Externalities as Commodities: Comment

By TALBOT PAGE AND JOHN FEREJOHN*

In this *Review* in 1967, F. Trenery Dolbear constructed a simple general equilibrium model which demonstrated, among other things, that Pigouvian unit effluent taxes could not be expected to be both Pareto optimal and exactly compensatory to pollution sufferers.[1] Calling this result Dolbear's "negative conclusion," Robert Meyer set out in his 1971 article to generalize Dolbear's model and derive conditions under which Pigouvian taxes would achieve both goals simultaneously.

Meyer established a test which he asserted would tell us when both goals are attainable simultaneously. And Meyer suggested that when Pigouvian taxes are unable to achieve both goals simultaneously, the failure is the result of some sort of nonconvexity, p. 737. The purpose of this note is first to point out that Meyer is mistaken in thinking that the source of Dolbear's negative conclusion is in lack of convexity conditions and then to point out some of the role convexity actually plays in pollution problems.

I. Necessity of Two Instruments

One might wonder how a single policy instrument, Pigouvian per unit taxes, could be expected to achieve two disparate policy goals, except fortuitously. And indeed the reason for Dolbear's negative conclusion is the necessity for two policy instruments and not some problem with convexity. Dolbear's model is well behaved with respect to convexity conditions, as can be seen by the indifference curves and production possibility frontier of his diagram.

In Dolbear's model there are two consumers. Consumer X divides his income between bread and heat and Y spends all his income on bread. In direct proportion to X's consumption of heat, smoke is produced which Y involuntarily and distastefully consumes. We limit ourselves to the case where we start with an endowment of OB bread for X and BF bread for Y. The model is driven by X who can trade bread for heat, incidentally increasing Y's consumption of smoke (in units heat). The role of Y is completely passive: he consumes just the bread he was endowed with and the smoke blown his way.

Inside the production possibility set, which is also an Edgeworth triangle, indifference curves are drawn for X and Y.[2] Points of tangency of indifference curves form the conventional "contract" curve CC'. With initial endowment B, X will trade himself down to E_1, carrying Y to OJ_1 units of smoke consumption willy-nilly and leaving Y's bread consumption unchanged. A Pigouvian effluent tax, whose revenue goes to Y in units of compensatory bread, steepens X's effective budget constraint. Relative to initial endowment B, different Pigouvian tax rates trace out Y's "price consumption" curve PP'.

Y's indifference curve through B specifies the exact compensation requirements. Clearly, it will not in general happen that BB', PP', and CC' intersect at the same point, in which case a Pigouvian tax could be both Pareto optimal and exactly compensatory. By the geometry, the exceptional case can only happen at corner solutions or if Y's relevant indifference curve is a straight line segment.

There are two considerations in our argument that Pigouvian taxes alone cannot be expected to achieve efficiency and compensation simultaneously. The first is basically

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[1] Exact compensation for a pollutant leaves a potential pollution sufferer indifferent between the pollutant's prohibition and its allowance with compensation.

[2] The origin for X's indifference curves is 0; the origin for Y's indifference curves is F. The allocation point E_1 specifies E_1R_1 bread to Y and E_1J_1 bread to X; OJ_1 smoke to Y and OJ_1 heat to X. In the diagram, Y is indifferent between (R_1E_1 bread, OJ_1 smoke) and (R_2E_2 bread, O_2E_2 smoke).

