DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

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PASADENA, CALIFORNIA 91125

STADIUM CAPACITIES AND ATTENDANCE IN PROFESSIONAL SPORTS

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SOCIAL SCIENCE WORKING PAPER 54

October 1974

STADIUM CAPACITIES AND ATTENDANCE IN PROFESSIONAL SPORTS *

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INTRODUCTION

Over the past fifteen years, professional team sports has become an increasingly important part of our social and economic life. While in 1959 there were just forty-two major league teams in five monopolistic leagues (AL, NL; NBA; NHL; NFL), by 1974 this had grown to 117 teams in eight major leagues (AL, NL; NBA, ABA; NHL, WHA; NFL, WFL). Accompanying this growth in professional team sports has been a wave of stadium construction, with new stadiums appearing in essentially every medium to large size city in the U.S. In contrast to stadiums in use in professional sports fifteen years ago these new stadiums have two distinguishing features -- with rare exceptions, they are publicly owned and they are multiple-use facilities, designed to accommodate several sports.

There are any number of issues raised for the economist by the growth of professional sports and by the role played by local governments and the federal government in encouraging that growth. The recent Brookings volume, Government and the Sports Industry, addresses many of the issues. The present paper restricts itself to questions concerning the relationships between attendance and stadium capacities in sports. The paper has two main objectives: first to present a summary picture of the patterns of attendance and

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* Tables referred to in the text are presented in the appendix to this paper.

capacities across and within sports leagues; and, second, to indicate some possible explanations for the patterns that emerge. It should be emphasized that this is at best only a preliminary study and the conclusions reached are still rather tentative.

The first few sections of the paper deal with a limited amount of empirical evidence concerning patterns of attendance and capacities in sports leagues. Most of the information comes either the league publications, sports guides, or from Sporting News. The later sections of the paper are concerned with the decision problems faced by team owners and government officials relative to "short run" and "long run" choices of attendance and capacity.

ATTENDANCE AND CAPACITY -- 1973-4 SEASON

Any knowledgeable sports buff is aware of the fact that NFL games are sellouts and that tickets to NHL games are next to impossible to obtain. On the other hand, baseball tickets are always available (except for playoffs and the World Series), and in most cities this is true for basketball as well. This raises the general question as to why such differences should exist among these various sports, something which will be pursued in some detail later.

But first it is worthwhile examining the actual attendance and capacity data for the major sports leagues to see how pronounced the differences in fact are. That differences exist is indicated by the following summary of attendance, capacity and attendance/capacity ratios for the 1973-4 season:

	Average Attendance Per game (000)	Capacity Per game (000)	A/C %
Hockey (1973-4)			
NHL	14.5	15.8	92 %
WHA	5.8	11.3	51 %
Football (1973			
NFL	55.9	62.0	90 %
Baseball (1973)			
NL	18.5	48.8	38 %
AL	14.8	49.0	30 %
Basketball (1973-4)			
NBA	8.8	14.1	62 %
ABA	5.5	10.5	52%

These data in turn represent overall averages of data for individual teams, as shown in Table 1 below. Certain comments are in order concerning the differences among sports, beyond those indicated with respect to attendance and capacity. In hockey, both the NHL and WHA play seventy-eight game seasons, while the NFL plays a fourteen game season (exclusive of exhibitions). The AL and NL play 162 games per year, the NBA eighty-two games and the ABA eighty-four games. It might well be the longer playing schedule of baseball that accounts in part for the lower A/C ratio for baseball relative to the other sports. On the other hand, this certainly is not an explanation for the marked difference between the A/C ratios in the NHL and the NBA. Baseball plays its regular season over a six-month period, football (NFL) over 3-1/2 months (plus 1-1/2 months of exhibitions and one month of playoffs plus Super Bowl), hockey stretches from October through March (plus one month of Stanley Cup action), and basketball has a comparable length of playing time. Thus each of the sports has roughly a six month exposure to the public per year. Finally, weather plays a role in determining attendance at football and baseball games but is less important so far as hockey and basketball is concerned.

Turning to the individual teams making up the seven major leagues during 1973-4, in the NHL we see that eleven of the seventeen teams played to over ninety percent of capacity for the season, with only Los Angeles and California falling below seventy percent of capacity. Following the 1973-4 season, the California club was purchased by the NHL from Charlie Finley. It is of interest that ten of the seventeen teams play in stadiums either owned by corporations that in turn own the teams or stadiums that are privately owned -- New York Rangers, Chicago, Montreal, Toronto, Boston, Detroit, Los Angeles, St. Louis, Minnsota and Philadelphia.

The remaining seven teams play in publicly owned facilities. Three privately owned arenas have been built in the recent past -- the new Madison Square Garden, the L. A. Forum and the Spectrum in Philadelphia. The New York Rangers (NHL) and New York Knicks (NBA) are owned by the Madison Square Garden Corporation, while the L. A. Kings (NHL) and L. A. Lakers (NBA) are owned by Jack Kent Cooke, who also owns the L. A. Forum.

In the WHA, only three teams play to over seventy percent of capacity -- Quebec, Houston and Edmonton. The Edmonton arena is clearly too small for major league hockey and a new facility is under construction there. Houston's success is certainly due to (1) a WHA championship and (2) the presence of Gordy Howe and his sons. Following the 1973-4 season, New England is moving from Boston to Hartford, the L. A. franchise is now in Detroit, and

Jersey has departed to San Diego. These three had the lowest A/C ratios in the league.

Attendance data for the NFL are not completely comparable with data for the other sports, representing actual attendance rather than tickets sold. During the 1973-4 season the blackout of home TV for sellout games was removed by the Congress, and actual attendance figures fell because of this, particularly in Miami, which had a season sellout. Only one team shows any pronounced weakness at the gate, namely Houston, which had the worst record in the NFL and was not affected by the removal of the blackout. Of the twenty-six teams in the NFL, only New England, Dallas and Green Bay (four of seven games) play in stadiums owned by the team. Interestingly, both New England and Dallas have built their stadiums within the past five years.

Data on baseball point out the superior drawing power of the NL versus the AL. Of the twelve NL teams, ten play in stadiums built since 1960; only Chicago and Montreal are in "old" stadiums. In contrast, five of the AL teams play in "new" stadiums -- Oakland, Kansas City, Minnesota, California and Texas -- while seven teams are in older facilities. Stadiums owned by teams appear in Los Angeles (NL), Chicago (NL), Boston (AL), Detroit (AL), St. Louis (NL) and Chicago (AL) while New York (AL) plays in a privately owned stadium. Two team owned stadiums have been built over the past twenty-five years, Dodger Stadium, and Busch Stadium (St. Louis). In contrast to football, there is a wide variance among teams in baseball in attendance, capacity and in A/C ratios, with A/C varying from twelve percent in Cleveland to sixty-one percent in Montreal.

Finally, there is the "feast or famine" sport -- basketball. While profit data for sports teams are not available on an across-the-board basis, there is general agreement that the New York Knicks are the most profitable team in sports, playing to virtual sellouts on a season-long basis, and charging the highest ticket prices in the NBA.

Two other teams in the NBA are also close to being virtual sellouts -- Seattle and Milwaukee -- with the L.A. Lakers not far behind. Note that the Boston Celtics, NBA champions for the 1973-4 season, drew only seventy percent of capacity. At the other extreme from the "feast" teams are Houston, Cleveland and Philadelphia, drawing less than an average ABA team.

The ABA also has its two tiers of teams, with the New York Nets (ABA champions), Kentucky, Utah and Indiana being the strong teams (though at best marginally profitable), while Virginia, Memphis and San Diego play to empty arenas. Given the "feast-famine" status of the NBA and the gap between the strong and weak teams in the ABA, congressional action to modify the proposed NBA-ABA merger terms becomes understandable. Only New York (NBA) and Los Angeles (NBA) play in team owned arenas. Boston, Chicago, Los Angeles, New York, Philadelphia, Buffalo, Golden State, all in the NBA, play in arenas that are also used for hockey.

DISTRIBUTION OF ATTENDANCE BY GAMES -- 1973-4 SEASON

Tables 2-7 show the distribution of attendance by games in all leagues, excluding the NFL. The tables were constructed in order to determine the number of games during a season when the capacity constraint was effective, as well as to determine the importanceof sellouts or near sellouts on season-long attendance. The contrasts among leagues with respect to sellouts are more pronounced than the constrasts with respect to A/C ratios, as the following summary indicates. (A sellout is defined as attendance at or above 95% of capacity).

League	Number of Sellouts	% Sellouts / Total Games
NHL	396	63%
WHA	38	8%
NBA	143	21%
ABA	18	4%
NL	29	3%
AL	26	3%

(Data for the NFL were not tabulated, because data available from Sporting News gives attendance and not ticket sales).

Within the NHL, nine teams sold out thirty-eight or thirty-nine of their thirty-nine home games, with five teams having sellouts for eleven or fewer games. In the WHA, only Edmonton has a record of frequent sellouts. In the National League, even Cincinnati sold out only seven games, with the Dodgers and the Expos selling out five times during the year. Boston and Kansas City, both with relatively small parks, led the sellouts in the AL, with seven and five respectively. In the NBA, the Knicks sold out thirty-six of forty-one games, with Milwaukee and Seattle selling out twenty-six and twenty-eight times. It should be noted that both of these latter teams have relatively small arenas. Los Angeles (12), Boston (10) and Detroit (9) are the only other teams with a sizeable number of sellouts. In the ABA, Indiana (5), San Antonio (5) and Kentucky (4) account for most of the sellouts. The Kentucky figure might be overstated, since a number of their games were played in Freedom Hall with a 20,000 capacity.

