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MARKETS FOR CONTRACTS: EXPERIMENTS EXPLORING THE COMPATIBILITY OF GAMES AND MARKETS FOR GAMES

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Abstract

The research presented here explores the relationship between games and the economic environment in which the game might be embedded. In particular, the focus is on a market institution in which agents buy and sell rights to participate in a follow-on stage of strategic interaction. Many instruments found in markets, such as insurance contracts and warranties, have that property and motivate the study. The central question posed is how the game and the market, two different types of processes, interact. Traditionally two sets of theory are applied to each of the separate processes: one relates to price formation in the market and the other involves modeling the follow-on stage of interaction as a game. However the application becomes ambiguous when the game has multiple solutions. Consequently, it is not clear from theory how or if joint convergence of the two processes might evolve. The study focuses on that issue and the results demonstrate that outcomes in the game are systematically linked to outcomes in the market. The game outcomes can be characterized by traditional game-theoretic solution concepts. Moreover, the market converges to a competitive equilibrium consistent with the Nash equilibrium that obtains in the game.

Markets for Contracts:
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of Games and Markets for Games

Charles R. Plott† Dean V. Williamson‡

1 Introduction

The research presented here examines the compatibility of game theoretical models with the classical model of market equilibrium. The experimental approach is ``exploratory'' in the sense that it is motivated by questions of economic behavior in the context of institutions even though there is neither good theory nor a clear line of previous experiments that point to what might be expected. Yet, the experiments seem to be central to both the thrust of theory and applications of theory. Thus, we will report on the outcomes of data generated in a particular institutional setting. While the models we apply are very suggestive, we leave open to speculation and further theory a more fundamental explanation of what we report.

The experimental design links a market process to a contract process. A contract is modeled as a game, and the purchase and sale of contracts (or games) is modeled as a market. The market involves the purchase and sale of rights to participate in a follow-on stage of strategic interaction (the game). Compatibility concerns how the two different processes - games and markets -- interact when they exist side by side as subsystems in economic environments. Are the equilibria in the two processes reinforcing, each promoting the convergence of the other process, or do they confound convergence? Is the selection of equilibrium in one process systematically linked to the selection of equilibrium in the other? Of course, the focus on institutions produces a deeper question about which we can only speculate: what dynamic process drives the *joint* selection of equilibrium in the two processes.

Clearly there are many alternative environments in which a study such as this could be initiated. The setting we chose reflects many arbitrary components in part, because there seemed to be no obviously unique place to begin. Contracting problems and even markets for contracts emerge naturally and abundantly in the context of industrial organization. Procurement contracts, for example, engage buyers and suppliers in

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strategic interactions that can be modeled as principal-agent games, and such contracts are often awarded to suppliers via market mechanisms such as auctions. Equity and debt certificates (``stocks and bonds'') constitute contracts that confer various rights over the control of corporations and can thus be modeled as games. Obviously these contracts are exchanged in markets. Futures contracts engage buyers and suppliers in a game that involves follow-on delivery of goods and services. Insurance contracts are clearly a case of markets for games. The insurance contract involves the insurer and the insured in classic relationships of moral hazard. Wage contracts frequently can be viewed as games involving principle-agent components, and of course the labor market is for the contract. Guarantees and warranties that accompany sales are in fact simply games. Thus the purchase and sale of games is common and arise naturally in the course of commerce. In all such cases two equilibrating processes exist side by side -- the market process and the game. The problems are pervasive.[1]

It is interesting to note that even though markets for games appear frequently in the applied literature, there is no explicit study of their compatibility. It is typically assumed that equilibria in both markets and games must be jointly determined and that the joint convergence of the two processes would conform to the convergence each of the two processes would obtain in isolation. The possibility that conduct in one process might confound convergence in the other process is not raised. For example, previous realizations of conduct in the game might setup price dynamics in the market that in turn would disrupt agents' expectations of successive conduct in the game. Alternatively, agents might maintain diverse expectations. In contexts involving multiple equilibria, expectations might fail to become aligned on particular equilibria. Furthermore markets have properties, such as the fact that they are continuous time processes, that are abstracted away in most theory. The influence that such variables might have is not obvious.

The results reported here are striking. Equilibria in the game are found to be systematically linked to competitive equilibria in the market. The convergence of the market to equilibria lags the convergence of behaviors in the game to equilibria, and the games move quickly to a solution of the game even though the market was in substantial disequilibrium. The markets then settled into the equilibrium that would exist if rational expectations prevailed, i.e., if agents were able to predict perfectly what

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[1] For example the original research questions were motivated by contracting problems that pertain to the financing of Mediterranean trade in the late Middle Ages. The prominent features of the organization of the Mediterranean trade, it turns out, emerge in a natural class of problems. The Mediterranean trade problem involves the joint equilibration of markets for traded commodities, of markets for agency contracts, and of principal-agent games. Specifically, the Mediterranean trade implicates the simultaneous equilibration of (geographically dispersed) markets. Coordination between these markets involves inputs from dispersed agents, and the contracting of these inputs generates a sequence of strategic interactions (games). Multiple equilibria may emerge in the games that obtain between the various functionaries of trade.

the outcome of the game would be. Thus, the research demonstrates that equilibria in games and in the institutional contexts in which they are embedded can be jointly determined and of course that fact suggests that institutional context might influence selection.

The paper proceeds in 5 parts. The next segment, Part II, relates the research to other work on equilibrium selection. Parts III and IV detail the experimental design, and Part V presents the models and predictions. Results are presented in Part VI, and Part VII concludes.

2 Background and Experimental Research

The research was motivated chiefly by the two observations. 1) Markets for contracts emerge in a broad range of applied contexts, and 2) the success with which certain types of solution concepts are able to characterize play in a game seems to depend on the institutional context in which the game is embedded. In some contexts, for example, traditional solution concepts help characterize play in a particular game whereas in other contexts they fail to characterize play in that same game. These contexts generally involve multiplicity of equilibria in games and the attendant problem of equilibrium selection. Much experimental research on games has approached problems such as the selection of equilibria by examining behaviors in increasingly simple environments, and yet other experimental research suggests that institutional context helps resolve the selection of equilibria in games.

Three branches of research suggest that the behavior in games can be influenced by placing the game in particular institutional environments. First, Cooper, DeJong, Forsythe, and Ross 1993, 1994 and Van Huyck, Battalio, and Beil 1990, 1991, and 1993 study special ways of allocating rights to participate in games. They suggest that prices might play a role in a process of forward induction that facilitates equilibrium selection. Secondly, research on ``cheap talk'' suggests that embedding particular games in contexts that permit specially tailored pre-game communication can promote the selection of equilibria. This research also points to a concept of forward. (See the survey of Crawford 1998. Examples include Cooper, DeJong, Forsythe, and Ross 1989 and 1992.)

A third body of research points to the possibility of very different institutional influences. It suggests that in properly structured environments variables exist that are unanticipated by game theory and may dominate or even confound convergence. Examples abound of games in which behaviors fail to line up with predictions in even starkly simple strategic contexts. (See Camerer and Thaler 1995 for examples.) The resulting paradoxes have motivated exciting lines of research. Implicit in some of this research is that the shortcomings of theory results from 1) the failure of theory to incorporate psychology that routinely emerges in strategic interactions, or 2) the failure of human actors to satisfy the cognitive competencies required of traditional conceptualizations of ``rationality.'' Behavioral game theory, for example, has involved efforts to innovate new equilibrium solution concepts, such as Mathew Rabin's ``Fairness Equilibrium,'' that are informed by research in psychology. (See Rabin 1993 and Camerer 1997.) Related research has responded by operationalizing concepts such as ``altruism'' (See, for example, McKelvey and Palfrey 1992, Eckel and Grossman 1996, Cooper, DeJong, Forsythe 1996), ``spitefulness''(Levine 1997), ``manners''(Camerer and Thaler 1995), ``fairness norms'' (Kagel, Kim and Moser 1996), and ``trust'' (Berg, Dickhaut, McCabe 1995). Other research more closely linked to the evolutionary game theory literature has involved efforts to operationalize concepts of bounded rationality (Stahl and Wilson 1995, Stahl 1996) and by articulating dynamic processes by which ``conventions'' or ``stable'' outcomes emerge. (See, for example, Crawford 1991, 1995 Van Huyck et. al. 1995, Van Huyck, Cook, Battalio 1997.)

The research presented here takes up a tack that is parallel to the research on special theories of equilibrium selection in games and the role of institutions in facilitating equilibrium selection and is complementary to the research that merges economics with psychological considerations and cognitive processes. Rather than focus on increasingly simple environments we focus on games that take place in a more institutional context -- a context that incorporates many of the features of situations for which applications of the theory are intended. In particular, we examine an environment in which 1) agents assume diverse roles and 2) payoffs are private information. In contrast, previous research on the role of institutions in equilibrium selection involved environments in which agents were symmetric and in which payoffs could be credibly modeled as common knowledge. Such environments could support processes suggestive of forward induction. In the research presented here, the structure of strategic interactions does not obviously accommodate behaviors that can be characterized by forward induction. Prices do not provide an unambiguous means of signaling behaviors in the follow-on game. Instead, equilibria of the game map into equilibria of the market process in an intuitively accessible way that can also be characterized by backward induction. As will become obvious, the compatibility of the concept of a market and a game theoretic equilibrium will be established, whether or not the vehicle was forward induction or backward induction is left open to speculation.

