describe (angular) momentum, a partial Legendre transformation of the effective Lagrangian with respect to the spin tensors yields an effective Routhian. In this framework, the equations of motion for the orbital dynamics and the spin dynamics are found by treating the Routhian as a Lagrangian and as a Hamiltonian, respectively [60].

The study of spin effects in the EFT framework was initiated in [22,23] in terms of an effective action approach, and subsequently elaborated in [24–30] where the Routhian formalism was developed. (See [86] for earlier work in the
context of motion in an external gravitational field). For computational reasons (e.g., to include spin-dependent finite size effects), the covariant SSC is more convenient, in which case the conservative dynamics of spinning bodies can be obtained from the Routhian\(^3\)

\[
\mathcal{R} = -\left( m\sqrt{u^2 + \frac{1}{2} \omega_{\mu}^{\,ab} S_{\mu}^{\,ab} u^\nu + \frac{1}{2m} R_{\nu\rho\sigma\tau} S^{\sigma\tau} u^\nu S^{\rho\sigma} u_\rho + \cdots } \right),
\]

(2.15)

with \( S^{\mu \nu} \) the spin tensor in a locally Minkowski frame described by the tetrad \( e_a^\nu \), \( S^{\mu \nu} \equiv S^{\mu \nu} e_a^\nu e_b^\mu \). The equations of motion follow from

\[
\frac{\delta}{\delta \sigma} \int \mathcal{R} \, d\sigma = 0, \quad \frac{d S^{\mu \nu}}{d\sigma} = \{ S^{\mu \nu}, \mathcal{R} \},
\]

(2.16)

where the orbital motion is derived with \( \mathcal{R} \) playing the role of a Lagrangian (see the left equation above), while the spin dynamics is derived as if \( \mathcal{R} \) were a Hamiltonian (see the right equation above). Here, \( \sigma \) may be chosen as the coordinate time, \( t \), and the spin algebra is given by

\[
\{ S^{\mu \nu}, S^{\rho \sigma} \} = \eta^{\mu \rho} S^{\nu \sigma} + \eta^{\mu \sigma} S^{\nu \rho} - \eta^{\nu \rho} S^{\mu \sigma} - \eta^{\nu \sigma} S^{\mu \rho},
\]

(2.17)

The last term(s) in (2.15) ensures that the SSC is conserved during evolution.

Because of the explicit breaking of Lorentz invariance in the Newton-Wigner SSC, the form of the resulting Routhian is less compact. Moreover, the extra term to ensure the preservation of the SSC contributes already at leading order, unlike the one in (2.15) which enters at next-to-leading order (and only in the \( S_3^3 \) sector). However, when these terms (quadratic in the spin) are ignored, such as finite size effects, it turns out the Newton-Wigner SSC may be enforced prior to using (2.16). The reason being that the resulting Dirac brackets turn into the canonical form, for instance for the spin\(^4\)

\[
\{ S^{(\text{NW})}_i, S^j_{(\text{NW})} \} = -\epsilon^{ijk} S^k_{(\text{NW})},
\]

(2.18)

unlike what occurs in the covariant case. This means that at linear order in the spin we may proceed as usual, after reducing the spin vector using (2.14) with \( \kappa = 1/2 \). See [60] and references therein for more details.

In what follows we will find it convenient to split the spin tensor into 3-vector components,

\[
S^i = \frac{1}{2} \epsilon^{ijk} S^{jk}, \quad S^{(0)} = S^{00},
\]

(2.19)

which are related via the SSC. For the covariant SSC, we will also have to consider the time variation of \( S^{(0)} \), which may be obtained directly from the preservation of the SSC upon time evolution,

\[
\dot{S}^{(0)} \overset{\text{cov SSC}}{\longrightarrow} (a_x \times S_x)^i + \cdots.
\]

(2.20)

2. Conservative dynamics

In order to include all the spin-orbit effects at the desired PN order, we also need to account for the conservative part of the relative acceleration at linear order in the spin, which can be obtained in either SSC [22,28,87],

\[
a^{\text{SO(NW)}}_{\text{cons}} = \frac{1}{r^3} \left\{ \frac{3}{2} \mathbf{n} \cdot \left( 7 \mathbf{S} + 3 \frac{\delta m}{m} \mathbf{\Sigma} \right) - \mathbf{n} \times \left( 7 \mathbf{S} + 3 \frac{\delta m}{m} \mathbf{\Sigma} \right) \right\},
\]

(2.21)

\[
a^{\text{SO(cov)}}_{\text{cons}} = \frac{1}{r^3} \left\{ 6 \mathbf{n} \cdot \left( 2 \mathbf{S} + \frac{\delta m}{m} \mathbf{\Sigma} \right) - \mathbf{n} \times \left( 7 \mathbf{S} + 3 \frac{\delta m}{m} \mathbf{\Sigma} \right) + 3 i \mathbf{n} \times \left( 3 \mathbf{S} + \frac{\delta m}{m} \mathbf{\Sigma} \right) \right\},
\]