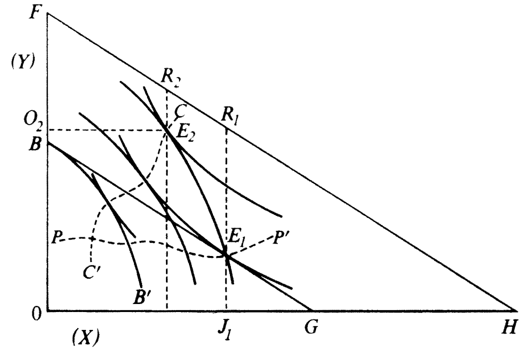

FIGURE 1

definitional. It is customary to think of a Pigouvian tax as having just a single degree of freedom, the rate itself. In Dolbear's model the set of possible Pigouvian taxes is specified by line segments hinged at B and falling between O and G. In a model of only two people, setting the tax rate for the polluter also determines the distribution of the proceeds for both participants. With a model more complicated than Dolbear's, in order to have no more than the single degree of freedom afforded by the Pigouvian tax rate, one needs to specify a fixed rule of distribution. We are not arguing that Pigouvian taxes in collection and lump sum transfers in distribution cannot achieve two policy goals, but we would count this arrangement as two policy instruments.

The second consideration is one of incentives. William Baumol has pointed out that in the case of no negotiations between polluters and pollution receivers (the "normal case") the proceeds of the effluent tax should not in general be paid to the pollution receivers in accordance with their marginal suffering. To do so would remove the incentive from pollution receivers from adopting defensive strategies which help to minimize the total social cost of pollution.[3] Consequently, even if we wanted to add to the Pigouvian tax by fine tuning its disbursement to marginal suffering, the goal of compensation would intefere with welfare efficiency. Attempting exact compensation would prevent efficiency unless payments to pollution sufferers are made conditional upon the pollution receivers taking best defensive strategies or unless there are no possible defensive strategies (the case of "pervasive pollution," which is the case in Dolbear's model). It is much simpler and more practical to use the Pigouvian tax proceeds to lower the general tax bill in a diffuse way so as to avoid perverse incentive effects in the tax distribution.

Meyer has no rule for distribution of the Pigouvian tax proceeds, and without a distribution rule his model remains incomplete as a general equilibrium model. Apparently the lack of a distribution mechanism misled Meyer in his test condition. To test whether or not per unit Pigouvian taxes can achieve exact compensation and efficiency, Meyer wrote " . . . one need only replace constraints (5) $[U^i \geq \overline{U}^i]$ by equality constraints of the form $U^i = \overline{U}^i$ . . . " (p. 739).

Meyer is maximizing a Lagrangian first with inequality constraints and then with equality constraints. It can be seen from Dolbear's model that either way this maximization leads to a definition of the same contract curve (CC' in Figure 1). With neoclassical utility functions and a negatively sloped utility frontier between X and Y, there is no need to "waste" extra utility on U^Y beyond the bare minimum $\overline{U}^Y$. Maximization of the Lagrangian will happen on the boundary $U^Y = \overline{U}^Y$ even if interior solutions are allowed. Dolbear's model passes Meyer's test and thus the test incorrectly predicts that exact compensation is possible. By omitting a distribution mechanism, Meyer seems to be confusing conditions for defining Pareto optimality (conditions independent from distribution mechanism) with solutions to a general equilibrium model (requiring tax distribution mechanisms).

There is an interesting complication. Suppose that in the presence of externalities the utility frontier bowed out sufficiently so that part of the frontier sloped upward as shown in Figure 2. Suppose also that $\overline{U}^Y$ corresponds to B, where no pollution is allowed.

[3] For an elaboration, see Baumol, pp. 309–12. Ronald Coase also has the example of farmers who, if they are compensated for losses due to sparks from railroad trains, have a tendency to plant too close to the tracks.

