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EXTERNALITIES AS COMMODITIES: COMMENT

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SOCIAL SCIENCE WORKING PAPER 20

January 1973

EXTERNALITIES AS COMMODITIES: COMMENT

by Toby Page and John Ferejohn*

In the A.E.R., in 1967 Trenery Dolbear [1] constructed a simple general equilibrium model which demonstrated, among other things, that Pigovian unit effluent taxes could not be expected to be both Pareto optimal and exactly compensatory to pollution sufferers.[1] Calling this result Dolbear's "negative conclusion," Robert Meyer [3] set out to generalize Dolbear's model and derive conditions under which Pigovian taxes would "simultaneously achieve a Pareto optimum and yield exact compensation."

In his communication Meyer wrote, "The negative conclusion of Dolbear is presumably predicated on some type of non-convexity which is not stated explicitly. It is shown here that the absence of non-convexities is the prime assumption which yields a conclusion contrary to Dolbear's" [3, p. 737]. Meyer's idea is that with the appropriate convexity assumptions Pigovian taxes will, in general, allow both welfare efficiency and exact compensation. The purpose of this note is first to point out that Meyer is mistaken in thinking that the source of Dolbear's "negative conclusion" is in lack of convexity conditions and then to point out the actual role of convexity for environmental transfer functions.

Dolbear's model is well behaved with respect to convexity. Instead of non-convexities causing the general impossibility of Pareto- optimal

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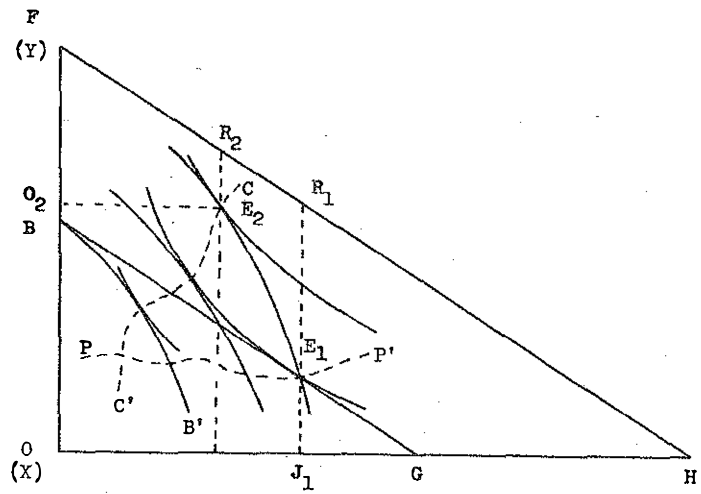
Pigovian taxes being exactly compensatory, the difficulty is an example of a regularity in economics: it generally takes at least two policy instruments (per-unit taxes and lump-sum transfers) to achieve two policy goals (Pareto optimality and exact compensation).

I. Necessity of Two Instruments

In Dolbear's model there are two consumers, X and Y: "X lives by bread and heat, Y lives by bread alone", although Y also involuntarily consumes smoke from X's heat; Y's smoke consumption is proportional to X's use of heat; the production possibility frontier between heat and bread is linear. We limit ourselves to the case where we start with an endowment of OB bread for X and BF bread for Y. Mr. X can trade bread for heat, incidentally increasing Y's consumption of smoke (in units heat). Y's role is completely passive: he consumes just the bread he was endowed with and the smoke blown his way.

Figure 1



Inside the production possibility set, which is also an Edgeworth triangle, indifference curves are drawn for X and Y.[2/] Points of tangency of indifference curves form the conventional "contract" curve CC'. With initial endowment B, X will trade himself down to E_1, carrying Y to OJ_1 units of smoke consumption willy-nilly and leaving Y's bread consumption unchanged. A Pigovian effluent tax, whose revenue goes to Y in units of compensatory bread, steepens X's effective budget constraint. Relative to initial endowment B, different Pigovian tax rates trace out a Y's "price consumption" curve PP'.