(2.22)

where \( \delta m = m_1 - m_2 \), and the spin variables on the right-hand side represent the Newton-Wigner and covariant SSC, respectively. These expressions enter at 1.5PN order in the conservative dynamics. For the spin we find the (conservative) evolution equations [22,28,87],

\[
\dot{S}^{\text{SO(NW)}}_{\text{cons}} = \frac{1}{r^3} \left[ 2 \left( 1 + \frac{3 m_3}{4 m_1} \right) \mathbf{L} \times S_{(\text{NW})} \right],
\]

(2.23)

\[
\dot{S}^{\text{SO(cov)}}_{\text{cons}} = \frac{1}{r} \left[ 2 \left( 1 + \frac{m_3}{m_1} \right) \mathbf{L} \times S_{(\text{cov})} - m_2 (S_{(\text{cov})} \mathbf{x} \times \mathbf{r}) \right],
\]

(2.24)

at linear order in the spins, for both the Newton-Wigner and covariant SSC, respectively. Notice in both cases the spin evolution involves an extra factor of \( v^2, \dot{S} \sim (v^3/r) S \) (since

---

\(^3\)The \( \omega_{\mu}^{\,ab} \) are the Ricci rotation coefficients and \( R^{\nu \rho \sigma \tau} \) is the Riemann tensor.

\(^4\)The minus sign is related to the mostly minus metric convention [60].
The equation of motion for spin will then be given by\(^6\)

\[
\dot{S}_{RR} = \{\{S^i_+, S^j_+\}_{(0)\pm} \}_{PL}.
\]  

where the physical limit PL includes \(S^-_0 \rightarrow 0\) and \(S^i_0 \rightarrow S^i\).

### III. SPIN-ORBIT EFFECTS

#### A. Source multipoles

The multipole moments that are used to compute the spin-orbit radiation-reaction effects are given in [29,60]. In terms of the \(\pm\) variables these are given by

\[
\mathbf{a} = \frac{1}{mv} \left[ \frac{\partial R_{RR}^{\text{eff}} (r_\pm, v_\pm, S_\mu)}{\partial r_\mp (t)} - \frac{d}{dt} \left( \frac{\partial R_{RR}^{\text{eff}} (r_\pm, v_\pm, S_\mu)}{\partial v_\mp (t)} \right) \right]_{\text{PL}}.
\]  

(2.30)

In order to fully describe the spin dynamics from the Routhian (2.29) we will need to extend the spin algebra to the \(\pm\) variables. As discussed in [72], to incorporate generic nonconservative effects the usual Poisson brackets must be generalized. In terms of the doubled phase space variables \((q_\pm, p_\pm)\), the new Poisson brackets are [72]

\[
\{ \{ f, g \} \} = \frac{\partial f}{\partial q_+} p_+ - \frac{\partial f}{\partial p_+} q_+ + \frac{\partial f}{\partial q_-} p_- - \frac{\partial f}{\partial p_-} q_-.
\]  

(2.31)

To obtain the spin algebra we can proceed in two ways. Either we can return to the original formulation in [22] (in terms of the angular velocity) and work out the steps to construct explicitly the spin brackets in the phase space or we can simply use (2.31) to find the algebra for an angular momentum variable, as a generator of the Lorentz group, which must also be satisfied by a spin variable. In both cases we arrive at the following algebra\(^5\):

\[
\{ \{ S^i_+, S^j_+ \} \} = -\frac{1}{4} \epsilon^{ijk} S^k_+.
\]  

\[
\{ \{ S^i_-, S^j_- \} \} = -\epsilon^{ijk} S^k_-;
\]  

\[
\{ \{ S^i_+, S^j_- \} \} = -\epsilon^{ijk} S^k_+;
\]  

\[
\{ \{ S^i_-, S^j_+ \} \} = -\epsilon^{ijk} S^k_{(0)\mp}.
\]  

(2.32)

As mentioned before, the minus sign is due to our convention for the Minkowski metric.