Briefly, it appears that capacity operates as an effective constraint on attendance in most NHL cities, in Edmonton of the WHA, is of little importance in baseball or in the ABA, and is of relevance for three to six cities in the NBA. In the absence of deterioration of the desirable/undesirable seat ratio as capacity increases, NFL attendance would certainly rise if capacities were increased for that league as well.

The importance of sellouts or near sellouts to league attendance may also be seen from the following data.

PERCENT OF LEAGUE ATTENDANCE

	Games At Less Than 1/3 Capacity	Games At 1/3 - 2/3 Capacity	Games At 2/3 Capacity & Above	Games At 90% Capacity & Above
NHL	1 %	10 %	89 %	78 %
WHA	15 %	57 %	28 %	67 %
NBA	11 %	34 %	55 %	35 %
ABA	10 %	35 %	55 %	30 %
NL	26 %	64 %	20 %	10 %
AL	48 %	29 %	13 %	6 %

ATTENDANCE AND STADIUM CAPACITIES, NATIONAL LEAGUE 1960 - 1974

Another dimension of the attendance-capacity issue is the intertemporal changes that have occurred. Table 8 below summarizes the capacity and ownership characteristics of stadiums in organized baseball for 1960 and 1974. Over this fifteen year period, eight new teams were added to the major leagues, and six franchise moves occurred. There were fourteen new stadiums built, of which twelve are publicly owned. Except for the Dodgers' move from the Coliseum to Chavez Ravine, every new stadium has a larger capacity than the stadium it replaced. On average, the new stadiums seat roughly 20,000 more spectators than the stadiums being replaced. Where the average 1960 stadiums had a capacity in the 30-50,000 range, the new stadiums are in the 40-60,000 range. This shift occurred despite the fact that just seven percent of the AL attendance in 1973 occurred for gates of 40,000 and over, and only eleven percent of NL attendance. There is no pronounced secular increase in per game attendance in baseball, hence the larger stadiums do not reflect a building for the future. Instead the basic explanation for the higher capacities must lie in the joint occupancy of the stadiums by football and baseball; the higher capacities for baseball are a side "benefit" derived from the fact that the NFL teams actually need larger stadiums.

For example, in the NL, only Chicago, San Diego and Los Angeles play in single purpose stadiums (and Wrigley Field was the home of the Bears until just the last few years). In the AL, California, Chicago, Kansas City, Texas and Boston all play in single purpose stadiums (but the WFL is now using Anaheim Stadium). It will be noted that stadiums that are for baseball alone have smaller capacities on average than the multi-purpose stadiums -- both because there is no need for the larger capacity and because smaller stadiums, like Fenway Park or Wrigley Field, make baseball watching more enjoyable, and hence enhance attendance.

Table 9 presents time series on attendance and stadium capacities for the NL from 1960-74. Two aspects of these data are worth noting: first, A/C ratios have generally remained relatively constant over time at roughly the forty percent level, despite a general upward trend in attendance per game; second, new stadiums generally encourage an increase in attendance for several years following the opening of the stadium. In the table, asterisks mark the first year of operation for a new stadium. Cincinnati attendance jumped from 13,500 to 23,400 per game; Houston from 9,200 to 26,900; Los Angeles from 24,100 to 35,300; the Mets from 15,600 to 26,100; the Phillies, from 9,700 to 20,100; Pittsburgh from 11,000 to 17,700; St. Louis from 16,600 to 23,800. Only San Diego (8,000 to 9,200) showed no pronounced increase. Typically, this first year bonanza attendance tails off as the stadium becomes more familiar but it still tends to be higher on average than in the older facility. Hence a part at least of the success of the NL relative to the AL must be ascribed to the numbers of new stadiums in the NL as compared to the AL.

We next turn to a brief discussion of the characteristics of stadiums being used in the various professional sports.

Public versus Private Ownership of Sport Facilities

This section merely summarizes some of the results of Okner's paper [3]. The issue is: whether cities do subsidize sport activity by building sport facilities. There are, obviously, problems of measurement involved in estimating the amount of subsidy to a team. For instance, do indirect effects of having a "big time" sport activity count as revenues to the city? And if the amount of subsidy is positive, is that not in payment for "community welfare" derived from having these activities? If we avoid such questions and look at a stadium as an independent operation, we find that the majority of stadiums (83% in 1970-1971) are operating at a net loss[(1)]. At least some of these losses are due to the general inefficiency that accompanies the existence of an almost free good, so that not all net losses may be counted as subsidies.

To illustrate the extent of subsidies we consider the following table[(2)] of private versus public ownership of sport facilities:

Distribution of Sports Facilities, by Ownership and Sport, 1970-71 Season

	Number of facilities				
Ownership	Baseball	Football	Basketball	Hockey	Total[a]
Public	16	20	32	4	53
Private	7	6	7	8	18
University	0	1	4	0	5
Total	23	27	43	12	76

Source: Author's compilation from public records.
a. Sum of individual rows may exceed total because some facilities are used for more than one sport.

(1) See table 6 Okner [3].
(2) Table 1 of Okner [3].

We note that the public owns 70% of the all sport facilities. Why is this the case? How should these facilities be rented to the teams?

We cannot give a complete theoretical account of these questions. After all, rationality must be assumed if we are to carry an economic analysis of these decisions and history is full of examples where communities have not exactly followed a rational path. On the other hand it may turn out that the decision making bodies of communities have been dominated by forces for which it was best that the communities build these facilities. These, however, are factual questions and there is no room for speculation about them.

At any rate, one form of subsidy is in terms of capital expenditure involved in building the facilities. Another form is in terms of particular rental agreements. In order to estimate the magnitude of this last type of subsidy we must find the market rental for the sport facility. In the short run, this is a very hard question in view of the immobility and the instantaneous perishability of the service that is produced, and in view of the stronger position of the buyers of the service relative to that of the sellers.

The approach we will adopt is the following. The next two sections consider the "short run" and "long run" decision problems of a profit oriented owner, so far as choosing attendance for a fixed capacity and so far as choosing a capacity level are concerned. The succeeding section then examines the capacity issue in terms of decision making by a municipality.

Short Run Decision Making

First we consider the short run decision from the owner's point of view, that of the "optimal" attendance for a given stadium capacity. The basic notions are most easily seen in the context of a "one game season," that is, where the decision is made on the basis of prospective attendance for one game.

Let p denote the average ticket price. Then the demand for tickets y is related to p through a demand function of the usual type, that is,

$$y = f(p) \quad \text{where } f'(p) < 0.$$

To keep things really simple, assume that the short run marginal cost is zero, where short run marginal cost is the added expense of serving one more customer within a given capacity. Then the profit maximizing team owner acts to maximize revenue. That is, the owner solves the problem

$$\max pf(p)$$

$$\text{subject to } f(p) \leq \bar{x}$$

Let p^* denote the profit maximizing choice of p. Then p^* is determined by the conditions

$$(1) \quad p^* f'(p^*) + f(p^*) \geq 0 \quad (= 0 \text{ if } f(p^*) < \bar{x})$$

$$(2) \quad \bar{x} - f(p^*) \geq 0$$

Thus it might well be the case that the owner finds it optimal to have an attendance that is less than a sellout. In particular, attendance less than capacity implies from (1) that

$$\frac{df(p^*)}{f(p^*)} \Big/ \frac{dp}{p^*} = -1;$$

that is, at $p^*(f(p^*) < \overline{x})$, the demand curve is of unitary elasticity.

In fact, choosing the ticket price (and thus attendance) at the capacity level implies that demand is elastic (or of unitary elasticity) at $f(p^*) = \overline{x}$. Hence for given capacity, choice of a sellout attendance is more likely the more elastic is the demand curve for the sport.

Before leaving this simple case, a comment is in order concerning the effect of the gate sharing arrangements on the attendance/capacity ratio. If the gate sharing arrangement is of the fixed percentage type (60 percent home, 40 percent visitor as in the NFL; 80-20 home-visitor as in the AL), then this has no effect on the choice of the optimal ticket price and attendance, since the objective function above is simply replaced by $\alpha pf(p)$ where α is the home team's share of the gate. Clearly p^* is unaffected by such a sharing arrangement, given that marginal cost is zero.

On the other hand, the NL employs a fixed fee per ticket arrangement, the visiting team getting 27-1/2¢ per ticket sold. Under such an arrangement, the objective function becomes

$$(p - \beta)f(p)$$

where β is the amount given to the visiting team per ticket sold.

The first order conditions become

$$(1') \quad (\overline{p} - \beta)f''(\overline{p}) + f(\overline{p}) \geq 0$$

$$(2') \quad \overline{x} - f(\overline{p}) \geq 0$$

Assuming a regular maximum $\left(\frac{d^2(p - \beta)f(p)}{dp^2}\right) < 0$, then $\overline{p} \geq p^*$ for $\beta > 0$. $\overline{p} = p^*$ if $f(\overline{p}) = \overline{x}$ and $\overline{p} > p^*$ if $f(\overline{p}) < \overline{x}$. Thus the fixed fee gate sharing arrangement generally leads to a higher price and lower attendance relative to capacity than a fixed percentage gate sharing arrangement.