Simpler environments have permitted sophisticated applications of theory. For example, Crawford and Broseta (1998) present a theoretical framework on learning dynamics that they apply to the experimental results of Van Huyck, Battalio, and Beil (1990, 1991, and 1993). (Also see Cheung and Friedman 1998 on learning dynamics.) Within these environments researchers have initiated a process of attempting to ``work up'' from basic principles to characterizations of dynamic processes that drive equilibration and selection in games. Of course, how one might go about generalizing theory developed in the special environments they studied to more complex settings is still in the realm of the unknown. The literature does contain suggestions such as provided by the spirit of Crawford and Broseta (1998), which may yet be generalized to accommodate environments that feature private information. By contrast, the research presented here works down from a more complex institutional environment -- an environment in which we lose the power of theories of learning dynamics. We provide an ``exploratory leap'' into a more complex setting and report what we see in hope that by working from ``both ends'' appropriate theory will be more rapidly developed.

3 Design Overview

The experimental design involves a market institution in which agents buy and sell rights to participate in a follow-on game. The questions to be posed are whether or not outcomes in the game are systematically linked to outcomes in the market, and whether or not the game outcomes can be characterized by traditional game-theoretic solution concepts. Moreover, the question is whether or not outcomes in the market correspond to competitive equilibria, and if the competitive equilibria in the market map uniquely into equilibria in the game.

Game buyers were given redemption values for the number of games that were purchased in a market. That is, games were considered a commodity with a declining value for marginal units. Accordingly, the buyer generated some utility for playing the game independent of any of the outcomes of the game. Of course if the game is interpreted as a contract it would mean that the contract had value independent of what the consequences of its execution might be.

Game sellers were given costs for the number of games sold. These cost schedules reflected increasing marginal costs. The seller could, in a sense, ``produce'' the game at a cost and then sell it in the market for a price. In the context of a contract sale of a unit my be interpreted as the seller selling a commodity or service under a contract and the production of the commodity or service having a cost independent of whatever might be involved in the execution of the associated contract.

If the game had no additional value to either the buyer or seller the demand and supply functions induced in the market by application of the competitive model would be as shown in Figure 1. The redemption values can be used to produce a market demand function and the marginal costs can be used to produce a market supply function.

All games produced and sold are the same Battle-of-the-Sexes game displayed in Figure 2. If row player chooses up and column player chooses left the payoff would be 700 to the row player and 300 to the column player. This matrix was common knowledge.

A market was opened for the purchase and sale of the games. Buyers and sellers both had the capacity to buy and sell units and so could generate some payoff buying trading units at different prices through the course of the market session. After the market closed all games were played buy the respective buyers and sellers. Buyers always chose among the rows of any game a buyer played and sellers always chose among the columns. Each buyer played all games that he or she purchased net of any sales. Similarly each seller played all games that he or she sold net of any purchases. However, the pairings of the play did not necessarily match the pairings of the market transactions. That is, a buyer who purchased two games would make two choices but the other player would not necessarily be the player who sold the games to this particular buyer. The pairings for play of the game were random and the identities of players were not known. The only things known at the time of play of the game were the number of game purchased and sold, the prices at which transactions took place in the market and the history of aggregate play in previous plays of the game. These aspects of information will be made more precise in the following section.

The total payoff of buyers was the profit from the market, the sum of the difference between the redemption values of units and the prices paid plus the sum of the payoffs from all games played. The total payoff to a seller was sum of the difference between selling prices and cost of units plus the payoffs from all games played. So, payoffs were determined jointly between the prices in the market and the subsequent patterns of play in the games that were bought and sold in the market.

4 Experimental Design and Procedures

Four experimental sessions were conducted. The sessions were conducted in 1996 on February 14, February 22, November 13, and November 14. Hereafter each experimental session will be identified as Feb 14, Feb 22, Nov 13, and Nov 14. Respectively, 8, 12, 10, and 14 subjects participated in these four sessions.

All subjects were students drawn from the Caltech student body. None had experience with this particular set of experiments but some may have had experience in either markets or games in other experiments. Each experiment employed an even number of subjects, half assigned the role of ``buyer'' and the other half assigned the role of ``seller.'' (See the following table):

Experimental Session	Number of Buyers	Number of Sellers
Feb 14	4	4
Feb 22	6	6
Nov 13	5	5
Nov 14	7	7

In each of four experimental sessions, an even number of agents participated in repeated rounds of a two-stage framework. In the first stage of each round agents bought or sold rights to participate in a second stage of strategic interaction. Exchange was organized via a double auction operationalized with standard software on a computer network. Agents were provided with sets of marginal cost schedules *or* with marginal redemption value schedules. Agents used a new schedule in each round. Costs and redemption values were denominated in an experimental currency called ``francs'' These marginal cost schedules determined the marginal cost of supplying to the market as many as 10 units (called ``assets''). Similarly, marginal redemption value schedules determined positive marginal payoffs for as many as 10 units that a buyer could acquire in the market. The experimenter provided half of the agents with marginal cost schedules and the other half with marginal redemption value schedules. Those agents with cost schedules were designated ``sellers,'' and the other agents were designated ``buyers.''

In the first stage of each round, agents purchased and sold assets in a double auction that lasted 5 minutes. Buyers and sellers were permitted to speculate or ``trade'' during the process of price formation. That is, buyers could both buy units and resell units and sellers could do the same. Total costs and total benefits were determined by the net inventories buyers and sellers maintained at the end of each double auction. These costs and benefits were recorded on record sheets, and in each round agents' inventories were restored to zero.

After each double auction closed, buyers assumed the right to choose the actions ``Up'' or ``Down'' in a binary choice process, the Battle-of-the-Sexes game. Sellers assumed the right to choose ``Left'' or ``Right''. Agents chose one action for each of the units they sold or purchased. These action choices were made without knowledge of other agents' choices. Agents recorded each of their choices on a separate piece of paper. The pieces of paper or ``tickets'' were color-coded and were labeled with the index number of the current round of the experiment and with the an index number assigned to each of the agents. Sellers' tickets were printed on green paper whereas buyer's tickets were printed on pink paper. Agents were supplied with ten tickets for each of as many as 22 rounds of interaction.

After all agents had chosen their actions, the experimenter shuffled the sellers' tickets (the green tickets) and paired each of a seller's tickets with a buyer's ticket. In this way sellers' and buyers' tickets were randomly and anonymously matched. The resulting pairs dictated one of four possible pairs of payoffs. (The actions and payoffs are arrayed in the 2x2 matrix below.) Payoffs were denominated in an experimental currency called ``francs.'' For example, a pair of action ``Left,'' ``Right'' determined payoffs of 700 francs to the buyer of that unit and 300 francs to the seller of that unit. The experimenter indicated payoffs of 100, 300, or 700, as appropriate on each of the buyers' and sellers' tickets. Tickets were re-distributed to the agents, and agents were permitted to account their payoffs on an account sheet.

The aggregate frequencies of choices were public information. After payoffs were distributed, the experimenter announced and posted aggregate results from the stage of strategic interaction. Posting the results entailed indicting the frequencies with which pairs occurred in each of the four cells of the 2x2 matrix. The four marginal frequencies were also posted. Thus both buyers and seller could see the relative frequencies with which strategies were chosen. Of course they could not identify the strategy of any particular person or persons.

For each agent, accounting for payoffs entailed 1) summing payoffs indicated on each of the returned tickets, 2) summing, as appropriate, total costs or total benefits, and 3) identifying revenues or expenditures from the double auction. In turn, summing these three quantities generated each agent's total payoff from participation in the two-stage process. Agents accounted payoffs at the end of each round. Payoffs from the entire experiment were generated by summing the payoffs from each round.

Notation

In the succeeding sections we articulate models, predictions, and results. The models operationalize mechanical adjustment processes. It is these processes that permit examination of the data. Before we proceed, however, we need some notation.

A realization of the entire two-stage process is a 3-tuple (P, Q, **f**), and an equilibrium is a 4-tuple (P*, Q*, p*, q*)[2] where

[2] Generically, the marginal distributions (p, 1-p) and (q, 1-q) are not independent, in which case expected payoffs must be evaluated with reference to the joint distribution (f_{UL}, f_{DL}, f_{UR}, f_{DL}). On the other hand, the equilibrium strategies (p*, 1-p*) and (q*, 1-q*) are independent.