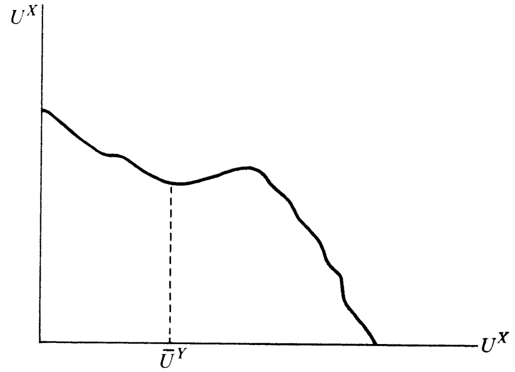

FIGURE 2

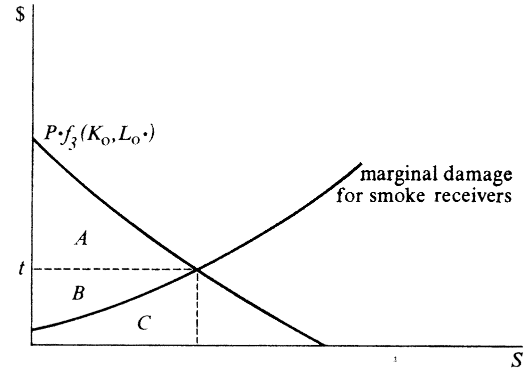

FIGURE 3

In this case the requirement of exact compensation makes welfare efficiency impossible. (Meyer's test would say that both goals are attainable because the Lagrangian max U^X subject to $U^Y \geq \bar{U}^Y$ can also be maximized on the boundary $U^Y = \bar{U}^Y$.) However, it seems that externality bads are more likely to bow the utility frontier in than out.[4]

II. Why Exact Compensation in Addition to Efficiency?

Having said this, we raise the question why a policy maker would want Pareto optimal Pigouvian taxes to have exact compensation. The point is that whether or not exact compensation suggests itself as a worthy policy goal depends in large part upon the formulation of the problem and the underlying conditions the formulation is trying to describe. In a benefit-cost analysis the question of distribution of benefits corresponds to the question of allocation of property rights in a Coasian analysis of externalities. The question of whether or not to compensate appears symmetrically. Demsetz would say the question is ethically symmetrical.

In a Pigouvian formulation, however, the situation *appears* less symmetric. In a Pigouvian analysis arising from the production sector, which is the most important case, it is natural to treat smoke as just another factor of production. For example, let quantity $Q = f(K, L, S)$, where K is capital, P the price of Q, L is labor, and S is smoke. Then the effluent tax t is analogous to the wage rate, or rental rate. For given $K = K_0$, $L = L_0$ we can draw the marginal revenue product of smoke and illustrate surpluses, as in Figure 3. Relative to a zero pollution status quo point, effluent tax t leads to a producer surplus A and consumer surplus B.

Given the practical difficulties of conditioning and distributing the proceeds $B+C$ to each individual pollution receiver on the basis of his marginal suffering, the question arises, why try?[5] For another factor of production, say L, exact compensation amounts to skimming off the labor surplus (corresponding to area B). For laborers, no one recommends exact compensation which would make them no better off working than unemployed. For laborers it is usually considered desirable not to interfere with laborers' surpluses, when they naturally appear. If we decide that a pollution receiver's surplus is not undesirable, especially since it is likely to be dispersed to consumers at large and not just to pollution sufferers, then we have no need to search for a second instrument to achieve exact compensation, nor to berate a Pigouvian tax for not doing two things at once.

[4] See Baumol, pp. 316–17.

[5] It seems obvious that attempting exact compensation in the typical case of several polluters and millions of receivers is attempting an administrative impossibility.

III. Convexity

In most neoclassical models, concave functions are used to insure stability and interior solutions.[6] Although Meyer has not placed any conditions on his transfer functions g^i, the behavior of the g^i turns out to be important and perhaps counterintuitive. Interestingly, in Meyer's model, *convex* transfer functions g^i assure interior solutions with Pigouvian taxes.

With $U^Y(B, S)$, Y's concave utility function between bread and smoke, and $S=g(H)$ the transfer function, linking smoke to Y's heat consumption $U^Y(B, g(H))$ may not be concave in B and H for concave function g.