Y's indifference curve through B specifies the exact compensation requirements. Clearly, it will not in general happen that BB', PP', and CC' intersect at the same point, in which case a Pigovian tax can be both Pareto optimal and exactly compensatory. By the geometry, the exceptional case can only happen at corner solutions or if Y's relevant indifference curve is a straight line segment.

What then led Meyer to think that one could generally find Pigovian taxes both Pareto optimal and exactly compensatory? It seems that Meyer has mistaken the set of Pareto solutions given by maximizing a Lagrangian with solutions of a general equilibrium model. In searching for the Pareto set we are not interested in the original income allocation or its later reallocation through tax transfers. In maximizing the Lagrangian, we can imagine a benevolent dictator making implicit lump-sum transfers which insure some utilities are held constant as we move from one allocation to another.

Meyer's model gives us conditions defining the Pareto set, but it is incomplete as a general equilibrium model because it neglects the income transfer aspects (and limitations) of the Pigovian taxes. Having neglected

income redistribution attendant in equilibrium models, Meyer attributes more flexibility to Pigovian taxes than they actually have.

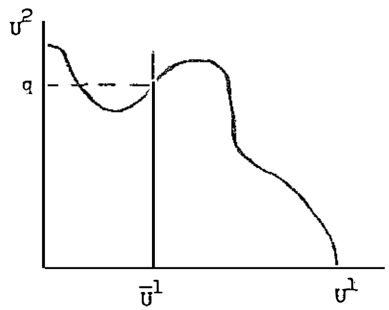
Dolbear's model, on the other hand, qualifies as a simple general equilibrium model. Distribution is determined by initial endowment and Pigovian taxes, which, as we have seen above, can distribute only according to a fixed rule. If out of all Pigovian taxes we choose the one with welfare efficiency, we will be stuck with the implied distribution. If a policy maker wanted Pareto-optimal Pigovian taxes to compensate exactly, he would need another degree of freedom, that is, another policy instrument. Dolbear recommended lump-sum transfers. The appropriate lump-sum transfer will move PP' just enough so that it will go through the intersection of CC' and BB'.

Where then is Meyer's formal mistake? In his more general model, for which Dolbear's is a special (but more complete) case, Meyer maximizes one man's utility subject to at least covering prescribed utility levels for the others and subject to production and externality interaction constraints. To test whether or not per unit Pigovian taxes can achieve exact compensation and efficiency, Meyer writes, ". . . one need only replace constraints (5) [$U^j \geq \bar{U}^j$] by equality constraints of the form $U^i \geq \bar{U}^i$. . ." [3, p. 739].

However, this is not a valid test. Narrowing the utility constraint has no effect on the maximum as long as the utility possibility frontier is negatively sloped (normal in positive orthant). With neoclassical utility functions and a negatively sloped utility frontier, there is no need to "waste" extra utility on U^i beyond the bare minimum $\bar{U}^i$. We can be sure there will be no slack U^i, that is $U^i > \bar{U}^i$, at the maximum.

In Dolbear's model the utility frontier is downward sloping. One can check this by looking at the contract curve. On the utility possibility frontier, X's utility goes down while Y's goes up. Changing the constraint $U^y \geq \bar{U}^y$ to $U^y = \bar{U}^y$ has no effect on the maximization of U^x. Dolbear's model passes Meyer's test, but there is almost always no exact compensation, as we have seen.

There is an interesting complication, though. In the presence of externalities it is conceivable that the utility frontier might look like :



In this case narrowing the constraint set from $U^1 \geq \bar{U}^1$ to $U^1 = \bar{U}^1$ does change max U^2. A model with this utility frontier would pass Meyer's test (the conditions satisfying max U^2 subject to $U^1 = \bar{U}^1$ would lead to q utility for individual 2) but Dolbear's "negative conclusion" would still hold. Not only do we have no income transfer mechanism to lead us to $(\bar{U}^1, q)$ but also $(\bar{U}^1, q)$ is not Pareto-optimal either. It seems, however, that externality bads are more likely to bow the utility frontier in than out.[3]

II. Why Exact Compensation in Addition to Efficiency?

Having said this, we raise the question why a policy maker would want Pareto-optimal Pigovian taxes to have exact compensation. Especially for pollution flowing from producers to consumers, pollutant emissions are analogous to (other) factors of production. Pareto-optimal Pigovian taxes, in other externality models besides Dolbear's, are likely to provide surpluses to pollution sufferers, as well as to polluters, over the situation in which pollution is prohibited. Just as it seems odd to devise lump-sum transfers to skim off a laborer's wage surplus, it seems odd that a policy maker should be interested in skimming off a pollution sufferer's surplus by insisting on exact compensation.