P = price in the double auction,
Q = volume in the double auction,
$\mathbf{f} = (f_{UL}, f_{DL}, f_{UR}, f_{DR})$
f_{rc} = relative frequency with which the outcome (r, c) emerges in the succeeding second stage of interaction; $r \in$ {Up, Down} and $c \in$ {Left , Right} and $f_{UL}+f_{UR}+f_{DL}+f_{DR} = 1$
$(p, 1-p) = (f_{UL}+f_{UR}, f_{DL}+f_{DR})$ = row player's strategy over the strategy set {Up, Down},
$(q, 1-q) = (f_{UL}+f_{DL}, f_{UR}+f_{DR})$ = column player's strategy over the strategy set {Left, Right},
(p^*, q^*) constitutes a static equilibrium, under some solution concept, of the second stage, and
(P^*, Q^*) conform to the equilibrium price and quantity in the market.

Each realization of the second stage is identified by a 4-tuple $(f_{UL}, f_{DL}, f_{UR}, f_{DL})$ of relative frequencies. Associated with these relative frequencies are marginal frequencies (p, q). In an equilibrium of the entire two stage process, (p^*, q^*) and (P^*, Q^*) are simultaneously determined.

The models of market equilibration share a common structure. *Given* unit payoffs (b, s) to buyers and sellers in the second stage (determined under a given solution concept), the models of market equilibration satisfy the following:

$$P \in [D(Q) + b(\cdot)] \cap [S(Q) - s(\cdot)]$$

where $D(Q)$ = inverse demand correspondence,
$S(Q)$ = inverse supply correspondence,
$b(\cdot)$ = unit payoff to buyers, under some solution concept, in the second stage,
$s(\cdot)$ = unit payoff to sellers in the second stage,

and $D(Q) + b(\cdot)$ = derived demand,
$S(Q) - s(\cdot)$ = derived supply.

5 Models and Predictions

A particular advantage of the experimental method is that it permits examination of dynamic processes by which equilibria (if any) emerge. Accordingly, analysis of the experimental data is organized around both static predictions and dynamic adjustment processes. The predictions pertain to the identification of static equilibria (P^*, Q^*, p^*, q^*) of the two-stage process, to the selection of equilibria, and to the evolution of prices.

Models

The models pertain to price formation, the determination of volume in the market and of action choices in the second stage of the two-stage apparatus, and to the relation (if any) of prices to action choices. Specifically, the models include two sets of partial equilibrium models: those that characterize equilibration in the second stage process, and those that characterize equilibration in the market process. The real power of the models, however, derives from the characterization of the simultaneous equilibration of the market process and second stage process. Such models link equilibria in the second stage with equilibria in the market and raise deep issues about the dynamic processes through which coordination between the market and second stage process is achieved (if at all).

The data are examined via three different classes of models. These models can be distinguished from each other by the nature of belief formation and the nature of the individual decision process. Two types of belief formation, labeled ``Cournot expectations'' and ``perfect foresight,'' are distinguished. Cournot expectations involve repeated interaction of the entire two-stage process. Agents generate beliefs with reference to immediate history of interaction. In particular, determine how to behave in the second stage of interaction by referring to the last realization of second stage interaction. Perfect foresight, on the other hand, does not implicate interaction across rounds. Rather, it is a static concept. Agents forecast (correctly) outcomes in the second stage of interaction. In a general equilibrium of the entire two-stage process, beliefs implicate the pricing of units.

Two types of decision-making processes are distinguished, and they are labeled ``dynamic'' and ``partial backward induction.'' These processes link outcomes in the market to outcomes in the second stage of interaction. Under the model of dynamic decision-making agents factor the structure of other agents actions into ones own choice of actions in the second stage and into pricing decisions. Behaviors that conform to partial backward induction, however, are less sophisticated. Under this behavioral model, agents fail to factor their own responses into their pricing decisions. Rather, agents lose their identities in the pricing decision and act as if they were a representative agent.

The two models about belief formation and the two models about individual decision-making identify four general equilibrium models, three of which we operationalize. We identify and label these models in the following table.

		Belief Formation	
		Cournot expectations	Perfect foresight
Individual Decision-making	partial backward induction	Myopia	Not Operationalized
	dynamic	Cournot	Rational Expectations (RE)

These models determine 1) unit payoffs b(·) and s(·), 2) market outcomes (P, Q), and 3) the nature of coordination (if any) between market outcomes and unit payoffs. The RE model, for example, incorporates the perfect foresight hypothesis with the dynamic behavioral hypothesis. Behaviors in the second stage of interaction conform to the Nash hypothesis, Nash outcomes of the second stage imply payoffs to buyers and sellers, and agents factor these payoffs into their pricing decisions. Nash equilibria of the second stage are thus linked to equilibria in the market.

a) A Static Rational Expectations model (RE)

RE operationalizes the concept of Simultaneous Equilibration. Under RE agents know the structure of the model and coordinate behaviors on outcomes of the two-stage process. Agents have perfect foresight: they anticipate behaviors will converge in the second stage of each round, and they anticipate *which* behaviors will obtain. More specifically, agents anticipate a Nash equilibrium (p^*, q^*) of the second stage, and they factor the payoffs that derive to them in their pricing decision. As under SEP, (p^*, q^*) map uniquely into equilibrium prices and volumes (P^*, Q^*). We label the predictions (P^*, Q^*, p^*, q^*) ``RE-consistent.''

Mechanical Adjustment Processes

b) Cournot (C)

The Cournot model assigns to agents a lesser degree of rationality than RE. Agents assume outcomes of the two-stage process result from the active strategizing of the other agents in the market; in the model agents best-reply to some subset of past realizations of second stage play. Agents factor into their pricing decision some subset of past marginal frequencies (p, q) and their own best-replies to that subset of frequencies. Under this model, however, agents do not factor other agents best-replies into their own best-replies. Rather, they assume that past frequencies characterize other agents' next-period choices.

Examined here is a particular case of ``Cournot expectations'' where agents craft best-replies with respect to the last realization of second-stage play. The model then predicts that second-stage frequencies converge on one of the RE-consistent predictions or that second-stage frequencies collapse into an infinite sequence of out-of-equilibrium play.

Formally, the pair

$(p_t, q_t) = (p_t(q_{t-1}), q_t(p_{t-1}))$, characterizes agents' strategies where

$g_t = g_t(\cdot)$ denote best-reply functions, and

$(P_t, Q_t) = (P_t(p_t, q_t), Q_t(p_t, q_t)$ denotes the equilibrium in the double auction implied by the anticipated frequencies (p_t, q_t).

Second-stage interaction converges on one of the two pure strategy equilibria of the two-stage process or collapses into an infinite, non-convergent sequence of out-of-equilibrium play. The frequencies $p_t(q_{t-1})$ and $q_t(p_{t-1})$ may not correspond to any Nash equilibrium of the second stage but may generate a non-convergent sequence of mis-coordinated play. A realization $(p_{t-1}, q_{t-1}) = (1, 0)$ generates $(p_t, q_t) = (0, 1)$ which in turn generates $(p_{t+1}, q_{t+1}) = (1, 0)$, and so on. On the other hand, $(p_2, q_2) = (1,1)$ or $(0, 0)$ generates a forward invariant sequence $(p_t, q_t) = (p_2, q_2)$.

The static predictions (P^*, Q^*, p^*, q^*) can be arrayed in the following table:

Price (P*)	Quantity (Q*)†	Strategies (p*, q*)	Surplus†
790 - 810	17	(1, 1)	8,540
760 - 780	4	alternately (1, 0) and (0, 1)	410
690 - 710	9	(0.75, 0.25)	1,980
390 - 410	17	(0, 0)	8,540

†Values correspond to the double auction conducted with 6 buyers and 6 sellers.

Observe that the RE-consistent predictions are also Cournot-consistent.

c) Myopia

Similar to the discussion of the Cournot model, Myopia assigns to agents a lesser degree of rationality than Cournot, and Myopia admits the Cournot-consistent (and therefore RE-consistent) predictions. Under the Cournot model, price and volume anticipate the frequencies (p,q). Under Myopia, prices and quantities respond to *previous* realizations of second stage interaction. Effectively, agents assume that observed frequencies are representative of the forthcoming frequencies, and they price their units accordingly. Formally, Cournot satisfies

$$P_t \in [D(Q_t) + b(p_t(q_{t-1}), q_{t-1})] \cap [S(Q_t) - s(p_{t-1}, q_t(p_{t-1}))]$$

whereas Myopia satisfies

$$P_t \in [D(Q_t) + b(p_{t-1}, q_{t-1})] \cap [S(Q_t) - s(p_{t-1}, q_{t-1})].$$

As under Cournot, agents best-reply in the second-stage of interaction to the previous realization (p_{t-1}, q_{t-1}).

Myopia does not pin down a countable set of predictions. Note however that the Cournot model can be distinguished from Myopia in the data in that Cournot predicts a larger period-on-period volume than Myopia, because under Cournot agents factor there own best-replies into their own decision to buy or sell units.