A more realistic model is one in which the team plays a schedule of games against opponents of varying drawing power. Let y_i denote the demand for tickets for a home game involving team i and the home team, with $y_i = f_i(p)$.

Then the decision problem becomes

$$\max p\sum_{i=1}^{n} f_i(p)$$

$$\text{subject to} \quad f_i(p) \leq \overline{x} \qquad i = 1, \ldots, n$$

$$\text{Let } L = p\sum_{i=1}^{n} f_i(p) + \sum_{i=1}^{n} \lambda_i(\overline{x} - f_i(p))$$

Let $Y = \sum_{i=1}^{n} f_i(p)$ denote the season-long demand for tickets. If p^* is a profit maximizing price, then p^* satisfies

(3) $p^* \frac{\partial Y(p^*)}{\partial p} + Y(p^*) - \sum_{i=1}^{n} \lambda_i \frac{\partial f_i(p^*)}{\partial p} = 0$

(4) $\lambda_i(\overline{x} - f_i(p^*)) = 0 \qquad \lambda_i \geq 0, \ \overline{x} - f_i(p^*) \geq 0 \qquad i = 1, \ldots, n$

Again note that $f_i(p^*) < \overline{x}$ for every i is a possible solution, so long as the season-long demand for tickets is of unitary elasticity at p^* where $f_i(p^*) < \overline{x}$ for every i. This follows from (3) and (4) since $f_i(p^*) < \overline{x}$ for every i implies $\lambda_i = 0$ for every i.

Choosing p^* so that at least one game during the season is sold out implies that the season-long demand is elastic (or of unitary elasticity) at p^* such that $f_i(p^*) = \overline{x}$ for some i. Again, the more inelastic the season-long demand, the less the chance that any game during the season will be sold out.

This raises the question as to why sports teams typically adopt a single schedule of ticket prices that apply to all regular season games, regardless of the drawing power of the opponent. Assuming no costs associated with a variable schedule of prices, it is clear that the team is better off to adjust ticket prices according to the demand functions $f_i(p)$ than to maintain a fixed schedule of prices that apply to all games.

The explanation no doubt rests in part on the uncertainty that would be created for potential customers by a variable ticket price plan; on advertising costs; on the reaction of owners of visiting teams and the gate sharing arrangement; and on the inability at the beginning of a season, to identify the high drawing teams for the season. But in fact there are devices that are employed by teams that have the effect of lowering ticket prices for certain games -- bat day, helmet day, free beer games, and doubleheaders. It would be surprising

to find these extras appearing in games involving the top drawing potential visiting teams.

The discussion thus far has ignored the complications involved with uncertainties as to the demand functions for tickets. We conclude this section by looking briefly at such problems.

Consider first the case of a "one game season." Assume that the demand for tickets is a random variable with a known probability distribution. y is the demand for tickets and q(y;p) is the probability density function for y given the average ticket price p.

Assuming that the team owner obeys the axioms of measurable utility theory and assuming that marginal costs are zero, the decision problem becomes the following.

$$\max_p \int_0^{\overline{x}} u(py)q(y;p)dy + \int_{\overline{x}}^{\infty} u(p\overline{x})q(y;p)dy$$

where u(.) is the measurable utility function of the owner. Let x denote the attendance. Then for $y \leq \overline{x}$, $x = y$ (attendance equals the demand for tickets), while for $y \geq \overline{x}$, $x = \overline{x}$.

At a maximum of profits we have

$$\int_0^{\overline{x}} \left[\frac{\partial u(py)}{\partial p} q(y;p) + u(py)\frac{\partial q}{\partial p}\right]dy + (1 - Q(\overline{x}))\frac{\partial u(p\overline{x})}{\partial p}$$

$$+ u(p\overline{x}) \int_{\overline{x}}^{\infty} \frac{\partial q}{\partial p} dy = 0,$$

where $Q(\overline{x}) = \int_0^{\overline{x}} q dy.$

In particular, if the owner is an expected income maximizer, the above condition reduces to

$$\int_0^{\overline{x}}\left(yq + py\frac{\partial q}{\partial p}\right)dy + \overline{x}(1 - Q(\overline{x})) + p\overline{x}\int_{\overline{x}}^{\infty}\frac{\partial q}{\partial p}dy = 0$$

Expected attendance, $E(x)$ is defined by

$$E(x) = \int_0^{\overline{x}} yq\,dy + \overline{x}(1 - Q(\overline{x}))$$

Hence under expected income maximization, the rule becomes: choose p so that

$$\frac{dE(x)}{E(x)}\bigg/\frac{dp}{p} = -1$$

Thus price (and attendance) is chosen so that the elasticity of expected attendance with respect to price is unity.

For a "many game" season, let $q_i(y_i;p)$ denote the probability density associated with a ticket demand y_i for games with team i, given the ticket price p. Under independence of probabilities, the choice problem becomes

$$\max_p \sum_{i=1}^{n}\left(\int_0^{\overline{x}} u(py_i)q_i(y_i;p)\,dy_i + \int_{\overline{x}}^{\infty} u(p\overline{x})q_i(y_i;p)\,dy_i\right)$$

with the following condition satisfied

$$\sum_{i=1}^{n}\left(\int_0^{\overline{x}}\left[\frac{\partial u(py_i)}{\partial p}q_i + u(py_i)\frac{\partial q_i}{\partial p}\right]dy_i + \frac{\partial u(p\overline{x})}{\partial p}(1 - Q_i(\overline{x}))\right.$$

$$\left. + u(p\overline{x})\int_{\overline{x}}^{\infty}\frac{\partial q_i}{\partial p}dy_i\right) = 0$$

where $Q_i(\overline{x}) = \int_0^{\overline{x}} q_i\,dy_i$.

Let x_i = attendance at a game involving team i, and let $X = \sum_{i=1}^{n} x_i$. Then

$$EX = \sum_{i=1}^{n}\int_0^{\overline{x}} y_i q_i\,dy_i + \overline{x}_i(1 - Q_i(\overline{x}))$$

If the owner is an expected income maximizer, then p is chosen so that

$$\frac{dEX}{EX}\bigg/\frac{dp}{p} = -1$$

Hence the conclusion derived above applies as well to a "many game" season, except that p is chosen so that the elasticity of the aggregate expected attendance schedule is set equal to unity.

One final comment is in order concerning ticket pricing policy and the resulting attendance/capacity ratio. The discussion in this section has concerned the choices by a profit maximizing team owner who takes into account the effect of his decisions on attendance because of his monopoly position in the sport so far as his city is concerned. If instead the question is asked, "what is the optimal price from the point of view of the society," clearly the answer is that that price should be chosen so that every game is a sellout assuming that demand is at least equal to capacity at a sufficiently low ticket price. The reason for this is the public good nature of sports contests. Given that marginal costs of serving additional customers is zero, restricting attendance to less than capacity is inefficient, since additional services (viewing the game) can be provided to consumers without the expenditure of additional resources. Under appropriate lump sum taxes and transfers, every individual in the society can be made better off (including the team owner) than under monopolistic pricing of tickets.

Long Run Decision Making

We next turn to the long runproblem for a team owner of choosing an optimal stadium size. Admittedly, few owners today actually build their own stadiums; instead, stadiums are typically publicly financed. But there are a few cases -- Dodger Stadium, Shaefer Stadium (New England Patriots), Texas Stadium (Dallas Cowboys). What can be said about the choice of a stadium capacity by a profit oriented owner?

Adopting the notation of the previous section, and looking first at the "one game" season, let

y_t = demand for tickets in year t

p_t = average ticket price in year t

$\overline{x}$ = stadium capacity

C_t = costs of operating the stadium in year t

I = capital cost of the stadium

T = life of the stadium

i = interest rate

The demand function for tickets can be written as

$$y_t = f(p_t, \overline{x}, t) \equiv f_t$$

with $\frac{\partial f}{\partial p_t} < 0$. $\frac{\partial f}{\partial \overline{x}}$ is somewhat more ambiguous. It might be the case that $\frac{\partial f}{\partial \overline{x}} > 0$ for $\overline{x}$ sufficiently small, reflecting a preference of fans for "breathing space." On the other hand, it is well known that as stadium size increases, the ratio of desirable to undesirable seats decreases, hence $\frac{\partial f}{\partial \overline{x}} < 0$ for $\overline{x}$ sufficiently large. We will take $\frac{\partial f}{\partial \overline{x}} < 0$ as the "typical" case.

Similarly, $C_t = C(y_t, \overline{x}, t)$, with $\frac{\partial C}{\partial y_t} \geq 0$, $\frac{\partial C}{\partial \overline{x}} > 0$, while $I = I(\overline{x})$, $I' > 0$.