III. Convexity

In most neo-classical models, concave functions are needed to insure stability and interior solutions. Although Meyer has not placed any conditions on his transfer functions g^i, the behavior of the g^i turns out to be important and perhaps slightly counter-intuitive. Interestingly, in Meyer's model, convex transfer functions g^i assure interior solutions with Pigovian taxes.

With

$U^x(B,S)$	X's concave utility function between bread and smoke and
$S = g(H)$	the transfer function, linking smoke to Y's heat consumption,
$U^x(B,g(H))$	may not be concave in B and H for concave function g.

A simple counter-example illustrates the point. Choose $U^x(B,S) = B-S$ as our concave function of B and S. For $g(H) = H^{1/2} = S$,

(1) $\qquad U(B,g(H)) = B-H^{1/2}$ is not concave in B and H.

and the indifference curves bend the "wrong" way. We note that if g had been convex, X's indifference curves between bread and heat would have been convex. The desirable convexity property of the environmental transfer function g can be stated more strongly as follows:

If U(B,S) is concave, $U_2 < 0$ (pollution is a bad), $g(H) = S$ an increasing, <u>convex</u> function, then $V(B,H) = U(B,g(H))$ is concave.

A little algebra on the minors of $\begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$ establishes the proposition.

Figure 2

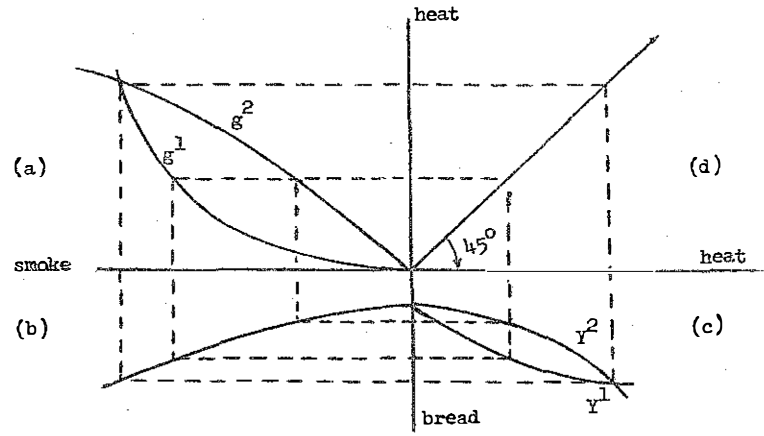


Figure 4

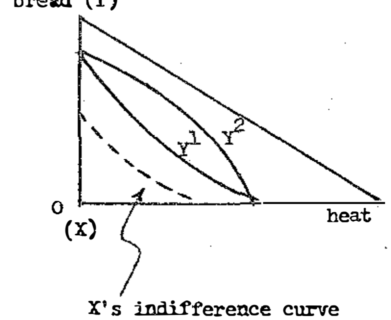
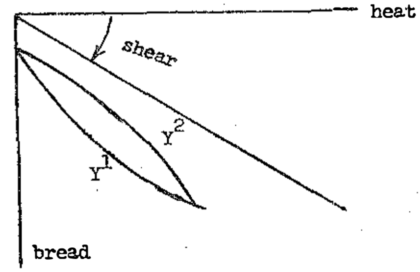


Figure 3



In Figure 2(a) the function smoke = g^1(heat) is concave and the function smoke = g^2(heat) is convex. Figure 2(b) shows one of Y's indifference curves between smoke and bread. It is bent downward in a well-behaved way. With the 45° line, Figure 2(d), this indifference curve is transformed into an indifference curve between heat and bread, depending on the functional form of g. Sufficiently concave g^2 leads to indifference curves bending up in Figure 2(c). The shear transformation between Figures 2 and 3 leaves the indifference curve convexities the same. Superimposed on Figure 4, Dolbear's Edgeworth triangle, Y^1 has the same type of convexity as indifference curves of Mr. X, a situation leading to corner solutions.