The models are summarized as follows:

Price formation:

1) RE: Agents factor anticipated frequencies (p_t, q_t) into pricing.
2) Cournot: Buyers factor $(p_t(q_{t-1}), q_t)$ and sellers factor $(p_{t-1}, q_t(p_{t-1}))$ into pricing decision.
3) Myopia: Prices respond to latest frequencies (p_{t-1}, q_{t-1}).

Determination of Volume:

1') RE: Agents factor anticipated frequencies (p_t, q_t) into volume.
2') Cournot: Buyers factor $(p_t(q_{t-1}), q_t)$ and sellers factor $(p_{t-1}, q_t(p_{t-1}))$ into volume.
3') Myopia:Volume responds to latest frequencies (p_{t-1}, q_{t-1}).

Determination of second-stage behaviors:

1'') RE: Agents' behaviors jointly correspond to a Nash equilibrium of the second-stage process.
2'') Cournot, Myopia: Agents best-reply to previous frequencies (p_{t-1}, q_{t-1}).

The Simultaneous Equilibration Prediction (SEP)

Under SEP (p^*, q^*) constitutes a static Nash equilibrium of the second stage, and the behaviors (p^*, q^*) imply expected payoffs $b(p^*, q^*)$ and $s(p^*, q^*)$ to buyers and sellers. Agents factor these payoffs into their pricing decision in the preceding double auction. The usual analysis of competitive markets provides an obvious way of factoring payoffs into the double auction: buyers and sellers derive their demand and supply schedules in the preceding double auction by factoring their payoffs $b(\cdot)$ and $s(\cdot)$into their marginal surplus computations. Specifically, buyers buy their *k*th units if they can secure prices which do not exceed the sum of b(p, q) and the marginal benefit of the *k*th unit. Sellers sell their *k*th units if they can secure prices which exceed the difference between the marginal cost of the *k*th unit and s(p, q). These derived demands and supplies in turn imply a unique competitive equilibrium in the double auction. The derived demand and supply schedules that are consistent with (p, q) = (1, 1), the Nash Equilibrium that corresponds to ``Up-Left'' are displayed in Figure 3.

In our notation, SEP satisfies

$$P^* \in [D(Q^*) + b(p^*, q^*)] \cap [S(Q^*) - s(p^*, q^*)]$$

As above, the static predictions (P^*, Q^*, p^*, q^*) can be arrayed in the following table:

Price (P*)	Quantity (Q*)†	Strategies (p*, q*)	Surplus†
790 - 810	17	(1, 1)	8,540
690 - 710	9	(0.75, 0.25)	1,980
390 - 410	17	(0, 0)	8,540

†Values correspond to the double auction conducted with 6 buyers and 6 sellers.

Whereas buyers and sellers oppositely rank the pure-strategy Nash equilibria (1, 1) and (0, 0) of the second stage, the surpluses available to each agent under each of the pure strategy equilibria (800, Q^*, 1, 1) and (400, Q^*, 0, 0) of the entire two-stage process are identical. All players rank the pure strategy equilibria above the mixed-strategy equilibrium (700, Q^*_{mixed}, 0.75, 0.25).

6 Results

The chief result of the paper is that prices, volumes, and behaviors systematically converge *and* that they converge to a state consistent with RE. Moreover, the data indicate that convergence of prices lags the convergence of realizations in the second stage. Also, the data indicate that agents some sophistication in their pricing decisions. Specifically, there is evidence that they factor their own action choices in the second stage into their pricing decision in the preceding double-auction.

To evaluate convergence of prices to predicted values, the following econometric model is enlisted: a second-order autoregressive process or AR(2) whereby

$$P_t = \alpha + \beta_1 P_{t-1} + \beta_2 P_{t-2} + \varepsilon_t,$$

and ε_t is assumed to be a ``white noise'' process.

Under this model a steady-state price P* corresponds to $\frac{\hat{\alpha}}{1-\hat{\beta}_1-\hat{\beta}_2}$ where $\hat{z}$ = the OLS estimate.[3]

Result 1: In each of the experimental sessions, behaviors in the second stage of strategic interaction converge to one of the two pure-strategy Nash equilibria.

Support: In 3 of 4 sessions behaviors converge to the pure-strategy Nash equilibrium that corresponds to ``Down, Right'' or (p, q) = (0, 0). In the one other session, behaviors converge on the other pure-strategy Nash equilibrium that corresponds to ``Up, Left'' or (p, q) = (1, 1).

Figure 4 exhibits the round-by-round convergence in session Feb 22 of behaviors in the Battle-of-the-Sexes to the pure-strategy Nash equilibrium conforming to ``Down-Right.'' In session Feb 22, behaviors converged on (0, 0) by round 6. A single buyer deviated with a single unit in both of the succeeding rounds, but in the remaining 7 rounds behaviors conformed to (p, q) = (0, 0).

In session Feb 14, behaviors converged on (0, 0) by round 5. (See ``Observed Frequencies'' in Table 2.1) A single buyer deviated from (0, 0) in round 6, but in the remaining 4 rounds behaviors conformed to (0, 0). Behaviors converged to the other equilibrium (1, 1) in session Nov 13. Behaviors converged by round 4. In rounds 6 through 11 some sellers deviated from the equilibrium, but behaviors conformed to (1, 1) in the closing 7 rounds of the session. Lastly, behaviors converged on (0, 0) by round 10 of session Nov 14. A single agent deviated with 8 units in round 13, and the same agent deviated with 3 units in round 14, but in the remaining 4 rounds behaviors conformed to (0, 0). (See ``Observed Frequencies'' in Tables 2.2, 2.3, and 2.4.)

Result 2: In each of the experimental sessions, prices converge to RE-consistent levels.

[3] Under the assumption that the error process corresponds to white noise, the assumptions of the Gauss-Markov theorem are satisfied, and, accordingly, OLS estimates are BLUE.

Support: In sessions Feb 14, Feb 22 and Nov 14, prices converged to 400, the price RE-consistent with the pure-strategy Nash equilibrium (0, 0). In session Nov 13, prices converged to 800, the price RE-consistent with the pure-strategy Nash equilibrium (1, 1).

Figures 5 and 6 exhibit the convergence of prices in sessions Feb 22 and Nov 13. Figure 5 exhibits the prices at which units were transacted over the course of 14 market sessions. After each market closed, test subjects participated in a round of strategic interaction. The round-on-round relative frequency with which the outcome ``Down-Right'' emerged is mapped against the right axis of Figure 5. By round 6, behaviors in the second stage of strategic interaction conformed to ``Down-Right'', and prices subsequently converged to the RE-consistent price of 400. In session Nov 13, prices converged to 800, the price that is RE-consistent with the outcome ``Up-Left'' to which behaviors converged.

Figure 7 presents prices and behaviors from all four experimental sessions. Average round-on-round prices are mapped against the left axis, and the statistic (p + q), the round-on-round sum of observed frequencies, is mapped against the right axis. The condition (p + q) = 2.00 indicates that behaviors in the second stage conformed to ``Up-Left,'' and (p + q) = 0.00 indicates that behaviors conformed to ``Down-Left.'' In the three sessions in which the statistic (p + q) converged to the value 0.00, prices converged to the RE-consistent price 400. In session Nov 13 (p + q) converged to 2.00, and average prices converged on the corresponding price of 800.

To substantiate the convergence of prices, we apply the AR(2) model to the price series that begin with the period succeeding the one in which behaviors in the second stage first lined up with a Nash equilibrium. For example, in the first session *all* action choices lined up with ``Down, Right'' for the first time in period 4. We argue that the data generating process shifts from one regime to another once agents begin to coordinate on an outcome in the second stage of strategic interaction. For the same reason, we apply the AR(2) to data commencing with period 7 in the second session, and to data commencing with periods 5 and 11 in sessions 3 and 4, respectively.

The AR(2) model $P_t = \alpha + \beta_1 P_{t-1} + \beta_2 P_{t-2} + \varepsilon_t$ generates estimates of the steady-state price that are consistent with the hypothesis that the prices are converging to 400 in sessions 1, 2, and 4, and to 800 in session Nov 13. The estimates of the steady-state prices from each session all lie within 1.33 standard deviations of the hypothesized steady-state prices. (See Table 4.)

Admittedly, we would prefer to model the entire data generating process either by endogenizing the regime shift or developing a different model that would incorporate all of the price data. However, our immediate purpose is to quantify the notion that prices ``converge.''

Figure 5 exhibits the convergence of prices to RE-consistent price of 400 in the experimental session Feb 22. Behaviors in the game converged to the pure-strategy Nash equilibrium corresponding to ``Down-Left,'' and prices subsequently converged.

To substantiate the notion of ``regime shift'' in the data generating process, we also include in Table 4 F-statistics corresponding to Chow's test for structural change in the price series. ``Large'' values support the hypothesis that the data generating process has changed from one segment of the price series to the next, and three our F-statistics are indeed significant at the 1% level. The fourth is significant at the 5% level.