\(^6\)Notice, since the brackets only affect the spin degrees of freedom, we can set to zero the minus variables associated with the orbital motion (e.g., \(r_\rightarrow 0\)) prior to computing (2.33).
\[ I^{ij}_{(0)} = m v [ r^+_i r^-_j + r^-_i r^+_j ]_{\text{TF}}, \]  
\[ I^{ij}_{(0)+} = m v [ r^+_i r^+_j ]_{\text{TF}}, \]  
\[ I^{ij}_{S(0)-} = 2 \nu \left\{ m \left[ \left( \frac{S^i_{(0)1} - S^i_{(0)2}}{m_1} \right) r^+_i + \left( \frac{S^i_{(0)2} - S^i_{(0)1}}{m_2} \right) r^-_i \right] + \frac{1}{3} \varepsilon^{ijk} \xi_{+}^{k} (v^+_i r^-_j - 2r^+_i v^+_j) \right\}_{\text{STF}}, \]  
\[ I^{ij}_{S(0)+} = 2 \nu \left\{ m \left[ \left( \frac{S^i_{(0)1} - S^i_{(0)2}}{m_1} \right) r^+_i + \frac{1}{3} \varepsilon^{ijk} \xi_{+}^{k} (v^+_i r^-_j - 2r^+_i v^+_j) \right] \right\}_{\text{STF}}, \]  
\[ J^{ij}_{(0)-} = -\nu \delta m [ \varepsilon^{ijk} (r^+_i v^+_j r^-_k - r^-_j v^-_k r^+_i + r^-_j v^+_k r^+_i - 2r^-_j v^-_k r^+_i) ]_{\text{STF}}, \]  
\[ J^{ij}_{(0)+} = -\nu \delta m [ \varepsilon^{ijk} (r^+_i v^+_j r^-_k - r^-_j v^-_k r^+_i + r^-_j v^+_k r^+_i - 2r^-_j v^-_k r^+_i) ]_{\text{STF}}, \]  
\[ J^{ij}_{S(0)-} = -\frac{3}{2} [ \Sigma^{i}_j r^+_j + \Sigma^{j}_i r^+_j ]_{\text{STF}}, \]  
\[ J^{ij}_{S(0)+} = -\frac{3}{2} [ \Sigma^{i}_j r^+_j + \Sigma^{j}_i r^+_j ]_{\text{STF}}, \]  
where “(S)TF” indicates the (symmetric) trace-free part of the quantity inside the brackets. We have ignored also terms involving products of minus variables (e.g., \( S^i_{(0)1} \)), which do not contribute to the equations of motion or other physical quantities [72]. Moreover, in the Newton-Wigner gauge we may apply the SSC prior to using (2.30) and (2.33), in which case we have
\[ I^{ij}_{S(0)} \overset{\text{NW SSC}}{\longrightarrow} \frac{\nu}{3} [ \varepsilon^{ijk} \xi_{+}^{l} (5v^+_i v^+_j r^+_l - 4v^+_i v^+_j r^-_l + 4v^+_i v^+_j r^+_l - 5v^+_i v^+_j r^-_l + 4v^+_i v^+_j r^+_l) + \varepsilon^{ijk} \xi_{+}^{l} (5v^+_i r^+_l - 4v^+_i r^+_l) ]_{\text{STF}}, \]  
\[ I^{ij}_{S(0)+} \overset{\text{NW SSC}}{\longrightarrow} \frac{\nu}{3} [ \varepsilon^{ijk} \xi_{+}^{l} (5v^+_i r^+_l - 4v^+_i r^+_l) ]_{\text{STF}}. \]  

**B. Acceleration**

In what follows we derive the accelerations in both the Newton-Wigner and the covariant SSCs. In the former the spin tensor is reduced prior to applying (2.30) and (2.33), whereas in the latter the reduction is performed only after the equations of motion are obtained.

**I. Newton-Wigner SSC**

We split the computation into pieces, as in [71]. The first term comes from the mass quadrupole,
\[ a_{RR(mq)}^m = -\frac{3}{5m} [ \varepsilon^{ijk} v^k \delta^{lm} + \varepsilon^{mi} \xi^{lj} / \nu ] I^{ij}_{(0)-} - \frac{2}{5} [ \rho^{ij} \delta^{lm} ] I^{ij}_{S(0)-} - \frac{1}{15m} [ \varepsilon^{mi} \xi^{lj} r^j + 4 \varepsilon^{ijk} \xi^l r^+ \delta^{ij} ] I^{ij}_{(0)-}. \]  
After applying the equations of motion, we find
\[ a_{RR(mq)}^m = -\frac{mv}{15} \left\{ (I \cdot \xi) \left[ 15r^2 \left( 42 \frac{m}{r} - 51v^2 + 119r^2 \right) - 2rv \left( 97 \frac{m}{r} - 81v^2 + 405r^2 \right) \right] \right\}. \]  
\[ - (r \times \xi) \left[ 40 \frac{m^2}{r} + 261mv^2 - 15r^2 \left( 41 \frac{m}{r} - 207v^2 \right) - 333rv^4 - 3360r^4 \right] \]  
\[ - \dot{r}^2 (v \times \xi) \left[ 596 \frac{m}{r} - 1233v^2 + 1905r^2 \right]. \]  

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The contribution from the current quadrupole is
\[
a_{RR(cq)}^m = \frac{8}{15m} \left[ \mathbf{\Sigma}^j \delta^i m \right] \mathbf{j}^{i(5)}_{(0)} + \frac{16\delta m}{45m} \left[ e^{ikl} \mathbf{r}^k \mathbf{v}^l \delta^i m + e^{imn} (2v^i \mathbf{r}^j + r^i \mathbf{v}^j) \right] \mathbf{J}^{j(5)}_{S(0)} + \frac{16\delta m}{45m} \left[ e^{imk} r^k \mathbf{r}^j \right] \mathbf{J}^{j(6)}_{S(0)},
\]
(3.13)
yielding
\[
a_{RR(cq)} = -\frac{4\delta m_{5}}{15r^5} \left\{ (\mathbf{\hat{L}} \cdot \mathbf{\Sigma}) \left[ 15i r \left( 12 \frac{m}{r} - 15 v^2 + 35 \mathbf{r}^2 \right) \right] - 4rv \left( 8 \frac{m}{r} - 9 v^2 + 45 \mathbf{r}^2 \right) \right\} \\
- (r \times \mathbf{\Sigma}) \left[ 22 \frac{m^2}{r} - 42mv^2 + 15r^2 \left( 8 \frac{m}{r} - 9 v^2 \right) + 18rv^4 + 105r^4 \right] \\
+ 15r^2 (v \times \mathbf{\Sigma}) \left[ 4 \frac{m}{r} - 3v^2 + 7 \mathbf{r}^2 \right].
\]
(3.14)