The owner's "long run" decision problem is the following:

$$\max \phi = \sum_{t=1}^{T} \frac{p_t f(p_t, \overline{x}, t) - C(y_t, \overline{x}, t)}{(1+i)^t} - I(\overline{x})$$

$$\text{subject to } y_t = f(p_t, \overline{x}, t) \leq \overline{x} \qquad t = 1, \ldots, T.$$

Let L denote the Lagrangeian, where

$$L = \phi + \sum_{t=1}^{T} \lambda_t (\overline{x} - f(p_t, \overline{x}, t))$$

At a maximum we have

(1) $$\frac{\partial L}{\partial p_t} = \left\{p_t \frac{\partial f_t}{\partial p_t} + f_t - \frac{\partial C_t}{\partial y_t}\frac{\partial f_t}{\partial p_t}\right\} \Big/ (1+i)^t - \lambda_t \frac{\partial f_t}{\partial p_t} = 0$$

(2) $$\frac{\partial L}{\partial \bar{x}} = \sum_{t=1}^{T}\left(p_t \frac{\partial f_t}{\partial \bar{x}} - \frac{\partial C_t}{\partial \bar{x}}\right) \Big/ (1+i)^t - I'(\bar{x}) + \sum_{t=1}^{T}\lambda_t\left(1 - \frac{\partial f_t}{\partial \bar{x}}\right) = 0$$

(3) $$\lambda_t(\bar{x} - f(p_t, \bar{x}, t)) = 0$$

$$\lambda_t \geq 0, \quad \bar{x} - f(p_t, \bar{x}, t) \geq 0$$

As usual, λ_t can be interpreted as the value of an additional unit of capacity in year t in terms of discounted net cash flows. In fact, solving (1) for λ_t, we obtain

$$\lambda_t = (MR_t - MC_t)/(1+i)^t$$

where $R_t = p_t y_t$, $MR_t = \frac{\partial R_t}{\partial y_t}$, $MC_t = \frac{\partial C_t}{\partial y_t}$. Let $\Pi_t = R_t - C_t$.

Then

$$\lambda_t = \frac{\partial \Pi_t}{\partial y_t} \Big/ (1+i)^t$$

Note that $y_t < \bar{x}$ implies $\lambda_t = 0$ which in turn implies $\frac{\partial \Pi_t}{\partial y_t} = 0$. If $MC_t = 0$, then $y_t < \bar{x}$ reduces to the case where price (and attendance) is chosen such that demand for that year is of unitary elasticity.

Is it ever optimal to choose a stadium size with "built in" permanent excess capacity? This would be the case where $\lambda_t = 0$ for every t. From (2) it is immediate that this can only occur if $\frac{\partial f_t}{\partial \bar{x}} > 0$ for some t. But for the "typical" case, $\frac{\partial f_t}{\partial \bar{x}} < 0$, hence it is only when the advantages of breathing space, availability of tickets, etc., outweigh the lower ratio of desirable to undesirable seats of an increase in capacity that a stadium would be built that is never intended to sellout.

Substituting for λ_t into (2), the rule for choosing stadium capacity becomes

$$\frac{dI}{d\bar{x}} = \sum_{t=1}^{T}\left\{\frac{\partial \Pi_t}{\partial y_t}\left(1 - \frac{\partial f_t}{\partial \bar{x}}\right) + p_t \frac{\partial f_t}{\partial \bar{x}} - \frac{\partial C_t}{\partial \bar{x}}\right\} \Big/ (1+i)^t$$

This is the usual condition: add capacity to the point where the increase in capital cost I, caused by adding one unit to capacity, is equal to the increase in the discounted present value of profits associated with the added unit of capacity. For those years in which attendance is less than capacity, the added capacity is only a source of costs, $p_t \frac{\partial f_t}{\partial \bar{x}}$ giving the loss due to the decrease in demand associated with added capacity and $\frac{\partial C_t}{\partial \bar{x}}$ giving the added costs of operating the stadium because of the higher capacity. When the stadium is sold out $\left(\frac{\partial \Pi_t}{\partial y_t} > 0\right)$, each additional seat increases profits by $\frac{\partial \Pi_t}{\partial y_t}\left(1 - \frac{\partial f_t}{\partial \bar{x}}\right)$, $\left(1 - \frac{\partial f_t}{\partial \bar{x}}\right)$ representing the net change in tickets sold due to adding one unit to capacity.

When we turn to the many game season with probabilistic demand, the decision problem becomes

$$\max_{p} \sum_{t=1}^{T}\left(\sum_{i=1}^{n}\left\{\int_0^{\overline{x}} u(\Pi_t^i)q_i(y_t^i;p_t;\overline{x})dy_t^i + \int_{\overline{x}}^{\infty} u(\overline{\Pi}_t^i)q_i dy_t^i\right\}\right)\Big/(1+i)^t - I(\overline{x})$$

where

y_t^i = demand for tickets in year t for a game with team i

$\Pi_t^i = p_t y_t^i - C(y_t^i, \overline{x}, t)$

$\overline{\Pi}_t^i = p_t\overline{x} - C(\overline{x}, \overline{x}, t)$

$q_i(y_t^i;p_t,\overline{x})$ = probability density of y_t^i, given p_t and $\overline{x}$.

For an expected income maximizing owner, the rules for choosing capacity reduce to

$$(1)\quad \sum_{i=1}^{n}\left\{\int_0^{\overline{x}}\left(y_t^i - \frac{\partial C}{\partial y_t^i}\right)q_i dy_t^i + \int_0^{\overline{x}} p_t y_t^i \frac{\partial q_i}{\partial p_t} dy_t^i + \overline{x}(1 - Q_t^i(\overline{x}))\right.$$

$$\left. + p\overline{x}\int_{\overline{x}}^{\infty}\frac{\partial q_i}{\partial p_t}dy_t^i\right\}\Big/(1+i)^t = 0 \qquad t = 1, \ldots, T$$

$$(2)\quad \sum_{t=1}^{T}\left(\sum_{i=1}^{n}\int_0^{\overline{x}}\left(-\frac{\partial C_t}{\partial \overline{x}}q_i + \Pi_t^i\frac{\partial q_i}{\partial \overline{x}}\right)dy_t^i + (1 - Q_t^i(\overline{x}))\left(p_t - \frac{\partial C_t}{\partial \overline{x}}\right)\right.$$

$$\left. + \int_{\overline{x}}^{\infty}\overline{\Pi}_t^i\frac{\partial q_i}{\partial \overline{x}}dy_t^i\right)\Big/(1+i)^t = \frac{dI}{d\overline{x}}$$

where

$$Q_t^i(\overline{x}) = \int_0^{\overline{x}} q_i(y_t^i;p_t,\overline{x})dy_t^i$$

Let x_t^i = attendance in year t for games with team i, and let $X_t \equiv \sum_{i=1}^{n} x_t^i$ = season attendance. Then

$$E(X_t) = \sum_{i=1}^{n}\left\{\int_0^{\overline{x}} y_t^i q_i dy_i + \overline{x}(1 - Q_t^i(\overline{x}))\right\}$$

When $MC_t = 0$, then condition (1) reduces to the familiar short run condition

$$\frac{dE(X_t)}{E(X_t)}\Big/\frac{dp_t}{p_t} = -1 \qquad t = 1, \ldots, T$$

Condition (2) asserts that, given the pricing policy of condition (1), $\overline{x}$ is set so that the added capital cost of the last unit of capacity is equal to the expected value of discounted profits attained by adding that last unit. In the "typical" case, we expect $\frac{\partial q_i}{\partial \overline{x}} > 0$ for low values of y_t^i and $\frac{\partial q_i}{\partial \overline{x}} < 0$ for high values of y_t^i, reflecting the undesirable properties of large stadiums so far as average ticket prices are concerned.

Finally consider the choice of an optimal stadium capacity by a profit oriented owner, when the stadium is to be used to house two sports teams operating in different sports. In the simplest case of a "one game" season, the choice problem for the owner becomes

$$\max \phi = \sum_{t=1}^{T} p_t f(p_t, \overline{x}, t) + \sum_{t=1}^{T} w_t g(w_t, \overline{x}, t) - C(y_t, z_t, \overline{x}, t) - I(\overline{x})$$

subject to

$$y_t \equiv f(p_t, \overline{x}, t) \leqq \overline{x}$$

$$z_t \equiv g(w_t, \overline{x}, t) \leqq \overline{x}$$

where y_t, z_t denote attendance at games in the two sports and p_t, w_t are the average ticket prices for games in the sports.

Let $f_t \equiv f(p_t, \overline{x}, t)$, $g_t = g(w_t, \overline{x}, t)$, $C_t = C(y_t, z_t, \overline{x}, t)$,

and let

$$L = \phi + \sum_{t=1}^{T} \lambda_t(\overline{x} - f) + \sum_{t=1}^{T} \mu_t(\overline{x} - g)$$

At a maximum we have

$$(1) \quad \frac{\partial L}{\partial p_t} = f_t + p_t \frac{\partial f_t}{\partial p_t} - \frac{\partial C_t}{\partial y_t}\frac{\partial f_t}{\partial p_t} - \lambda_t \frac{\partial f_t}{\partial p_t} = 0$$

$$(2) \quad \frac{\partial L}{\partial w_t} = g_t + w_t \frac{\partial g_t}{\partial w_t} - \frac{\partial C_t}{\partial z_t}\frac{\partial g_t}{\partial w_t} - \mu_t \frac{\partial g_t}{\partial w_t} = 0$$

$$(3) \quad \frac{\partial L}{\partial \overline{x}} = \sum_{t=1}^{T} p_t \frac{\partial f_t}{\partial \overline{x}} + \sum_{t=1}^{T} w_t \frac{\partial g_t}{\partial \overline{x}} - \frac{\partial C_t}{\partial \overline{x}} + \sum_{t=1}^{T} \lambda_t\left(1 - \frac{\partial f_t}{\partial \overline{x}}\right) + \sum_{t=1}^{T} \mu_t\left(1 - \frac{\partial g_t}{\partial \overline{x}}\right) - \frac{dI}{d\overline{x}} = 0$$

It is immediate that $\lambda_t = \frac{\partial \pi_t}{\partial y_t}$, $\mu_t = \frac{\partial \pi_t}{\partial z_t}$, with $\lambda_t > 0$ only if $y_t = \overline{x}$, $\mu_t > 0$ only if $z_t = \overline{x}$.