Result 3: In each of the experimental sessions, volumes converge to RE-consistent levels.

Support: We apply a statistical criterion that indicates that volumes in all four sessions converge to RE-consistent levels. We apply an ARMA(1, 2) of the form $Q_t = \alpha_1 d_1 + \alpha_2 d_2 + \alpha_3 d_3 + \alpha_4 d_4 + \beta Q_{t-1} + u_t$ where $u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t$, ε_t is assumed to conform to the usual ``white noise'' process, and the terms d_i are dummy variables identifying each of the four experimental sessions. The model generates steady-state predictions of volumes in each of the four sessions that cannot be statistically distinguished from the RE-consistent predictions of volumes. (See statistical tests in Table 6.)

The ARMA(1,2) employs 57 observations of volumes of 61 of the observations generated from all four experimental sessions. Four observations are omitted in order to accommodate the single lag on volume Q_{t-1}. The model pools the data to generate an estimate of the effect of the lag, but the four dummy variables permit estimation of an ``intercept'' α_i for each of the four experimental sessions.

The next result characterizes the evolution of prices in the system. Principally, the convergence of prices lags the convergence of behaviors in the game. The lag of prices itself suggests that prices respond to the equilibrium realizations that emerge in the second stage.

Two measures of the lag are articulated. First, it is observed that within the first 4 rounds of each experimental session behaviors on at least one side of the market make initial contact with the equilibrium of the second stage that subsequently obtains, and prices subsequently converge. Secondly, the AR(2) model of the evolution of prices employed in Result 1 substantiates the follow-on convergence of prices.

Result 4: In all four experimental sessions, the convergence of prices to RE-consistent values lags the convergence of behaviors to pure-strategy Nash equilibria.

Support: Figure 7 indicates the convergence of behaviors in the second stage and the follow-on convergence of prices. Within the first four rounds of all four sessions, behaviors on the side of the market receiving 700 in the equilibrium of the second stage game make initial contact with the equilibrium that eventually obtains. In sessions Feb 14, Feb 22, and Nov 14, sellers actions all conform to ``Right'' (q = 0) in the first round, the fourth round, and the fourth round, respectively. In each of the three sessions behaviors in the second stage subsequently converge to the equilibrium conforming to ``Down,'' ``Right''. By the fourth stage, prices have yet to converge. Average prices in the fourth round of each of these sessions were 780, 631, and 582. Prices in each of these sessions eventually converged to 400. In session Nov 13 the equilibrium conforming to ``Up,'' ``Left'' eventually obtained. In this case, buyers actions first made contact with the equilibrium behaviors (p = 1) by round 3. Average prices were 720, well short of the equilibrium price of 800.

The AR(2) model $P_t = \alpha + \beta_1 P_{t-1} + \beta_2 P_{t-2} + \varepsilon_t$ employed in the discussion of Result 1 articulates the remainder of the experience of prices. The data support the hypothesis that the price series in each of the four experimental sessions exhibit structural shift by the time behaviors on both sides of the market first make contact with a pure-strategy Nash equilibrium of the second stage. The estimates of the coefficients on the lagged prices (β_1 and β_2) after convergence in behaviors are statistically significant, indicating that prices experience substantial evolution after behaviors have already converged. (Once again, regression results are presented in Table 4.)

Result 5 provides a clue about the strategic sophistication agents exercise, and it suggests that their behaviors can be partially characterized by backward induction. First, the result suggest that agents use the latest realizations of the second stage game to form expectations about the next stage. More importantly, however, the result suggests that agents are sophisticated enough 1) to factor expected payoffs into their pricing decisions -- they apply backward induction -- and 2) to factor there own best-replies to expected behaviors into their calculations of expected payoffs.

Result 5: The Cournot model characterizes the evolution of prices with markedly more success than the RE and Myopia models.

Support: The data indicate that the Cournot model predicts price intervals that capture 82.46% of the prices on the last units transacted in each round. The Myopia and RE models generate predictions that are consistent with 35.09% and 24.59% of the same data, respectively.

To characterize the agreement of observed prices with the various price predictions, we tailor a series π_t to each the Cournot model, Myopia, and RE where

$$\pi_t = \frac{P_t - \hat{P}_t(\cdot)}{[\overline{P}_t(\cdot) - \underline{P}_t(\cdot)]/2},$$

P_t = the price of the *last* unit traded in period t,

$[\underline{P}_t(\cdot), \overline{P}_t(\cdot)]$ is the interval of prices consistent with the given model in period t,

$\hat{P}_t(\cdot) = \frac{\overline{P}_t(\cdot) + \underline{P}_t(\cdot)}{2}$ is the midpoint of the interval $[\underline{P}_t(\cdot), \overline{P}_t(\cdot)]$, and

functions $g_t(\cdot)$ are specific to each model.

Under RE $g_t(\cdot)$ is a function of the (anticipated) marginal frequencies (p_t, q_t), whereas under Cournot $g_t(\cdot)$ is a function of both the observed frequencies (p_{t-1}, q_{t-1}) *and* the best-replies $p_t(q_{t-1})$ and $q_t(p_{t-1})$ to them, and under Myopia $g_t(\cdot)$ is a function of the observed frequencies (p_{t-1}, q_{t-1}).

Observe that a value of π_t in the interval [-1, 1] indicates that the observed last price P_t is consistent with the models under examination. If we define $\pi_t \in [-1, 1]$ as a ``success'' for trial t, we find that the RE model registers 15 successes out of 61 trials (24.59%). The Cournot process, however, captures 47 out of 57 available observations (82.46%), and Myopia registers 20 successes out of 57 trials (35.09%). (See Table 5.1.)

Figure 8 presents the distribution of the series $|\pi_t|$ for each of the models. The cumulative density of $|\pi_t|$ generated under each model is mapped against $|\pi_t|$ itself. Each cumulative density is bounded above at 100%. At $|\pi_t| = 1$, the density generated under the Cournot model captures 82% of the data.

Under the Cournot model agents best-reply to previous frequencies (p_{t-1}, q_{t-1}), and they determine price and volume by factoring the last period's frequencies *and* their best-replies to those frequencies into their calculations. The ``Predicted Prices'' in Tables 3.1 through 3.4 indicate the price margins that are consistent with the realization that emerged.

The RE model generates a series π_t that is almost uniformly distributed around $\pi_t = 0$, the value of π_t that corresponds to predictions centered precisely on the observed last price P_t. The Cournot model, however, generates predictions that are more tightly packed. The median absolute divergence of π_t from zero under the RE and Myopia models exceeds 2. That is, under these models the mid-point of more than half the predicted prices diverges from the data by more than 1 price interval. Further, the standard deviation of the π_t series is nearly one-half or one-third the standard deviations of the series generated under the other three models. (See Table 5.2.)

One of the salient behavioral phenomena the models do not capture is the deviation, usually by a single agent, in the second stage of interaction from a pure strategy equilibrium of the second stage after convergence has initially been achieved. Nonetheless, the largest deviations of π_t from the observed prices do not correspond to the deviations from Nash equilibria of the second stage. In the experimental session of November 13, for example, the largest absolute values of π_t under the Cournot model correspond to periods 7 and 10. Neither of these periods corresponds to a period following a ``deviation.'' The average absolute deviation of π_t from 0 under the Cournot model is 1.49. That is, on average π_t diverges from 0 by more than half a Cournot-consistent price interval. The average deviation of the data less the two largest deviations is 0.68. That is, if we exclude the largest deviations from our calculations, we find that on average π_t diverges from 0 by a magnitude that is Cournot-consistent.

7 Conclusion

What happens when players buy the right to play a game? There is a body of experimental evidence, including the research presented here, that suggests that what happens depends systematically on features of the larger institutional context in which games are embedded.

Price coordination mechanisms have been the focus of attention of several studies. In the experiments of Van Huyck, Battalio and Beil, agents participate in a market through which the right to participate in a second stage of interaction is distributed. In their experiments market volume is predetermined and the structure of payoffs is common knowledge. In Cooper, Dejong, Forsythe, and Ross (1993), 18 agents bid for 9 units where each unit entitles the owner one action choice in a 9-player coordination game. In this framework, a forward-induction model entails mapping prices into Nash equilibria in an obvious way.[4]

The framework we present is less accommodating. In the framework presented here agents do not know the entire structure of the two-stage apparatus: they know their own marginal cost or marginal redemption value schedules, and they commonly know the structure of the second stage process, but they do not know the demand and supply schedules. Accordingly, it is not obvious how agents might use prices to signal

4. For references consult Van Huyck, Battalio and Beil (1990, 1991, 1993), Crawford (1990, 1995), Crawford and Broseta (1995), and Cooper, Dejong, Forsythe, and Ross (1993, 1994).

behaviors in the second stage. How would, for example, an agent know that a price of 800 is consistent (in our RE model) with the outcome (``Up,'' ``Left'')?