Finally, we have the “reduced” part from the leading order term in (2.29). Keeping only the contribution from (2.21) in the time derivatives, we find
\[
a_{RR(red)} = \frac{2mv}{5r^5} \left\{ (\mathbf{\hat{L}} \cdot \mathbf{\chi}) \left[ iv \left( -32 \frac{m}{r} - 255 v^2 + 455 \mathbf{r}^2 \right) \right] + 4rv \left( -7 \frac{m}{r} + 15 v^2 - 60 \mathbf{r}^2 \right) \right\} \\
- 3(r \times \mathbf{\chi}) \left[ 7mv^2 + 3r^2 \left( \frac{m}{r} + 45 v^2 \right) - 15rv^4 - 140r^4 \right] \\
+ 5r^2 (v \times \mathbf{\chi}) \left[ 4 \frac{m}{r} + 33v^2 - 45 \mathbf{r}^2 \right].
\]
(3.15)
The total 4PN radiation-reaction (relative) acceleration in the Newton-Wigner SSC is thus given by
\[
(a_{RR}^{SO(NW)})^m = \frac{3}{5m} \left[ e^{ikl} \mathbf{v}^k \mathbf{\xi}^l \delta^i m + e^{mi} \mathbf{g}^i \mathbf{v}^j \right] \mathbf{j}^{i(5)}_{(0)} - \frac{1}{15m} \left[ 5e^{mi} \mathbf{\xi}^i \mathbf{r}^j + 4e^{ikl} \mathbf{r}^k \mathbf{\xi}^l \delta^i m \right] \mathbf{j}^{i(6)}_{(0)} \\
- \frac{2}{5} \left[ \frac{1}{r^i} \delta^i m \right] \mathbf{j}^{i(5)}_{S(0)} - \frac{2}{5} \left[ \frac{1}{r^i} \mathbf{j}^{i(5)}_{S(0)} \right] \mathbf{\Sigma}^i \delta^j m \\
+ \frac{16\delta m}{45m} \left[ e^{ikl} \mathbf{r}^k \mathbf{v}^l \delta^i m + e^{imn} (2v^i \mathbf{r}^j + r^i \mathbf{v}^j) \right] \mathbf{J}^{j(5)}_{S(0)} + \frac{16\delta m}{45m} \left[ e^{imk} r^k \mathbf{r}^j \right] \mathbf{J}^{j(6)}_{S(0)},
\]
(3.16)
which, after some algebra, becomes
\[
a_{RR}^{SO(NW)} = \frac{2mv}{15r^5} \left\{ 3r \left[ 4(\mathbf{\hat{L}} \cdot \mathbf{S}) \left( 14 \frac{m}{r} - 165 v^2 + 315 \mathbf{r}^2 \right) - 9(\mathbf{\hat{L}} \cdot \mathbf{\xi}) \left( 7 \frac{m}{r} + 40v^2 - 70 \mathbf{r}^2 \right) \right] \\
- rv \left[ 8(\mathbf{\hat{L}} \cdot \mathbf{S}) \left( 29 \frac{m}{r} - 54v^2 + 225 \mathbf{r}^2 \right) + 3(\mathbf{\hat{L}} \cdot \mathbf{\xi}) \left( 53 \frac{m}{r} - 93v^2 + 375 \mathbf{r}^2 \right) \right] \\
- 2(r \times \mathbf{\Sigma}) \left[ 22 \frac{m^2}{r} + 21mv^2 + 3r^2 \left( 49 \frac{m}{r} + 360v^2 \right) - 117rv^4 - 1155r^4 \right] \\
+ 3(r \times \mathbf{\xi}) \left[ 8 \frac{m^2}{r} - 103mv^2 + r^2 \left( 169 \frac{m}{r} - 1215v^2 \right) + 135rv^4 + 1260r^4 \right] \\
+ 120r^2 (v \times \mathbf{\Sigma}) \left[ 9v^2 - 13 \mathbf{r}^2 \right] - r^2 (v \times \mathbf{\xi}) \left[ 88 \frac{m}{r} - 1269v^2 + 1755 \mathbf{r}^2 \right] \right\}.
\]
(3.17)