Hence (3) may be written as

$$\frac{dI}{d\overline{x}} = \sum_{t=1}^{T} \left\{ \frac{\partial \pi_t}{\partial y_t}\left(1 - \frac{\partial f_t}{\partial \overline{x}}\right) + \frac{\partial \pi_t}{\partial z_t}\left(1 - \frac{\partial g_t}{\partial z_t}\right) + p_t \frac{\partial f_t}{\partial \overline{x}} + w_t \frac{\partial g_t}{\partial \overline{x}} - \frac{\partial C_t}{\partial \overline{x}} \right\}$$

Note that it might well be the case that the optimal strategy is one in which the stadium is never sold out for one sport, because of the profitability of the added capacity for the second sport. This would occur, say, if $\frac{\partial \pi_t}{\partial y_t} = 0$ for every t but $\frac{\partial \pi_t}{\partial z_t} > 0$ for some t, assuming marginal cost $\left(\frac{\partial C_t}{\partial y_t}, \frac{\partial C_t}{\partial z_t}\right)$ is zero. This expresses the intuitive notion that in the "typical" case $\left(\frac{\partial f_t}{\partial \overline{x}} < 0, \frac{\partial g_t}{\partial \overline{x}} < 0\right)$ a single purpose stadium is never built with permanent excess capacity by a profit oriented owner, but a multiple purpose stadium might well have built-in excess capacity for one (but not all) sports.

Consider in contrast the situation in which a municipality arrives at the decision as to capacity levels for a sports stadium, as in the discussion of the next section.

Team versus Community Choices

The question asked by Okner [3] is: does a community subsidise a sports team when it builds a stadium? The question may be safely answered in the affirmative as Okner's paper [3] demonstrates.

We ask two related questions; should a community subsidise a sports activity? and should a profit maximizing team build its own facility? The, not surprising, answer to the first question is that the community would, on the margin, subsidize sports activity if its "marginal utility" with respect to that activity is positive. Subsidy is defined, here, as the excesses of maintenance, rental and gross investment costs over sport generated revenue. A team would not build its own facility if its imputed net returns on an additional capacity unit is less than the marginal cost of constructing such a unit. In computing the net returns, the team figures on a subsidy by the community. Obviously, the team's decision is influenced by the size of the subsidy.

In the discussion below, we concentrate on an extreme case. The city acts as a consumer and producer which controls all activities.

The team acts as monopolist as far as ticket pricing is concerned and considers rentals as given. In a real situation, there is a conflict which is settled according to the powers of the negotiators.

We outline a simple model for decisions by the team and community and state some implications of their choices.

Assume that the community's economic level is denoted by y^{J+1} and several sports activities whose levels are denoted by $y^j, j=1, 2, \ldots, J$. As a result of these activities, certain net revenues

are produced. Net revenues are related to the vector (y, y^{J+1}), where y is a vector whose components are y^j's, by way of J+1 functions $g(y, y^{J+1})$, $g^{J+1}(y, y^{J+1})$, each of which corresponding to an activity. The operation of these activities is, however, constrained by the availability of certain facilities that are required for their operation. Examples of such facilities are seating capacity of stadiums, transportation systems and housing. We shall aggregate the non-stadium facilities into one facility and denote its capacity by x^{J+1}. The seating capacities for various sport activities are denoted by $x^j, j=1, \ldots, J$. Each x^j represents an S^j-vector (a vector with S^j components), each of whose components is a seating capacity of a certain type for the j^{th} sport activity. The seating capacity requirement is given by functions: $h^j(y^j)$. Each of these functions is Sj-vector valued. The non-sport activity capacity requirement is given by a function $h^{J+1}(y, y^{J+1})$. The capacity constraints are written as:

$$(1) \qquad h^j(y^j) \leq x^j \quad , j=1, \ldots, J$$

$$h^{J+1}(y, y^{J+1}) \leq x^{J+1} .$$

The method of producing sport capacity is to build stadiums and we always think in terms of a multiple use stadium. Given the size of a stadium, denoted by z, we may determine the various seating capacities as functions:

$$(2) \qquad x^j = x^j(z), \quad j=1, \ldots, J.$$

Assuming some flexibility in seating arrangements it is possible to transform one capacity to another This is expressed by way of a transformation function:

(3) $f(x) \leq 0, \quad x = x^1, \ldots, x^J$.

The net addition to stadium size is given by:

(4) $\dot{Z} = I_1 - \alpha_1 Z$,

where I_1 denotes gross addition and where α_1 is the rate of depreciation. The net addition to non-sport capacity is given by:

(5) $\dot{x}^{J+1} = I_2 - \alpha_2 x^{J+1}$,

where I_2 and α_2 are gross addition and depreciation rates.

Suppose it costs $C_1(I_1)$ to add I_1 units of stadium size and it costs $C_2(I_2)$ to add one unit of non-sport capacity. Then the community's budget constraint may be written as:

(6) $$\int_0^T \left[\sum_{j=1}^{J+1} g^j(y, y^{J+1}) - C_1(I_1) - C_2(I_2) \right] e^{-rt} dt \geq 0 ,$$

where T is the length of the planning period and r is the interest rate. Side-stepping the interesting question of choice of a community criterion, we assume that the community maximizes a function:

$\int_0^T W(y, y^{J+1}) e^{-\rho t} dt$, where ρ is the discount rate.

Substituting from (2) in (1) and (3) our problem becomes:

Maximize $\int_0^T W(y, y^{J+1}) e^{-\rho t} dt$ subject to:

(7) $\dot{Z} = I_1 - \alpha_1 Z$

(8) $\dot{x}^{J+1} = I_2 - \alpha_2 x^{J+1}$

(9) $x^j(Z) - h^j(y^j) \geqq 0 , \quad j = 1, \ldots, J.$

(10) $x^{J+1} - h^{J+1}(y, y^{J+1}) \geqq \bullet$.

(11) $-f(x(Z)) \geqq 0.$

(12) $$\int^T \left[\sum^{J+1} g^j (y, y^{J+1}) - C_1(I_1) - C_2(I_2) \right] e^{-rt} dt \geq 0.$$

Assume concavity of W , of the left hand sides of 9) - 11) and of the integrand of 12). Assume also that all functions are continuously differentiable and assume normality. Then, see Mangasarian [2] and Hestenes [1], it is a necessary and sufficient condition for a maximum that there exist functions $\lambda_1, q, \lambda_2, \mu, \mu^{J+1}$, and a constant γ such that:

(13) $$W_{y^j} e^{-\rho t} - \mu^j h^j_{y^j} - \mu^{J+1} h^{J+1}_{y^j} + \gamma \sum_{j'=1}^{J+1} g^{j'}_{y^j} e^{-rt} = 0$$

(14) $$W_{y^{J+1}} e^{-\rho t} - \mu^{J+1} h^{J+1}_{y^{J+1}} + \gamma \sum_{j=1}^{J+1} g^j_{y^{J+1}} e^{-rt} = 0$$

(15) $$\lambda_1 = \gamma C_1' e^{-rt}$$

(16) $$\lambda_2 = \gamma C_2' e^{-rt}$$

(17) $$\dot{\lambda}_1 = (\lambda_1 - q f_x x') \alpha_1 - \sum_{j=1}^{J} \mu^j x^j_Z$$

(18) $$\dot{\lambda}_2 = \lambda_2 \alpha_2 - \mu^{J+1}$$

From 13) it follows that:

(19) $$W_{y^j} = \mu^j h^j_{y^j} + \mu^{J+1} h^{J+1}_{y^j} - \gamma \sum_j g^{j'}_{y^{j'}} e^{-rt}$$

The functions μ^j and μ^{J+1} may be interpreted as rental rates for capacities and the constant γ may, in view of 15 and 16, be interpreted as the rate of imputed return per unit of additional investment. This justifies interpreting the right hand side of (19) as marginal subsidy for sport activity j. To clarify this, assume $W_{yj} = o$, i.e. assume that there is no "pleasure" to be derived from running sport activity j. Then (19) becomes the usual marginal cost = marginal revenue condition which, in view of concavity, is necessary and sufficient for the community maximization of profits.

Turning our attention now to the team's point of view, we assume that the team's choice of optional stadium capacity is a choice of optimal mix between rented capacity and owned capacity. In real situations, a choice never exists where one could rent a positive amount of capacity and use a positive amount of one's own capacity. The real situation is represented either by choice where one uses zero rented capacity or zero owned capacity. We assume further that the team's objective is to minimize discounted net cash flow:

$$\int_o^T \left[p^j y^j (p^j) - R^j(y^j, \xi^j) - C^j(I^j) - L^j(y^j) \right] e^{-rt}, \tag{20}$$

where $y^j (p^j)$ is the demand function for the teams activity as a function of ticket price p^j, R^j is total rent of public facilities ξ_j, $C^j (I^j)$ is the total cost of gross investment I^j in adding to capacity and $L^j (y^j)$ is operating cost.