A simple counterexample illustrates the point. Choose $U^Y(B,S)=B-S$ as our concave function of B and S. For $g(H)=H^{1/2}=S$,

$$U(B, g(H)) = B - H^{1/2} \tag{1}$$

is not concave in B and H, and the indifference curves bend the "wrong" way. We note that if g had been convex, Y's indifference curves between bread and heat would have been convex. The desirable convexity property of the environmental transfer function g can be stated more strongly as follows:

If $U(B, S)$ is concave, $U_2<0$ (pollution is a bad), $g(H)=S$ an increasing, convex function, then $V(B, H)=U(B, g(H))$ is concave.

A little algebra on the minors of

$$\begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$$

establishes the proposition.

In Figure 4(a) the function smoke$=g^1$(heat) is concave with respect to its argument heat, and the function smoke$=g^2$(heat) is convex. Figure 4(b) shows one of Y's indifference curves between smoke and bread. It is bent downward in a well behaved way. With the 45° line, Figure 4(d), this indifference curve is transformed into an indifference curve between heat and bread, depending on the functional form of g. Sufficiently concave g^2 leads to indifference curves bending up in Figure 4(c). The shear transformation between Figures 4 and 5 leaves the indifference curve convexities the same. Superimposed on Figure 6, Dolbear's Edgeworth triangle, Y^1 has the same type of convexity as indifference curves of consumer X, a situation leading to corner solutions.

What one thinks about the convexity of actual transfer functions depends partly on where one cuts off the transfer. For example, if $S=g(H)$ describes street level densities of smoke as a function of heat emissions, we may consider g nearly linear. But if $S=g(H)$ describes health damage as a function of heat emission, the function may be convex, due to diminishing returns to biological defenses.

Important policy implications follow from the degree of convexity of the environmental transfer functions. The more concave are the g, the more likely we are to recommend all or nothing policy prescriptions. Suppose, for example, water pollution damage functions are concave. Then we may want some rivers to be trout-clean and some to be industrial sewers. If the damage function is convex, it is more likely that a little pollution in all rivers is a better policy. For more convex damage functions, interior (i.e., mixed) solutions are more likely than corner solutions. Similar considerations apply to the question of whether smoking should be permitted throughout a bus or whether it should be segregated to one end.

Robert Kohn has investigated another situation which also may lead into all or nothing policy prescriptions, p. 994. Suppose the S in $f(K, L, S)$ is a vector $S=(S_1, S_2)$. The two factors of production may be sufficiently joint so that an optimal standard for both may be reached by specifying a required standard for just one. Kohn showed that this problem can be analyzed by looking at the convexity of isocost curves for abatement. It should also be noted that the convexity of air pollution damage functions partly determines whether it is wiser to limit peak episodes, by emergency measures, or to emphasize control of the chronic average pollutant levels.

[6] Michael Farrell has shown that the normally invoked convexity conditions are far stronger than necessary.