The first three results indicate that behaviors converge to RE-consistent equilibria, and the remaining results begin to illuminate the dynamic processes by which behaviors converge. From Result 4 one sees that agents do not coordinate behaviors in the game via equilibrium prices. Rather, the convergence of prices in the double auction lags convergence of behaviors in the second stage. Result 5 indicates that prices respond to realizations in the game. Specifically, the data indicate that agents factor their own actions in the second stage into their pricing decisions in the first stage. Finally, the conjecture presented above suggests that agents expectations of realizations in the game become aligned after the opening stages of the game. Agents indicate that in the early rounds they anticipate a particular pure-strategy equilibrium of the second stage and, accordingly, they jointly bid up volumes in the early rounds and start coordinating on one of the two pure-strategy equilibria of the second stage. The sensitivity of agents to different payoffs remains and even after many periods some of them attempt to unilaterally motivate a shift from one equilibrium to the other. All of these attempts to unilaterally manipulate the selection of equilibria fail.

We speculate that both learning dynamics and solution concepts, whether new or traditional, can be systematically linked to attributes of the institutional context in which games are embedded. That is, we suggest that the issue of equilibrium can be naturally embedded within the larger issue of the institutional sensitivity of equilibria. In this context the research makes progress in characterizing a mapping from institutions into learning dynamics and into equilibria. First and foremost, the research demonstrates that agents' behaviors converge on static outcomes that are consistent not merely with a Nash prediction of behaviors in a static game but are consistent with a joint application of supply-demand analysis and Nash predictions. While none of the models completely characterize the learning and dynamics exhibited in the experimental data, one model out-performs all of the others. That model is one in which agents best-reply to some subset of past realizations of strategic interaction *and* in which agents factor their own actions into future realizations. This model outperforms those in which agents fail to factor their own actions into their calculus and those in which agents forecast (perfectly) other agents' actions.

Table 1: Static RE Predictions

Session	Buyers:Sellers	Price	Volume	Frequencies (p, q)	Nash Equilibrium of Second Stage	Per Unit Surplus		
						Buyers	Sellers	Total
1	4:4	790-810	12	(1, 1)	Up, Left	329.17	140.83	470.00
		690-710	6	(0.75, 0.25)	Mixed	143.33	58.33	201.67
		390-410	12	(0, 0)	Down, Right	329.17	140.83	470.00
2	6:6	790-810	17	(1, 1)	Up, Left	352.35	150.00	502.35
		690-710	9	(0.75, 0.25)	Mixed	156.67	63.33	220.00
		390-410	17	(0, 0)	Down, Right	352.35	150.00	502.35
3	5:5	790-810	14	(1, 1)	Up, Left	347.86	147.86	495.71
		690-710	7	(0.75, 0.25)	Mixed	167.14	67.14	234.29
		390-410	14	(0, 0)	Down, Right	347.86	147.86	495.71
4	7:7	790-810	21	(1, 1)	Up, Left	331.43	141.90	473.33
		690-710	11	(0.75, 0.25)	Mixed	142.73	58.18	200.91
		390-410	21	(0, 0)	Down, Right	331.43	141.90	473.33

Table 2.1: February 14 Results

Round	Observed Frequencies (p, q	Average Payoffs		Observed		Per Unit Surplus Extracted			System Efficiency
		Buyers	Sellers	Avg Price	Volume	Buyers	Sellers	Total	
1	(1.00 , 0.00)	100.00	100.00	836	5	-148	-4	-152	-13.48%
2	(0.50 , 0.33)	266.67	333.33	842	6	-35	280	245	26.06%
3	(0.63 , 0.25)	325.00	375.00	833	6	-133	315	181	19.28%
4	(0.50 , 0.00)	200.00	400.00	780	6	-16	563	547	58.16%
5	(0.00 , 0.00)	300.00	700.00	734	9	147	315	462	73.67%
6	(0.33 , 0.00)	233.33	500.00	644	9	114	168	282	45.04%
7	(0.00 , 0.00)	300.00	700.00	574	9	250	330	580	92.55%
8	(0.00 , 0.00)	300.00	700.00	480	9	344	194	538	85.82%
9	(0.00 , 0.00)	300.00	700.00	459	9	365	228	593	94.68%
10	(0.00 , 0.00)	300.00	700.00	411	11	345	161	505	98.58%

Table 2.2: February 22 Results

Round	Observed Frequencies (p, q)	Average Payoffs		Observed		Per Unit Surplus Extracted			System Efficiency
		Buyers	Sellers	Avg Price	Volume	Buyers	Sellers	Total	
1	(0.57 , 0.36)	285.71	314.29	742	14	-89	50	-39	-6.44%
2	(0.60 , 0.07)	166.67	300.00	725	15	-63	-43	-106	-18.65%
3	(0.27 , 0.09)	300.00	554.55	692	11	129	286	415	53.49%
4	(0.35 , 0.00)	229.41	488.24	631	17	1	96	96	19.20%
5	(0.08 , 0.00)	284.62	653.85	535	13	287	240	527	80.21%
6	(0.00 , 0.00)	300.00	700.00	425	11	406	194	600	77.28%
7	(0.07 , 0.00)	286.67	660.00	369	15	377	93	470	82.55%
8	(0.07 , 0.00)	286.67	660.00	344	15	389	64	453	79.63%
9	(0.00 , 0.00)	300.00	700.00	355	17	367	95	462	92.04%
10	(0.00 , 0.00)	300.00	700.00	361	17	389	101	489	97.42%
11	(0.00 , 0.00)	300.00	700.00	386	15	411	152	563	98.95%
12	(0.00 , 0.00)	300.00	700.00	398	16	376	156	533	99.77%
13	(0.00 , 0.00)	300.00	700.00	402	16	372	153	525	98.36%
14	(0.00 , 0.00)	300.00	700.00	402	16	372	157	529	99.06%

Table 2.3: November 13 Results

Round	Observed Frequencies (p, q)	Average Payoffs Buyers	Average Payoffs Sellers	Observed Avg Price	Observed Volume	Per Unit Surplus Extracted Buyers	Per Unit Surplus Extracted Sellers	Per Unit Surplus Extracted Total	System Efficiency
1	(0.43, 0.86)	271.43	157.14	838	7	48	93	141	14.27%
2	(0.90, 0.80)	600.00	320.00	728	10	264	125	389	56.05%
3	(0.78, 1.00)	566.67	255.56	720	9	346	9	354	45.97%
4	(1.00, 1.00)	700.00	300.00	724	11	375	85	460	72.91%
5	(1.00, 1.00)	700.00	300.00	730	17	282	11	293	71.76%
6	(0.73, 1.00)	536.36	245.45	746	11	221	53	275	43.52%
7	(0.71, 1.00)	528.57	242.86	739	14	226	2	228	45.97%
8	(0.64, 1.00)	481.82	227.27	751	11	127	34	161	25.58%
9	(0.73, 1.00)	536.36	245.45	755	11	200	73	273	43.23%
10	(0.75, 1.00)	550.00	250.00	751	12	216	71	287	49.57%
11	(0.77, 1.00)	561.54	253.85	752	13	188	60	248	46.54%
12	(1.00, 1.00)	700.00	300.00	751	15	344	89	433	93.52%
13	(1.00, 1.00)	700.00	300.00	753	15	355	81	436	94.24%
14	(1.00, 1.00)	700.00	300.00	765	14	364	107	471	95.10%
15	(1.00, 1.00)	700.00	300.00	770	12	410	124	534	92.36%
16	(1.00, 1.00)	700.00	300.00	782	12	408	152	560	96.83%
17	(1.00, 1.00)	700.00	300.00	786	16	297	101	398	91.79%
18	(1.00, 1.00)	700.00	300.00	790	15	319	123	442	95.53%
19	(1.00, 1.00)	700.00	300.00	805	15	302	140	442	95.53%

Table 2.4: November 14 Results

Round	Observed Frequencies (p, q)	Average Payoffs Buyers	Average Payoffs Sellers	Observed Avg Price	Observed Volume	Per Unit Surplus Extracted Buyers	Per Unit Surplus Extracted Sellers	Per Unit Surplus Extracted Total	System Efficiency
1	(0.63, 0.19)	287.50	362.50	555	16	248	-94	154	24.75%
2	(0.33, 0.40)	206.67	313.33	588	15	138	-28	111	16.70%
3	(0.63, 0.13)	200.00	300.00	576	16	131	-97	34	5.53%
4	(0.44, 0.00)	212.50	437.50	582	16	141	46	187	30.08%
5	(0.29, 0.00)	241.18	523.53	546	17	187	82	269	45.98%
6	(0.24, 0.06)	241.18	523.53	545	17	198	122	320	54.73%
7	(0.06, 0.00)	288.89	666.67	533	18	236	226	462	83.60%
8	(0.20, 0.00)	260.00	580.00	510	20	180	110	290	58.25%
9	(0.15, 0.00)	270.00	610.00	465	20	250	100	350	70.32%
10	(0.00, 0.00)	300.00	700.00	453	18	317	168	485	87.83%
11	(0.00, 0.00)	300.00	700.00	440	20	304	159	463	93.16%
12	(0.00, 0.00)	300.00	700.00	438	21	288	144	432	91.35%
13	(0.38, 0.00)	223.81	471.43	441	21	183	-50	134	28.27%
14	(0.12, 0.00)	276.47	629.41	442	17	331	117	447	76.46%
15	(0.00, 0.00)	300.00	700.00	443	18	342	179	521	94.37%
16	(0.00, 0.00)	300.00	700.00	436	20	309	156	465	93.46%
17	(0.00, 0.00)	300.00	700.00	412	22	304	143	447	98.89%
18	(0.00, 0.00)	300.00	700.00	431	19	333	165	498	95.27%