The net addition to capacity, x^j, is given by:

$$\dot{x}^j = I^j - \alpha^j x^j \ , \quad I^j \geq o \tag{21}$$

and the capacity constraint is given by:

$$y^j (p^j) \leqslant x^j + \xi^j \ . \tag{22}$$

Assuming concavity of the integrand of (20) and assuming normality it is necessary and sufficient for an optimum that there exist functions λ_j and μ_j such that:

$$\dot{\lambda}_j = \lambda \ \alpha^j - \mu_j \tag{23}$$

$$\left[p^j y^j_{p^j} + y^j - R^j_{y^j} y^j_{p^j} - L^j_{y^j} y^j_{p^j} \right] e^{-\rho t} - \mu_j \ y^j_{p^j} = o \tag{24}$$

$$- C^j_{I^j} \ e^{-rt} + \lambda_j \leqslant o \tag{25}$$

$$- R^j_{\xi j} + \mu_j \leqslant o \tag{26}$$

The team will follow a path of zero gross investment in capacity if:

$$\lambda_j < C^j_{I^j} \ e^{-rt} \ . \tag{27}$$

By (26), which holds as an equation in this situation,

$$\mu_j = R^j_{\xi j} \tag{28}$$

Thus the gross instantanious return on owned capacity equals the cost of renting an additional unit of publicly owned capacity.

By 27), 23) and 28), the team will follow a rent all path if the imputed marginal cost of rental is less than the marginal cost of construction.

The relation between the community's choice and the team's choices may be outlined as follows:

The city maximizes its utility and thus determines rental terms and subsidies to the teams. With this as data, the teams make a profit maximizing behaviour. Under certain conditions on the communities welfare function, the two sets of choices will coincide.

This manner of dealing with the problem, however, avoids the conflict of interest situation that exists in real life.

Concluding Remarks

The theoretical sections of this paper have emphasized the importance of the marginal costs of construction and of marginal revenue (in terms of discounted present value) as determinants of "optimal" stadium capacity, whether from the point of view of a profit oriented team owner or from the point of view of a municipality. If data were readily available on marginal construction cost and on marginal revenue, it would be possible to arrive at relatively straightforward answers to the questions concerning apparent excess capacity in baseball, and apparent lack of capacity for football and NHL hockey. Unfortunately, such data are not available, at least to the knowledge of the authors.

Another approach that suggests itself is to take a team like the L. A. Dodgers, which owns its own single-purpose facility, as a standard against which other baseball teams can be judged. At a minimum, the Dodgers provide a guide as to whether or not privately owned single purpose facilities in baseball would duplicate the pattern of capacity presently found in the major leagues. The Dodgers play to roughly 48% capacity (average over the 12 years Dodger stadium has been in existence), and had 5 sellouts during the 1973 season. The only NL team with more sellouts is Cincinnati; the only teams with higher A/C records over an extended period of time are Montreal and the New York Mets. St. Louis, with a privately owned dual-purpose stadium, runs at around 45% of capacity, with 3 sellouts in 1973. A very crude calculation suggests that the marginal revenue for the Dodgers is quite small; that is, the added revenue received for one additional seat in Dodger Stadium. An additional customer brings in roughly \$3 in ticket price, less 27 1/2 ¢ visiting share, plus \$1 in parking, plus perhaps 40¢ in concession profits, adding up to around \$4.10. But this is average revenue per customer. The last seat in the Stadium is sold only 5 times a year, for a season long revenue of $20.50.

Marginal revenue can be calculated using the formula

$$MR = (1 - 1/e)AR,$$

where e is the elasticity of demand. If e is close to unity, then MR is close to zero, given that AR = $20.50. Noll's study in Government and the Sports Business finds e for baseball not significantly different from one. At e = 1.1, MR = $2.30 on a season long basis. Given a discount rate of 10% and assuming that Dodger Stadium has a life of 20 years or more, the discounted present value of revenue of the last seat in the Stadium is in the neighborhood of $20 - $25. For stadiums that are sold out less than 5 times per year, marginal revenue is proportionately less. Further, the figure of $20 - $25 must be decreased to take into account the fact that adding seats reduces the ratio of desirable to undesirable seats, thus lowering demand as a function of the average ticket price. In any case, equating marginal profitability to marginal investment cost means that the marginal cost of adding a seat to Dodger Stadium is in the $20 range.

This figure might be contrasted with average costs of seats in baseball stadiums. Data on three stadiums constructed in the mid-1960's give an average cost per seat in the range between $400 and $500. This reflects the very heavy fixed costs associated with stadium construction, including such things as land acquisition, etc.

The basic conclusion that is reached is that baseball stadiums are generally larger than would be the case if they were operated as single-purpose facilities by a profit maximizing owner. There are fewer sellouts and a lower ratio of attendance to capacity than would be regarded as optimal from the owner's point of view. On the other hand, NFL games tend to be sellouts. At a $20 marginal cost for adding a seat, are football stadiums smaller than would be constructed by a profit maximizing owner?

The revenue received per ticket by an NFL team is probably in the $7 to $7.50 range, representing the 60% home share of a $10 ticket, plus concession income. With 7 sellouts per year, this adds up to a revenue of around $50 per seat. Given an elasticity of demand of 1.1, MR is approximately $5 on a season long basis, which converts to a discounted present value of near $50, at an interest rate of 10%. If $20 is the appropriate value of marginal cost of construction in 1962, when Dodger Stadium was built, $50 is certainly not an unreasonable figure for 1974. What this suggests is that in fact NFL seating capacity is probably not all that short, if viewed in terms of pure profit maximization. An additional indication is provided by the seating capacities of Shaefer Stadium and Texas Stadium, both in the 55 - 60,000 range, as contrasted with the new public stadiums in Kansas City and Buffalo, both at around 80,000 capacity.

The situation with respect to basketball and hockey arenas is much more complex, since such arenas have year-round use for ice shows, circuses, conventions, etc. Basketball and hockey even combined (as in Madison Square Garden and the L. A. Forum) only account for some 80 nights of the year, leaving 285 dates to be filled by non-sports events. Hence the relevant consideration in choosing capacity for an arena, unless it is specialized to 100% sports use, is probably not the attendance at sports events, which means the analysis of this paper is less relevant for basketball and hockey than for football and baseball.

REFERENCES

1. Hestenes. "On Variational Theory and Optimal Control Theory." SIAM Journal on Control 3 (1965) : 23-48.

2. Mangasarian. "Sufficient Conditions for the Optimal Control of Nonlinear Systems." SIAM Journal on Control 4 (1966): 139-152.

3. Okner. "Subsidies of Stadiums and Arenas." In Government and the Sports Business, edited by Roger Noll. Washington D. C.: Brookings Institution, 1974.

TABLE 1. ATTENDANCE AND CAPACITIES

ALL SPORTS, 1973-4 SEASON

(Attendance and Capacity in Thousands)

HOCKEY

NHL	Attend Per Game	Capacity	A/C	WHA	Attend Per Game	Capacity	A/C
St. Louis	18.4	18.0	102%	Vancouver	9.1	15.6	58 %
New York R.	17.9	17.5	102	Quebec	7.9	10.0	79
Chicago	17.4	18.0	97	Houston	6.7	9.3	72
Montreal	17.2	18.4	94	Minnesota	6.7	16.2	41
Philadelphia	16.6	16.6	100	Winnipeg	6.5	11.0	59
Toronto	16.0	16.5	97	Cleveland	6.2	9.5	65
Buffalo	16.0	15.2	105	New England	5.4	15.0	36
Vancouver	15.6	15.6	100	Los Angeles	5.3	14.7	36
Boston	15.0	15.0	100	Chicago	4.7	9.0	52
Minnesota	15.0	15.0	100	Edmonton	4.3	5.2	83
Atlanta	14.2	15.3	93	Toronto	4.2	9.3	45
Detroit	13.2	15.8	84	Jersey	2.7	11.0	25
New York Is.	12.8	14.7	87				
Los Angeles	11.1	16.0	69				
Pittsburgh	10.0	12.6	79				
California	4.8	12.5	38				
NHL	14.5	15.8	92%	WHA	5.8	11.3	51%

BASKETBALL

NBA	Attend Per Game	Capacity	A/C
New York	19.3	19.6	99%
Los Angeles	14.8	17.5	85
Seattle	12.2	12.7	96
Boston	10 7	15.3	70
Milwaukee	10.2	10.7	95
Buffalo	10.0	17.3	58
Capitol	9.3	17.5	53
Chicago	8.3	17.4	48
Portland	8.0	11.8	68
Atlanta	7.4	16.8	44
Detroit	7.2	11.0	66
Phoenix	7.1	12.5	57
Golden State	6.5	13.5	48
Kansas City-Omaha	5.6	9.6	58
Philadelphia	4.7	15.3	31
Cleveland	4.0	11.0	36
Houston	4.0	10.2	39
NBA	8.8	14.1	62%

ABA	Attend Per Game	Capacity	A/C
New York	8.9	16.0	56%
Kentucky	8.2	11.5	71
Indiana	7.3	9.1	80
Utah	7.0	12.2	57
San Antonio	6.3	10.1	62
Carolina	6.1	13.2	46
Denver	4.2	6.8	62
Virginia	3.0	10.4	29
Memphis	2.2	10.9	20
San Diego	1.8	4.5	40
ABA	5.5	10.5	52%