What one thinks about the convexity of actual transfer functions depends partly on where one cuts off the transfer. For example, if S = g(H) describes street-level densities of smoke as a function of heat emissions, we may consider g nearly linear. But if S = g(H) describes health damage as a function of heat emission, the function may be convex, due to diminishing returns to biological defenses.

Important policy implications follow from the degree of convexity of the environmental transfer functions. The more concave are the g, the more likely we are to recommend all or nothing policy prescriptions. Suppose, for example, water pollution damage functions are concave. Then we may want some rivers to be trout-clean and some industrial sewers. If the damage function is convex, it is more likely that a little pollution in all rivers is a better policy. For more convex damage functions interior, mixed solutions are more likely than corner solutions. Similar considerations apply to the question of whether smoking should be permitted throughout a bus or whether it should be segregated to one end. And again the convexity of air pollution damage functions partly determines whether it is wiser to limit

peak episodes, by emergency measures, or to emphasize control of the chronic average pollutant levels.

At this time there are few empirical studies which shed light on the question of convexity for g. Using SMSA's, Lester Lave [2] regressed total mortality rate on air pollution variables (minimum and maximum two-week averages of sulfates and particulates) and sociological variables. With both piecewise linear and quadratic specifications of the pollution variables, there was some evidence that g, here a damage function, is concave for long-run air pollution effects. The evidence is weak because the coefficients are nearly all insignificant and the sociological variable, percent over 65, seems to be carrying the equations.

For cases where the damage function is sufficiently concave, the policy prescription will be non-Pigovian. Depending on which corner solution is better, pollution should be either outlawed altogether or allowed without any constraint.

However, we think that in many cases g will be convex, due to diminishing returns to environmental capacity. For example, congestion is more than proportional to the number of cars on a highway.

An air pollution study provides a fragment of evidence of this second alternative. Page estimated the impact of daily levels of sulfur dioxide and particulates, in London, along with meteorological and psychological variables on a measure of perceived health [4]. The nonlinear equation

$$H_t = \beta_0 + \beta_1 \sum \lambda^i S_{t-1}^{\sigma} + \beta_2 V_t + e_t \quad (2)$$

was estimated, where

H_t is the number of people who feel worse on day t,

S_t is sulfur dioxide on day t,

V_t is visibility on day t,

α is non-linearity "stretching" coefficient, and

λ is Koyck lag coefficient.

e_t is the error term

The stretching coefficient α was 1.5, indicating that this measure of perceived health is a convex function of the daily sulfur dioxide load. In another approach the same linear equation of health as a function of pollution and meteorological variables was estimated for years of high, medium, and low pollution. Decline in the pollution coefficients from the high to low years also suggested a convex damage function for pollution variables.

Most likely, transfer functions exist in a variety of forms, from concave to convex. While little has been done empirically to estimate transfer functions, from polluting sender to suffering receiver, this is an area ripe for econometric work. It is also an area ripe for theorists, for the relation of transfer functions to changes in location of receivers and senders lies at the heart of the externality problem, especially for the most interesting case of large numbers of polluters and receivers.

FOOTNOTES

1. Exact compensation for a pollutant leaves a potential pollution sufferer indifferent between the pollutant's prohibition and its allowance with compensation.

2. The origin for X's indifference curves is O; the origin for Y's indifference curves is F. The allocation point E_1 specifies E_1R_1 bread to Y and E_1J_1 bread to X; OJ_1 smoke to Y and OJ_1 heat to X. In the diagram, Y is indifferent between (R_1E_1 bread, OJ_1 smoke) and (R_2E_2 bread, O_2E_2 smoke).

3. For more on this point see Baumol [5].

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