2. Covariant SSC

Once again we split the radiation-reaction acceleration into separate terms. The only new expression relative to what we computed in the Newton-Wigner (NW) SSC, in terms of the source multipole moments, is given by the mass quadrupole
which becomes, after applying the equations of motion (and the covariant SSC prior to taking the time derivatives),

\[ a_{RR(mq)}^{m} = - \frac{2}{5m} \left( \frac{m}{m_1} S_{(0)}^i - \frac{m}{m_2} S_{(0)}^i \right) \delta^{im} + e^{imk} \xi^k v^j + e^{ijk} \xi^k \delta^{jm} \right] f_{(0)}^{j(5)} + \frac{2}{15m} \left[ e^{imk} \xi^k r^j + 2e^{ijk} \xi^k \delta^{jm} \right] f_{(0)}^{j(6)} + \frac{2}{5} \left[ r^i \delta^{jm} \right] f_{(0)}^{j(5)}, \tag{3.18} \]

The current quadrupole and reduced term remain formally the same as in the Newton-Wigner SSC. However, for the reduced part we input (2.22) instead of (2.21) and obtain

\[ a_{RR(red)}^{m} = - \frac{4mv}{15r^6} \left\{ \left( \mathbf{L} \cdot \xi \right) \left[ 15ir \left( -6 \frac{m}{r} - 3v^2 + 7r^2 \right) \right] + 2r \left( 7 \frac{m}{r} + 9uv^2 - 45r^2 \right) \right\} + \left( r \times \xi \right) \left[ 8 \frac{m^2}{r} + 47mv^2 + 15r^2 \left( -7 \frac{m}{r} + 39v^2 \right) - 63rv^4 - 630r^4 \right] + 5r^2 (v \times \xi) \left[ 116 \frac{m}{r} - 243v^2 + 375r^2 \right]. \tag{3.19} \]

Collecting all the pieces together, we find

\[ a_{RR}^{SO(cov)} = \frac{2mv}{15r^6} \left\{ 3ir \left[ 4 \left( \mathbf{L} \cdot S \right) \left( 14 \frac{m}{r} - 165v^2 + 315r^2 \right) + \left( \mathbf{L} \cdot \xi \right) \left( \frac{m}{r} - 540v^2 + 980r^2 \right) \right] - r \left[ 8 \left( \mathbf{L} \cdot S \right) \left( 29 \frac{m}{r} - 54v^2 + 225r^2 \right) + 9 \left( \mathbf{L} \cdot \xi \right) \left( 31 \frac{m}{r} - 43v^2 + 175r^2 \right) \right] \right\} - 2 \left( r \times S \right) \left[ 22 \frac{m^2}{r} + 21mv^2 + 3r^2 \left( 49 \frac{m}{r} + 360v^2 \right) - 117rv^4 - 1155r^4 \right] + \left( r \times \xi \right) \left[ 28 \frac{m^2}{r} - 205mv^2 + 9r^2 \left( 33 \frac{m}{r} - 295v^2 \right) + 297rv^4 + 2730r^4 \right] + 120r^2 (v \times S) \left[ 9v^2 - 13r^2 \right] + r^2 (v \times \xi) \left[ 68 \frac{m}{r} + 981v^2 - 1305r^2 \right]. \tag{3.21} \]

**C. Spin evolution**

For the spin evolution we use (2.33) together with the Routhian in (2.29) and find

\[ \dot{S}_{aRR}^{k} = \frac{1}{9} \left[ f_{(0)}^{j(5)} \left\{ \{ S_{a}^{k}, I_{S(0)-}^{j} \} \right\} + \frac{16}{9} f_{(0)}^{j(5)} \left\{ \{ S_{a}^{k}, J_{S(0)-}^{j} \} \right\} \right]_{PL}. \tag{3.22} \]

---

7This is allowed since the SSC is conserved upon evolution.
To evaluate the right-hand side, we use the multipole moments from Sec. III A and, depending on the SSC, the spin algebra in (2.32). As we show next, there is no radiation reaction in the evolution for spin-orbit effects since the spin evolution equation can be written entirely as the time derivative of a redefined spin vector.

1. Newton-Wigner SSC

In this case we can enforce the SSC prior to applying the brackets. Using (3.7) and (3.9), we have (e.g., for particle 1)

\[
(S_{1,RR}^{(SW)})^k = -\frac{\nu}{15} \left[ \Gamma_{ij}^{(S)} r^i r^j - 4 e^{ij} r^i r^j \right] \{ \{ S_{1,i}^{(S)}, \mathbf{e}_j^S \} \} - 8 j_{ij}^{(S)} r^i \{ \{ S_{1,i}^{(S)}, \mathbf{e}_j^S \} \} \],
\]

for the radiation-reaction evolution equation to linear order in the spins. Using the spin-independent equations of motion to reduce the derivatives on the multipole we obtain

\[
S_{1,RR}^{(NW)} = \frac{4 m v^3}{15 r^2} (L \times S_1) \left[ \left( -22 \frac{m}{r} + 36 v^2 - 60 \dot{r}^2 \right) + \frac{m_2}{m_1} \left( 16 \frac{m}{r} - 48 v^2 + 75 \dot{r}^2 \right) \right].
\]

It is easy to show the right-hand side is a total derivative that can be absorbed into a redefinition of the spin

\[
S_1 \rightarrow S_1 - \frac{2 m v^3}{15 r^2} (L \times S_1) \left[ \left( 3 \frac{m}{r} - 8 v^2 + 24 \dot{r}^2 \right) + \frac{m_2}{m_1} \left( 6 \frac{m}{r} + 12 v^2 - 30 \dot{r}^2 \right) \right]
\]

such that the new spin variable is insensitive to radiation-reaction effects to 4PN order.