FOOTBALL

NFL	Attend Per Game	Capacity	A/C
New England	58.6	61.0	96%
Buffalo	79.0	80.0	99
Baltimore	54.2	60.0	90
Cincinnati	55.3	56.2	98
Cleveland	70.0	79.3	88
Denver	50.8	51.7	98
Houston	31.3	50.0	63
Kansas City	65.6	78.0	84
Miami	63.2	80.0	79
New York Jets	49.7	60.0	83
Oakland	52.2	56.9	93
Pittsburgh	46.9	50.0	92
San Diego	45.4	53.2	85
AFC	55.6	62.8	90
Total NFL	55.9	62.0	90

	Attend Per Game	Capacity	A/C
Atlanta	52.5	58.9	89%
Chicago	47.7	55.7	86
Dallas	58.9	65.1	90
Detroit	54.0	54.4	99
Green Bay	50.4	52.9	95
Los Angeles	74.6	76.0	98
Minnesota	46.9	49.8	94
New Orleans	64.5	81.0	80
New York Giants	65.9	70.9	93
Philadelphia	59.9	65.4	92
St. Louis	47.0	51.2	92
San Francisco	54.4	61.0	89
Washington	53.0	53.0	100
NFC	56.1	61.2	92

BASEBALL

National League	Attend Per Game	Capacity	A/C	American League	Attend Per Game	Capacity	A/C
Cincinnati	26.9	51.7	52%	Baltimore	13.1	52.2	25%
Los Angeles	26.7	56.0	48	Boston	19.3	33.4	58
New York Mets	26.2	55.3	47	Detroit	22.3	54.2	41
St. Louis	19.9	50.1	40	New York	16.8	65.0	26
Philadelphia	19.7	55.7	35	Milwaukee	15.8	46.0	34
Chicago	18.5	37.7	49	Cleveland	8.9	77.0	12
Pittsburgh	18.3	50.2	36	Oakland	13.0	50.0	26
Houston	17.4	45.0	39	Kansas City	17.8	40.6	44
Montreal	17.1	28.0	61	Minnesota	11.8	45.9	26
Atlanta	11.8	52.7	22	California	13.5	43.2	31
San Francisco	11.3	58.0	19	Chicago	16.6	46.6	36
San Diego	8.1	44.8	18	Texas	9.0	33.7	27
NL	18.5	48.8	38%	AL	14.8	49.0	30%

Source: Attendance, Sporting News, various issues 1973-4
Stadium capacities, league guides, 1973-4

TABLE 2

HOME ATTENDANCE BY GAMES, NHL, 1973-4 SEASON

(Attendance in Thousands)

Team \ Attend	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15.	16	17	18	19	Average Per Game	Sellouts
St. Louis																	1	1	19	18	18.4	38
New York R																1	-	1	37		17.9	38
Chicago																	4	34	—	1	17.4	39
Montreal																	7	19	10	3	17.2	39
Philadelphia																		39			17.0	39
Toronto																	39				16.0	39
Buffalo																	39				16.0	39
Vancouver																	39				16.0	39
Boston																39					15.0	39
Minnesota															1	36	2				15.0	39
Atlanta													1	10	11	17					14.2	28
Detroit												10	5	7	6	8	3				13.2	11
New York Is									1	2	9	8	3	5	11						12.8	11
Los Angeles									8	2	7	8	4	3	3	1	3				11.1	4
Pittsburgh						-		4	8	5	5	8	4	4	1						10.0	9
California			3	6	8	11	3	2	2	2	-	1									4.8	--
Total			3	6	8	11	3	6	19	11	21	35	17	29	33	102	137	89	66	22		396

Source: Sporting News, various issues, 1973-4

TABLE 3

HOME ATTENDANCE by GAMES, WHA, 1973-4 SEASON

(Attendance in Thousands)

Team \ Attend	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Avg. Per Game	Sellouts
Vancouver	-	-	-	-	2	-	1	1	8	12	9	3	2	-	1	-	9.1	-
Quebec	-	-	-	-	1	-	4	14	9	6	2	2	-	-	1	-	7.9	8
Houston	-	-	-	1	4	2	9	11	9	2	1	-	-	-	-	-	6.7	3
Minnesota	-	-	-	1	6	5	10	5	6	2	-	2	2	-	-	-	6.7	-
Winnipeg	-	-	-	-	1	11	13	6	3	1	4	-	-	-	-	-	6.5	4
Cleveland	-	-	1	3	3	5	9	9	6	2	1	-	-	-	-	-	6.2	3
Chicago	-	-	-	8	8	15	6	1	1	-	-	-	-	-	-	-	5.3	-
New England	-	-	-	-	8	13	6	6	5	-	-	-	-	-	-	1	5.4	1
Los Angeles	-	-	-	3	10	12	5	6	2	1	-	-	-	-	-	-	5.3	-
Edmonton	-	-	-	7	14	18	-	-	-	-	-	-	-	-	-	-	4.3	18
Toronto	1	-	1	5	20	9	1	1	-	1	-	-	-	-	-	-	4.2	1
Jersey	-	9	12	9	5	1	2	-	1	-	-	-	-	-	-	-	2.7	-
Total	1	9	14	37	82	91	66	60	50	27	17	7	4	-	2	1		38

Source: Sporting News, various issues 1973-4

TABLE 4

HOME ATTENDANCE BY GAMES, NATIONAL LEAGUE 1973 SEASON

(Attendance in Thousands)

Team \ Attend	0-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60	Average Per Game	Sell outs
Cincinnati	1	3	17	12	5	8	11	5	3	5	5		26.9	7
Los Angeles	-	-	12	15	19	12	8	2	2	5	5		26.7	5
New York Mets	-	5	9	10	9	18	10	7	3	1	2		26.2	2
St. Louis	-	12	25	15	3	11	4	3	2	3			19.9	3
Philadelphia	1	17	16	14	10	3	5	3	2	1	--	2	19.7	2
Chicago	5	12	14	9	16	9	3	3	1				18.5	4
Pittsburgh	2	12	17	14	13	6	4	1	1	--	1		18.3	1
Houston	4	9	22	14	13	13	3	1					17.4	-
Montreal	1	9	23	22	13	3	2						17.1	5
Atlanta	13	26	9	10	5	3			1				11.8	-
San Francisco	26	21	10	7	3	4	2		1				11.3	-
San Diego	28	30	8	4	1	1	1						8.1	-
Total	81	156	182	146	110	91	53	27	16	15	13	2		29

Source: Sporting News, various issues, 1973

TABLE 5

HOME ATTENDANCE by GAMES, AMERICAN LEAGUE, 1973 SEASON

(Attendance in Thousands)

Team \ Attend	0-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66	Avg per Game	Sellouts
Detroit	1	7	19	18	8	9	5	-	6	3	2				22.3	2
Boston	-	7	24	16	10	14	7								19.3	7
K. City	1	14	19	14	21	1	1	5							17.8	5
New York	3	20	21	11	4	4	3	1	1	1	-	1	1		16.8	1
Chicago	4	16	16	13	8	7	4	2	2	-	-	1			16.6	3
Milwaukee	7	22	17	8	8	2	5	2	1	-	1				15.8	2
California	6	25	27	10	5	5	1	-	1						13.5	1
Baltimore	4	29	18	11	6	2	1	-	1						13.1	-
Oakland	13	32	14	6	3	3	2	2	2	1					13.0	1
Minnesota	15	22	24	6	3	3	-	1	-	1					11.8	1
Texas	28	25	10	4	3	2	1	1							9.0	2
Cleveland	29	22	10	5		1	1							1	8.9	1
Total	111	240	219	122	79	53	31	14	14	6	3	2	1	1		26

Source: Sporting News, various issues, 1973-4

TABLE 6

HOME ATTENDANCE BY GAMES, NBA, 1973-4 SEASON

(Attendance in Thousands)

Team \ Attend	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Avg. per Game	Sellouts
New York																	3	6	7	25	19.3	36
Los Angeles												4	10	10	3	3	2	8	1		14.8	12
Seattle						1		1	1	6	5	8	4	13	1		1				12.2	28
Boston					1	7	4	1	1	6	2	4	3	3	9						10.?	10
Milwaukee							3	5	9	11	15										10.2	26
Buffalo		1		2	1	8	3	2	1	7	2		4	3	1	3	1	2			10.0	4
Capitol			2		6	4	2	5	5	2	3	5	1	2	1		2	1			9.3	3
Chicago			2		9	6	6	1	6	3	1	2	1			1	1		2		8.3	3
Portland				1	2	10	6	6	7	2	2	5									8.0	5
Atlanta				1	1	10	9	8	9	2	1										7.4	--
Detroit			2	4	9	5	3	4	3	2	9										7.2	9
Phoenix					8	12	5	6	8		1										7.1	--
Golden State		1	5	9	6	5	1	3	3	1	3	1	2	1							6.5	3
Kansas City-Omaha			2	10	12	8	1	4	4												5.6	4
Philadelphia		1	9	10	9	7	3	1	1												4.7	--
Cleveland		4	18	8	5	2	1	1	2												4.0	--
Houston		8	11	8	8	4	1	1													4.0	--
Total		15	51	53	77	89	53	46	60	42	44	29	25	32	15	7	10	17	10	25		143