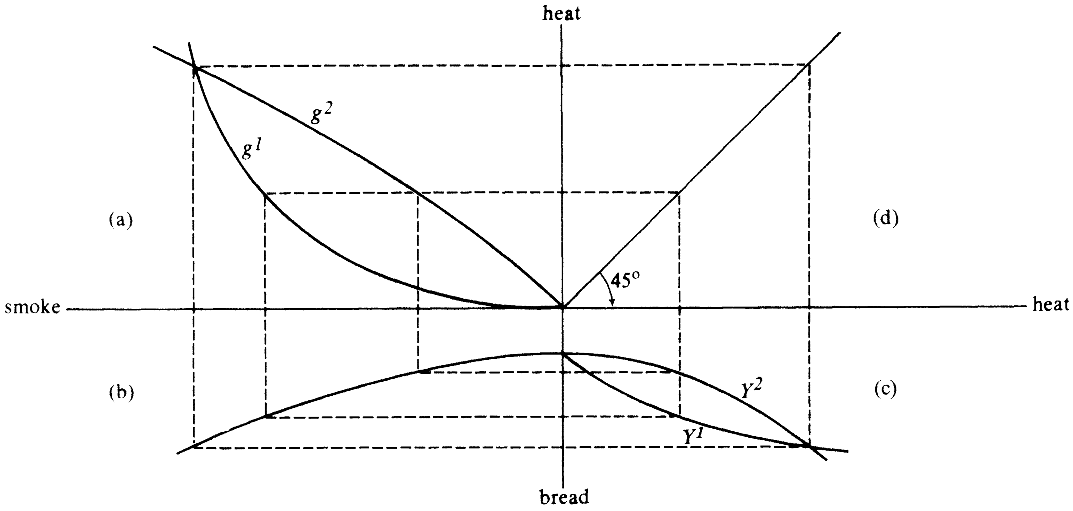

FIGURE 4

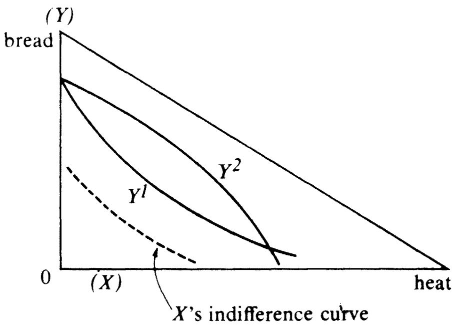

FIGURE 5

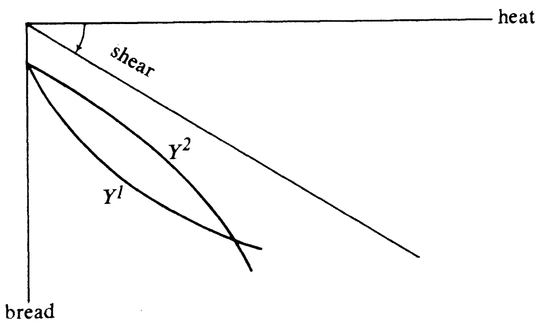

FIGURE 6

At this time there are few empirical studies which shed light on the question of convexity for g. Using cross-section data on urban areas, Lester Lave regressed total mortality rate on air pollution variables (minimum and maximum two-week averages of sulfates and particulates) and sociological variables. With both piecewise linear and quadratic specifications of the pollution variables, there was some evidence that g, here a damage function, is concave for long-run air pollution effects. The evidence is weak because the coefficients are nearly all insignificant and the sociological variable, percent over age 65, seems to be carrying the equations.

For cases where the damage function is sufficiently concave, the policy prescription will be non-Pigouvian. Depending on which corner solution is better, pollution should be either outlawed altogether or allowed without any constraint.

However, we think that in many cases g will be convex, due to diminishing returns to environmental capacity. For example, congestion is more than proportional to the number of cars on a highway.

An air pollution study provides a fragment of evidence for this second alternative. Page estimated the impact of daily levels of sulfur dioxide and particulates, in London, along with meteorological and psychological variables on a measure of perceived health, pp.

107–13. The non-linear equation

$$(2) \quad H_t = \beta_0 + \beta_1 \sum_i \lambda^i S_{t-i}^{\alpha} + \beta_2 V_t + e_t$$

was estimated where H_t is the number of people who feel worse on day t, S_t is sulfur dioxide on day t, V_t is visibility on day t, α is nonlinearity "stretching" coefficient, λ is Koyck lag coefficient, and e_t is the error term. The stretching coefficient α was 1.5, indicating that this measure of perceived health is a convex function of the daily sulfur dioxide load. In another approach the same linear equation of health as a function of pollution and meteorological variables was estimated for years of high, medium, and low pollution. Decline in the pollution coefficients from the high to low years also suggested a convex damage function for pollution variables.

Most likely, transfer functions exist in a variety of forms, from concave to convex. While little has been done empirically to estimate transfer functions, from pollution sender to suffering receiver, this is an area ripe for econometric work. It is also an area ripe for theorists, for the relation of transfer functions to changes in location of receivers and senders lies at the heart of the externality problem, especially for the most interesting case of large numbers of polluters and receivers.

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