2. Covariant SSC

The spin evolution equation can also be obtained in the covariant SSC, using (3.22) and the spin algebra in (2.32). Applying (2.20) after computing the brackets we obtain

\[
(S_{1,RR}^{(c)})^k = -\frac{2 \nu}{15} \left\{ \Gamma_{ij}^{(S)} m_2 m_1 \left[ 3 S_{1,i}^{(c)} v^i r^j - 4 v^k S_{1,i}^{(c)} r^j + 2 S_{1,i}^{(c)} r^k v^j + 8 \delta^i j S_{1}^{(c)} (v^i r^j - 2 r^i v^j) - 4 m_1 j_{ij}^{(S)} \right] e^{ij} r^i S_{1}^{(c)} \right\},
\]

leading to

\[
S_{1,RR}^{(c)} = \frac{4 m v^3}{15 r^2} \left\{ \dot{r} (L \times S_1) \left[ -22 \frac{m}{r} + 36 v^2 - 60 \dot{r}^2 + \frac{m_2}{m_1} \left( 16 \frac{m}{r} - 48 v^2 + 75 \dot{r}^2 \right) \right] \right. \\
- \frac{m_2}{m} \left[ -2 S_1 \left( 6 m v^2 + r \dot{r}^2 \left( 2 \frac{m}{r} + 99 v^2 \right) - 18 r v^4 - 75 r \dot{r}^4 \right) \\
+ \dot{r} (r S_1 \cdot v) + v (S_1 \cdot r) \left( 2 \frac{m}{r} + 54 v^2 - 75 \dot{r}^2 \right) + 6 v (S_1 \cdot v) \left( 2 \frac{m}{r} - 6 v^2 + 15 \dot{r}^2 \right) \right\}. 
\]

The spin dynamics in the covariant SSC is not a total derivative due to the spin definition in this gauge. However, it is easy to see the above result is equivalent to our derivation in the Newton-Wigner case. Recall that the transformation between the two spin variables is given by [see (2.25)]

\[
S_{1,(NW)} = S_{1,(c)} + \frac{1}{2} (v_1 (v_1 \cdot S_{1,(c)}) - S_{1,(c)} v_1^2) + \cdots,
\]

which implies

\footnote{This is not surprising since already at leading order the spin evolution equation for the covariant SSC does not conserve the norm of the vector, unlike the Newton-Wigner case. See (2.24) and (2.23).}
\[ \dot{S}_{1}(NW) = \dot{S}_{1}(c) - \frac{1}{2} \left( \frac{m_{2}}{m} \right)^{2} [2S_{1}(c) \cdot (a \cdot \nu)] + \cdots. \]  

(3.29)

Hence, inputting the leading order dissipative part of the relative acceleration [see (2.10)],

\[ (a_{\text{LO}}^{\text{RR}})^{i} = -\frac{2}{5} \rho^{i(j(5)}_{(0)}, \]  

(3.30)

into the terms in the square brackets on the right-hand side, and using (3.26), we recover (3.23) as expected.

**IV. CONSISTENCY TEST**

In this section we prove the equivalence between the power emitted at infinity via the multipole formula and the power induced by the radiation-reaction force, up to contributions that can be shown to be total time derivatives that average to zero for bound orbits.

**A. Schott terms**

The radiated total power can be obtained from the effective action in (2.1) [60,83,84],

\[ P_{\text{RR}} = -\frac{1}{5} \nu^{i} \left\{ \left[ \frac{\partial}{\partial r_{-}} - \frac{d}{dt} \frac{\partial}{\partial \nu_{-}} \right] \left( I_{+}^{ij(t)}(t) + \frac{16}{9} J_{+}^{ij(5)}(t) \right) \right\}_{\text{PL}}. \]  

(4.4)

Crucially, the derivatives only act on the minus variables. Let us now take a time average of the above expression,

\[ \langle P_{\text{RR}} \rangle = -\frac{1}{5} \left\{ \left[ \nu^{i} \frac{\partial}{\partial r_{-}} + a^{i} \frac{\partial}{\partial \nu_{-}} \right] I_{+}^{ij(t)}(t) \right\}_{\text{PL}} - \frac{16}{9} \left\{ \left[ \nu^{i} \frac{\partial}{\partial r_{-}} + a^{i} \frac{\partial}{\partial \nu_{-}} \right] J_{+}^{ij(5)}(t) \right\}_{\text{PL}}, \]  

(4.5)

where we integrated by parts the time derivative in the second term in both of the square brackets. Noticing that

\[ \left[ \nu^{i} \frac{\partial}{\partial r_{-}} + a^{i} \frac{\partial}{\partial \nu_{-}} \right] I_{+}^{ij(t)}(t) = I_{+}^{ij(1)}, \]  