Source: Sporting News, various issues 1973-4

TABLE 7

HOME ATTENDANCE by GAMES, ABA, 1973-4 SEASON

(Attendance in Thousands)

Team \ Attend	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Avg. Per Game	Sellouts
New York	-	-	-	-	-	-	3	12	4	11	6	-	2	2	1	1	-	8.9	1
Kentucky	-	-	2	5	-	1	3	6	7	6	2	4	1	1	1	2	1	8.2	4
Indiana	-	1	1	-	2	7	10	12	5	4	-	-	-	-	-	-	-	7.3	5
Utah	-	-	-	-	-	6	14	13	4	4	1	-	-	-	-	-	-	7.0	-
San Antonio	-	-	1	3	8	6	7	6	2	5	3	1	-	-	-	-	-	6.3	5
Carolina	-	-	-	1	11	5	14	5	1	3	1	-	-	-	1	-	-	6.1	1
Denver	-	-	-	11	20	5	4	2	-	-	-	-	-	-	-	-	-	4.2	2
Virginia	-	1	10	19	8	2	1	-	-	-	-		-	-	-	-	-	3.0	-
Memphis	-	17	12	8	-	2	2	1	-	-	-	-	-	-	-	-	-	2.2	-
San Diego	-	14	21	7	-	-	-	-	-	-	-	-	-	-	-	-	-	1.8	-
Total	-	33	47	54	49	34	58	57	23	33	13	5	3	3	3	3	1		18

Source: Sporting News, various issues, 1973-4

TABLE 8

STADIUM CAPACITIES, BASEBALL, 1960 vs. 1974

NATIONAL LEAGUE

	1960		1974	
	Ownership	Capacity	Ownership	Capacity
Cincinnati	private	29, 600	public	51, 700
Chicago	private	36, 700	private	37, 700
Los Angeles	public	94. 600	private	56, 000
San Francisco	private	23, 000	public	58, 000
Milwaukee-Atlanta	public	43, 800	public	52, 700
Pittsburgh	private	35, 000	public	50, 200
St. Louis	private	30, 500	private	50, 100
Philadelphia	private	33, 600	public	56, 600
New York Mets		______	public	55, 300
Houston		______	public	45, 000
San Diego		______	public	44, 800
Montreal		______	public	28, 000

Source: National League Green Book (1960, 1974)

TABLE 8 (cont'd.)

STADIUM CAPACITIES, BASEBALL, 1960 vs. 1974

AMERICAN LEAGUE

	1960		1974	
	Ownership	Capacity	Ownership	Capacity
California		______	public	43, 200
Chicago	private	46, 600	private	46, 600
Kansas City		______	public	40, 800
Washington-Minnesota	private	28, 700	public	45, 900
Kansas City-Oakland	public	31, 200	public	50, 000
Texas		______	public	35, 700
Baltimore	public	52, 100	public	52, 100
Cleveland	public	77, 000	public	77, 000
Detroit	private	54, 200	private	54, 200
Milwaukee		______	public	46, 000
New York	private	65, 000	private	65, 000*
Boston	private	33, 400	private	33, 400

* Temporarily (74-75 seasons) New York Yankees are playing in Shea stadium (public), capacity 55, 300.

Source: American League Red Book, (1960, 1974)

TABLE 9: ATTENDANCE AND CAPACITY PER GAME, NATIONAL LEAGUE 1960-1973
(Attendance and Capacity in Thousands)

	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	Average Old	Average New
Milwaukee-Atlanta																
Attendance	21.1	14.4	10.4	9.9	12.5	7.6	*20.3	17.4	14.1	19.7	14.4	13.2	11.1	11.8		
Capacity	43.8	43.8	43.8	43.8	43.8	43.8	50.9	50.9	51.4	51.4	51.4	52.7	52.7	52.7		
%A/C	48.2	32.9	23.7	22.6	28.5	17.4	39.9	34.2	27.4	38.3	28.0	25.0	21.1	22.4	28.8	24.6
Chicago																
Attendance	11.7	10.2	8.1	12.7	10.7	9.2	9.1	14.2	14.7	22.0	22.8	21.5	17.6	18.5		
Capacity	36.8	36.8	36.8	36.8	36.8	36.6	36.6	36.6	36.6	36.6	36.6	36.6	37.7	37.7		
% A/C	31.8	27.7	22.0	34.5	29.1	25.1	24.9	38.8	40.2	60.1	62.2	58.7	46.7	49.1	39.4	
Cincinnati																
Attendance	9.7	16.8	13.7	12.1	12.2	14.4	10.8	13.1	9.8	13.5	*23.4	19.8	22.7	26.9		
Capacity	30.3	30.3	30.3	30.3	29.6	29.6	29.6	29.6	29.6	29.6	51.7	51.7	51.7	51.7		
% A/C	32.0	55.4	45.2	39.9	41.2	48.6	36.5	44.3	33.1	45.6	45.3	38.3	43.9	52.0	42.2	44.9
Houston																
Attendance	----	----	12.2	9.2	9.2	*26.9	23.4	16.9	16.2	18.0	15.7	15.9	19.3	17.4		
Capacity	----	----	32.0	32.0	32.6	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0		
% A/C	----	----	38.1	28.8	28.2	59.8	52.0	37.6	36.0	40.0	34.9	35.3	42.9	38.7	31.7	41.9
Los Angeles																
Attendance	30.1	24.1	*35.3	33.4	28.9	32.7	33.6	21.6	20.3	22.9	21.8	26.4	23.9	26.7		
Capacity	94.6	94.6	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0	56.0		
% A/C	31.8	25.5	63.0	59.6	51.6	58.4	60.0	38.6	36.3	40.9	38.9	47.1	42.7	47.7	28.7	48.7
Montreal																
Attendance	----	----	----	----	----	----	----	----	----	16.9	19.3	18.0	15.9	17.1		
Capacity	----	----	----	----	----	----	----	----	----	30.0	28.5	28.5	28.0	28.0		
% A/C	----	----	----	----	----	----	----	----	----	56.3	67.7	63.2	56.8	61.1	60.1	
New York Mets																
Attendance	----	----	14.7	15.6	*26.1	26.0	28.0	23.4	25.8	31.1	36.4	30.6	29.6	26.2		
Capacity	----	----	55.0	55.0	55.3	55.3	55.3	55.3	55.3	55.3	55.3	55.3	55.3	55.3		
% A/C	----	----	26.7	28.4	47.2	47.0	50.6	42.3	46.7	56.2	65.8	55.4	53.5	47.4	27.6	51.2
Philadelphia																
Attendance	13.5	8.8	10.5	13.3	19.5	16.0	14.9	11.7	9.5	7.3	9.7	*20.1	18.7	19.7		
Capacity	33.6	33.6	33.6	33.6	33.6	33.6	33.6	33.6	33.6	48.0	48.0	56.6	56.4	55.7		
% A/C	40.2	26.2	31.3	39.6	58.0	47.6	44.3	34.8	28.3	15.2	20.2	35.5	33.2	35.4	35.1	34.7
Pittsburgh																
Attendance	26.6	17.9	16.5	11.5	10.8	12.5	15.9	12.1	9.9	11.0	*17.7	19.8	20.4	18.3		
Capacity	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	35.0	50.2	50.2	50.2	50.2		
% A/C	76.0	51.1	47.1	32.9	30.9	35.7	45.4	34.6	28.3	31.4	35.3	39.4	40.6	36.5	41.3	38.0
St. Louis																
Attendance	15.7	10.6	13.4	16.0	15.0	16.6	*23.8	27.9	26.8	22.2	20.8	20.3	16.5	19.9		
Capacity	30.5	30.5	30.5	30.5	30.5	30.5	49.5	49.5	49.5	49.5	50.1	50.1	50.1	50.1		
% A/C	51.5	34.8	43.9	52.5	49.2	54.4	48.1	56.4	54.1	44.8	41.5	40.5	32.9	39.7	47.7	44.8
San Diego																
Attendance	----	----	----	----	----	----	----	----	----	6.9	8.8	8.0	*9.2	8.1		
Capacity	----	----	----	----	----	----	----	----	----	50.0	50.0	50.0	44.8	44.8		
% A/C	----	----	----	----	----	----	----	----	----	13.8	17.6	16.0	20.5	18.1	15.8	19.3
San Francisco																
Attendance	*23.9	18.8	20.7	20.7	19.8	20.3	21.5	16.8	11.2	10.8	10.3	15.4	9.4	11.3		
Capacity	42.5	42.5	42.5	42.5	42.5	42.5	42.5	42.5	42.5	42.5	42.5	42.5	58.0	58.0		
% A/C	56.2	44.2	48.7	48.7	46.6	47.8	50.6	39.5	26.4	25.4	24.2	36.2	16.2	19.5	----	37.9
Average for National League																
Attendance	19.0	15.1	15.6	15.4	16.5	18.3	20.1	17.5	15.8	16.9	18.4	19.1	17.9	18.5		
Capacity	43.4	43.4	39.6	39.6	39.6	40.8	43.5	43.5	43.5	41.9	44.9	45.7	46.6	46.7		
% A/C	43.8	34.8	39.4	38.9	41.7	44.9	46.2	40.2	36.3	40.3	41.0	41.8	38.4	39.6	----	40.5

Source: National League Green Books, 1960-1973

* New stadium opened.