(4.6)

we find

\[ \frac{dE}{dt} = -\sum_{\ell=2}^{\infty} \frac{(\ell + 1)(\ell + 2)}{\ell(\ell - 1)(\ell + 1)!!} (I^{(\ell+1)})^{2} + \frac{4\ell(\ell + 2)}{(\ell - 1)(\ell + 1)!!} (J^{(\ell+1)})^{2}, \]  

(4.1)

which yields, to the order we work here,

\[ \frac{dE}{dt} = -\frac{1}{5} \left[ I^{(3)}(I^{(3)} + \frac{16}{9} J^{(3)}J^{(3)}) + \cdots. \right. \]  

(4.2)

The energy flux can also be obtained directly by computing the power induced by the radiation-reaction force,

\[ P_{\text{RR}} \equiv m \dot{a}_{\text{RR}} \cdot \nu. \]  

(4.3)

However, in one case the power is computed using the radiation field in the far zone, obtaining (4.1), whereas the radiation-reaction force acts “instantaneously” on the dynamics of the bodies. The difference, nevertheless, is a local redefinition of the conserved energy (i.e., a total time derivative) that will not affect the radiated power in the far region. These effects are often denoted as Schott terms, since they also appear in electrodynamics and, consequently, at leading order in the radiation-reaction force (e.g., see [19]).

For nonrotating bodies the equivalence is almost straightforward. For instance, let us consider the leading order effective Lagrangian in (2.7). Then, according to the definition in (4.3),

\[ \langle P_{\text{RR}} \rangle = \left\langle \frac{dE}{dt} \right\rangle, \]  

(4.7)

which implies

\[ P_{\text{RR}} = \frac{d\tilde{E}}{dt}, \quad \tilde{E} \equiv E - E_{S}. \]  

(4.8)

with \( E_{S} \) the Schott terms [19]. The latter are given by

\[ E_{S} = \frac{1}{5} \left( I^{(1)}(I^{(4)} - I^{(2)}I^{(3)}) + \frac{16}{45} (J^{(1)}J^{(4)} - J^{(2)}J^{(3)}) + \cdots. \right. \]  

(4.9)

The equivalence, which can be proved to all orders, becomes a nontrivial consistency check when translated to the final expressions in terms of the variables of the problem.
B. Spinning bodies

When spin is incorporated, the equivalence becomes a little less straightforward. The effective Lagrangian now depends on the spin tensor, $S^{ab}$, which is a new dynamical variable. However, at leading order, the spin vector simply plays the role of a constant source.\footnote{At lowest PN order the time variation of the spin vector is 1PN order higher than the naive power counting would suggest \cite{22,60}.} Hence, for the case of the Newton-Wigner SSC (where the spin tensor is reduced prior to obtaining the equations of motion) the above proof applies unaltered. This is not the case in the covariant gauge.

where the $S^{0i}$ components must be kept until the end of the calculation and the equation of motion for $S^{0i}$, which follows from the conservation of the SSC (2.20), cannot be ignored. We explicitly demonstrate below how the consistency check applies in both cases.

1. Newton-Wigner SSC

The calculation of the spin-orbit radiation-reaction power in (4.3) is straightforward. Using (3.16) we obtain

\begin{equation}
\frac{dE}{dt}_{\text{SO(NW)}} = \frac{8m^2 \nu}{15r^6} \left[ (L \cdot S) \left( \frac{12 m}{r} + 37 v^2 - 27 \dot{v}^2 \right) + (L \cdot \xi) \left( \frac{8 m}{r} + 19 v^2 - 18 \dot{v}^2 \right) \right],
\end{equation}

which agrees with the literature \cite{87}. Comparing both expressions, we find

\begin{equation}
\frac{dE}{dt}_{\text{SO(NW)}} - \frac{dE}{dt}_{\text{RR}} = \frac{4m^2 \nu}{15r^6} \left\{ (L \cdot S) \left[ 2 \frac{m^2}{r} + 169mv^2 + 15r\dot{r} \left( -19 \frac{m}{r} + 54 v^2 \right) - 99rv^4 - 735r\dot{r}^4 \right] + (L \cdot \xi) \left[ 28 \frac{m^2}{r} - 37mv^2 + 24r\dot{r} \left( 13 \frac{m}{r} - 30 \dot{v}^2 \right) + 63rv^4 + 945r\dot{r}^4 \right] \right\}.
\end{equation}

This can be shown to be a total time derivative, such that the expression in \cite{80} holds with a Schott term given by\footnote{The second derivative of the positions entering in the leading order multipole moment ($I^{(i)}_{(0)}$ in the first term) must be replaced here only by the spin-orbit acceleration in (2.21).}

\begin{equation}
E_{S}^{\text{SO(NW)}} = \frac{1}{5} \left[ I_{(0)}^{ij(1)}I_{(0)}^{ij(4)} - I_{(0)}^{ij(2)}I_{(0)}^{ij(3)} \right] + mv(L \cdot S) \left( \frac{88 m\ddot{r}}{15 r^3} - \frac{72 v^2 \dot{r}}{5 r^4} + \frac{24 \dot{r}^2}{r^3} \right) + mv(L \cdot \xi) \left( -\frac{8 m\ddot{r}}{3 r^3} + \frac{129 v^2 \dot{r}}{5 r^4} - \frac{39 \dot{r}^2}{r^3} \right).
\end{equation}

2. Covariant SSC

The covariant case is a little more involved, since the binding energy now depends on $S_{(0)}$, which cannot be taken as a constant at leading order, unlike the spin 3-vector. Therefore, the energy balance acquires an extra term compared to the Newton-Wigner calculation. At linear order in the spin we have

\begin{equation}
\left\langle \frac{dE}{dt}_{\text{SO}} \right\rangle = \left\langle \left[ \frac{\partial E}{\partial v^i} \frac{\partial}{\partial v^a} + \sum_{a} \frac{\partial E}{\partial S_{(0)a}^{i}} \frac{\dot{S}_{(0)a}^{i}}{a_{L,a}} \right] \right\rangle = \left\langle mv a_{L}^{\text{RR}} + \sum_{a} \frac{\partial E}{\partial S_{(0)a}^{i}} \dot{S}_{(0)a}^{i} \right\rangle.
\end{equation}

Preserving the covariant SSC during evolution [see (2.20)] implies

\begin{equation}
\dot{S}_{(0)a}^{i} = a_{L,a}^{\text{RR}} \times S_{a} + \cdots,
\end{equation}
where we used only the nonspinning radiation-reaction part of the acceleration in (2.10) since all other (conservative) terms cancel out in (4.14). On the other hand, the spin-orbit energy can be written as [22,28,60]

\[ E_{SO} = -\sum a_{Na} \cdot S_{(0)a} + \cdots, \tag{4.16} \]

with \( a_{Na} \) the Newtonian acceleration for each body, which gives

\[ \frac{\partial E_{SO}}{\partial S_{(0)a}} = -a_{Na}^t + \cdots. \tag{4.17} \]

From here we find

\[ a_{RR}^{SO(cov)} \cdot v = 4 \frac{1}{15r^6} \left\{ (L \cdot S) \left[ \frac{22}{r} \frac{m^2}{r} - 95m v^2 + 3r \left( 77 \frac{m}{r} - 270 v^2 \right) + 99r v^4 + 735r^4 \right] + (L \cdot \xi) \left[ -14 \frac{m^2}{r} - 37r v^2 - 3r \left( 49 \frac{m}{r} + 90 v^2 \right) + 45r v^4 + 105r^4 \right] \right\}, \tag{4.19} \]

and

\[ \frac{2\nu}{5} \epsilon^{kli} a^k a^l r_i i^{(5)} = -\frac{8m^2 \nu}{5r^6} (L \cdot \xi) \left( 2 \frac{m}{r} - 6v^2 + 15r^2 \right). \tag{4.20} \]

On the other hand, the computation of the total radiated power using (4.2) yields

\[ \left( \frac{dE}{dt} \right)_{SO(cov)} = \frac{8m^2 \nu}{15r^6} \left[ (L \cdot S) \left( 12 \frac{m}{r} + 37v^2 - 27r^2 \right) + (L \cdot \xi) \left( -4 \frac{m}{r} + 43v^2 - 51r^2 \right) \right], \tag{4.21} \]

also in agreement with the literature [87]. It is then straightforward to show that the difference is a total time derivative, such that the expression in (4.18) holds with

\[ E_{SO}^{SO(cov)} = \frac{1}{5} \left[ i^{(1)} _{(0)} i^{(2)} _{(0)} - i^{(1)} _{(0)} i^{(3)} _{(0)} \right] + m v (L \cdot S) \left( \frac{88m \nu}{15r^5} - \frac{72v^2 \nu}{5r^4} + 24 \frac{\nu^2}{r^2} \right) + m v (L \cdot \xi) \left( \frac{8m \nu}{5r^5} + 36 \frac{v^2 \nu}{r^4} - 52 \frac{\nu^2}{r^2} \right), \tag{4.22} \]

for the Schott term.

V. CONCLUSIONS

We incorporated radiation-reaction effects due to spin in the dynamics of compact binary systems within the EFT formalism [60]. We extended the nonconservative approach developed in [71–73] to spinning bodies, and we computed the spin-orbit contributions to the acceleration and spin evolution to 4PN order from first principles, in both the covariant and the Newton-Wigner SSCs. In order to test the consistency of our results, we explicitly showed that the induced power resulting from the radiation-reaction force is equivalent to the total radiated emission computed in the far zone, using the standard multipole expansion [4]. We find there is no net effect on the spin evolution from radiation reaction at this order, which is consistent with the findings in [75]. The results presented here complete the knowledge of the binary’s dynamics to 4PN order within an EFT framework [60], which will play a key role in the

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forthcoming era of gravitational wave observations. Our work also paves the way for higher order calculations. We present the leading contributions from radiation reaction to the binary’s dynamics due to spin-spin effects in a companion paper [82].

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