Table 3.1: February 14 Predictions

Round	Price Predictions			Volume Predictions		
	RE	Cournot	Myopia	RE	Cournot	Myopia
1	760 - 780			2		
2	627 - 707	660 - 740	760 - 780	8	8	2
3	660 - 690	560 - 740	627 - 707	8	8	8
4	390 - 410	560 - 740	660 - 690	12	8	8
5	460 - 673	390 - 410	390 - 410	8	12	12
6	460 - 673	520 - 580	460 - 673	8	9	8
7	390 - 410	520 - 580	460 - 673	12	9	8
8	390 - 410	390 - 410	390 - 410	12	12	12
9	390 - 410	390 - 410	390 - 410	12	12	12
10	390 - 410	390 - 410	390 - 410	12	12	12

Table 3.2: February 22 Predictions

Round	Price Predictions			Volume Predictions		
	RE	Cournot	Myopia	RE	Cournot	Myopia
1	654 - 718			11		
2	628 - 639	633 - 674	654 - 718	11	12	11
3	518 - 527	620 - 727	628 - 639	14	11	11
4	502 - 589	484 - 562	518 - 527	12	14	14
5	396 - 485	532 - 580	502 - 589	15	14	12
6	390 - 410	396 - 500	396 - 485	17	15	15
7	390 - 487	390 - 410	390 - 410	15	17	17
8	390 - 487	390 - 500	390 - 487	15	15	15
9	390 - 410	390 - 500	390 - 487	17	15	15
10	390 - 410	390 - 410	390 - 410	17	17	17
11	390 - 410	390 - 410	390 - 410	17	17	17
12	390 - 410	390 - 410	390 - 410	17	17	17
13	390 - 410	390 - 410	390 - 410	17	17	17
14	390 - 410	390 - 410	390 - 410	17	17	17

Table 3.3: November 13 Predictions

Round	Price Predictions RE	Price Predictions Cournot	Price Predictions Myopia	Volume Predictions RE	Volume Predictions Cournot	Volume Predictions Myopia
1	738 - 777			9		
2	764 - 816	647 - 724	738 - 777	11	16	9
3	764 - 847	770 - 780	764 - 816	11	12	11
4	790 - 810	794 - 900	764 - 847	14	12	11
5	790 - 810	790 - 810	790 - 810	14	14	14
6	775 - 816	790 - 810	790 - 810	11	14	14
7	777 - 809	816 - 820	775 - 816	11	13	11
8	763 - 842	809 - 820	777 - 809	10	13	11
9	775 - 816	772 - 810	763 - 842	11	14	10
10	770 - 830	819 - 820	775 - 816	11	13	11
11	766 - 842	800 - 900	770 - 830	11	12	11
12	790 - 810	796 - 900	766 - 842	14	12	11
13	790 - 810	790 - 810	790 - 810	14	14	14
14	790 - 810	790 - 810	790 - 810	14	14	14
15	790 - 810	790 - 810	790 - 810	14	14	14
16	790 - 810	790 - 810	790 - 810	14	14	14
17	790 - 810	790 - 810	790 - 810	14	14	14
18	790 - 810	790 - 810	790 - 810	14	14	14
19	790 - 810	790 - 810	790 - 810	14	14	14

Table 3.4: November 14 Predictions

Round	Price Predictions RE	Price Predictions Cournot	Price Predictions Myopia	Volume Predictions RE	Volume Predictions Cournot	Volume Predictions Myopia
1	654 - 671			13		
2	593 - 700	635 - 703	654 - 671	13	13	13
3	648 - 653	520 - 620	593 - 700	13	16	13
4	553 - 573	635 - 715	648 - 653	14	13	13
5	496 - 521	553 - 660	553 - 573	16	14	14
6	485 - 532	496 - 580	496 - 521	16	16	16
7	383 - 489	461 - 568	485 - 532	18	16	16
8	440 - 540	414 - 420	383 - 489	16	19	18
9	440 - 470	470 - 500	440 - 540	18	18	16
10	390 - 410	440 - 500	440 - 470	21	18	18
11	390 - 410	390 - 410	390 - 410	21	21	21
12	390 - 410	390 - 410	390 - 410	21	21	21
13	519 - 584	390 - 410	390 - 410	14	21	21
14	421 - 476	549 - 580	519 - 584	18	16	14
15	390 - 410	421 - 500	421 - 476	21	18	18
16	390 - 410	390 - 410	390 - 410	21	21	21
17	390 - 410	390 - 410	390 - 410	21	21	21
18	390 - 410	390 - 410	390 - 410	21	21	21

Table 4: The Convergence of Prices

Experimental Session	Periods	Observations	$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	P*	$\hat{P} = \frac{\hat{\alpha}}{1-\hat{\beta}_1-\hat{\beta}_2}$	Standard Error σ_P	t-statistic $t_{n-3} = \frac{\hat{P}-P^*}{\sigma_P}$	DW statistic	Chow test $F_{3,n-6}$
Feb 14	**1 - 3**	**1 - 28**	**415.51** (2.82)	**0.70** (3.43)	**-0.23** (-1.14)					**2.08**	**6.68** ($F_{3,92\ 1\%}$ = 4.02)
	4 - 10	**29 - 98**	**21.44** (1.01)	**0.67** (5.70)	**0.27** (2.36)	**400**	**406.37**	**39.56**	**0.16**	**2.16**	
Feb 22	**1 - 4**	**1 - 58**	**592.55** (4.42)	**0.03** (0.24)	**0.09** (0.69)					**2.02**	**20.03** ($F_{3,203\ 1\%}$ = 3.78)
	5 - 15	**59 - 209**	**46.28** (3.16)	**0.64** (8.11)	**0.24** (3.05)	**400**	**384.13**	**26.15**	**-0.61**	**2.02**	
Nov 13	**1 - 3**	**1 - 28**	**660.79** (2.61)	**0.16** (0.21)	**-0.06** (0.31)					**2.01**	**4.46** ($F_{3,227\ 1\%}$ = 3.78)
	4 - 19	**29 - 273**	**329.72** (6.96)	**0.34** (5.59)	**0.23** (4.24)	**800**	**763.78**	**28.65**	**-1.26**	**2.16**	
Nov 14	**1 - 8**	**1 - 178**	**298.31** (6.11)	**0.23** (3.08)	**0.23** (3.21)					**2.01**	**19.45** ($F_{3,397\ 1\%}$ = 3.78)
	9 - 18	**179 - 405**	**211.74** (6.52)	**0.25** (3.94)	**0.27** (4.16)	**400**	**440.76**	**25.59**	**1.59**	**1.92**	

Table 5.1

	π_t			$\lvert\pi_t\rvert$		
	Median	mean	std. dev.	median	mean	std. dev.
RE	0.00	-0.07	10.81	2.22	4.85	9.65
Cournot	-0.15	-0.90	4.61	0.48	1.49	4.45
Myopia	-0.57	-0.83	8.55	2.03	3.93	7.62

Table 5.2

	"trials"	"successes" $\lvert\pi_t\rvert \leq 1$	Deviations from Observed Volume 0 units	$\leq$ 1 unit	$\leq$ 2 units	$\leq$ 3 units	"successes" sign(ΔQ_t)
RE	61	24.59%	14.75%	45.90%	68.85%	90.16%	52.63%
Cournot	57	82.46%	19.30%	50.88%	75.44%	96.49%	40.74%
Myopia	57	35.09%	17.54%	47.37%	71.93%	91.23%	37.04%

Table 6

Equations:

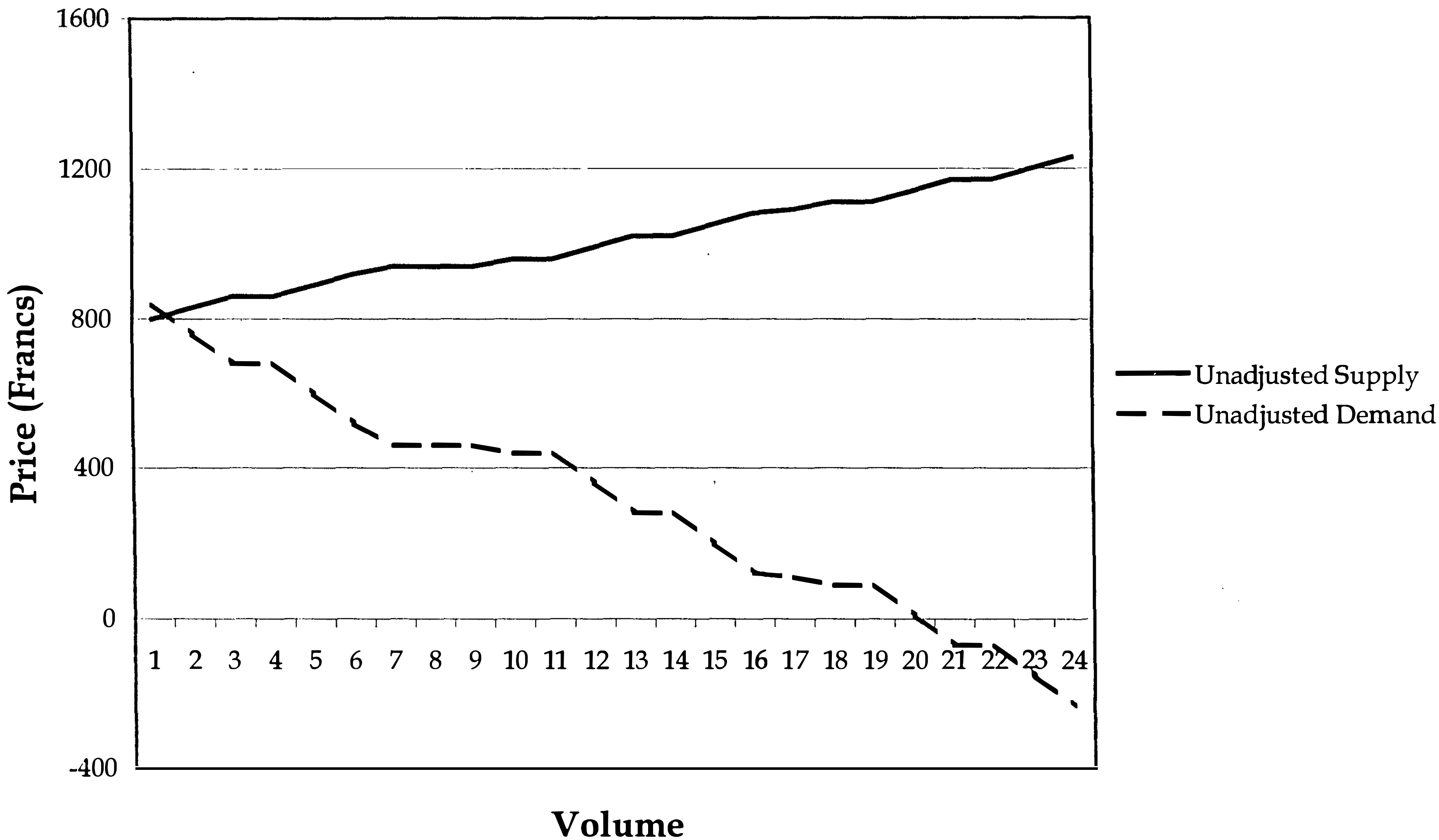
$$\text{Volume}_t = \alpha_1 d_1 + \alpha_2 d_2 + \alpha_3 d_3 + \alpha_4 d_4 + \beta \text{Volume}_{t-1} + u_t,$$

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \varepsilon_t, \quad \varepsilon_t \sim N(0,\sigma^2)$$

Regressor	ML Coefficent Estimates $\hat{\alpha}_i, \hat{\beta}, \hat{\rho}_i$	Std. Error	Estimated Session Volume $\hat{V}_i = \frac{\hat{\alpha}_i}{1-\hat{\beta}}$	Predicted Session Volume V_i	t-ratio: $\left(\frac{\hat{V}_i - V_i}{\sigma_V}\right)$
Feb 14 Dummy: d_1	3.16	0.7737	9.43	12	-1.41
Feb 22 Dummy: d_2	5.16	1.3363	15.38	17	-0.89
Nov 13 Dummy: d_3	4.60	1.1416	13.72	14	-0.15
Nov 14 Dummy: d_4	6.41	1.6541	19.09	21	-1.04
Volume_{t-1}	0.66	0.0889			
u_{t-1}	-0.46	0.1273			
u_{t-2}	-0.28	0.1273			
Std. Error of Volume_t, σ_V	1.83				
DW-statistic	2.09				

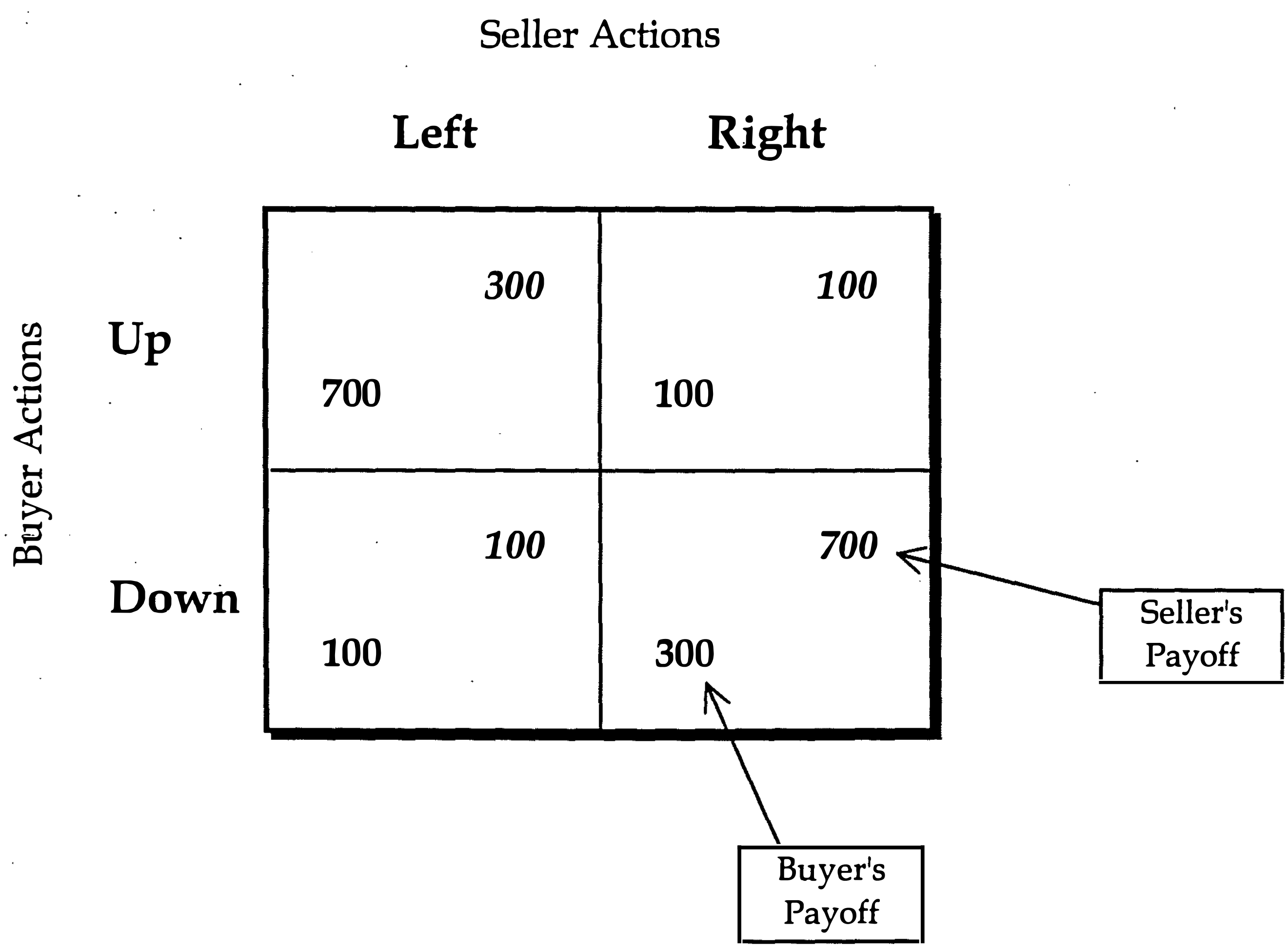
Figure 1

Supply and Demand Schedules



These are the schedules that would obtain with a configuration of 6 buyers and 6 sellers.

Figure 2



Seller Actions
Left
Right
Buyer Actions
Up
Down
300
100
700
100
100
700
100
300
Seller's Payoff
Buyer's Payoff

Figure 3

Derived Supply and Demand Schedules RE-consistent with the "Up-Left" Equilibrium of the Second Stage Game

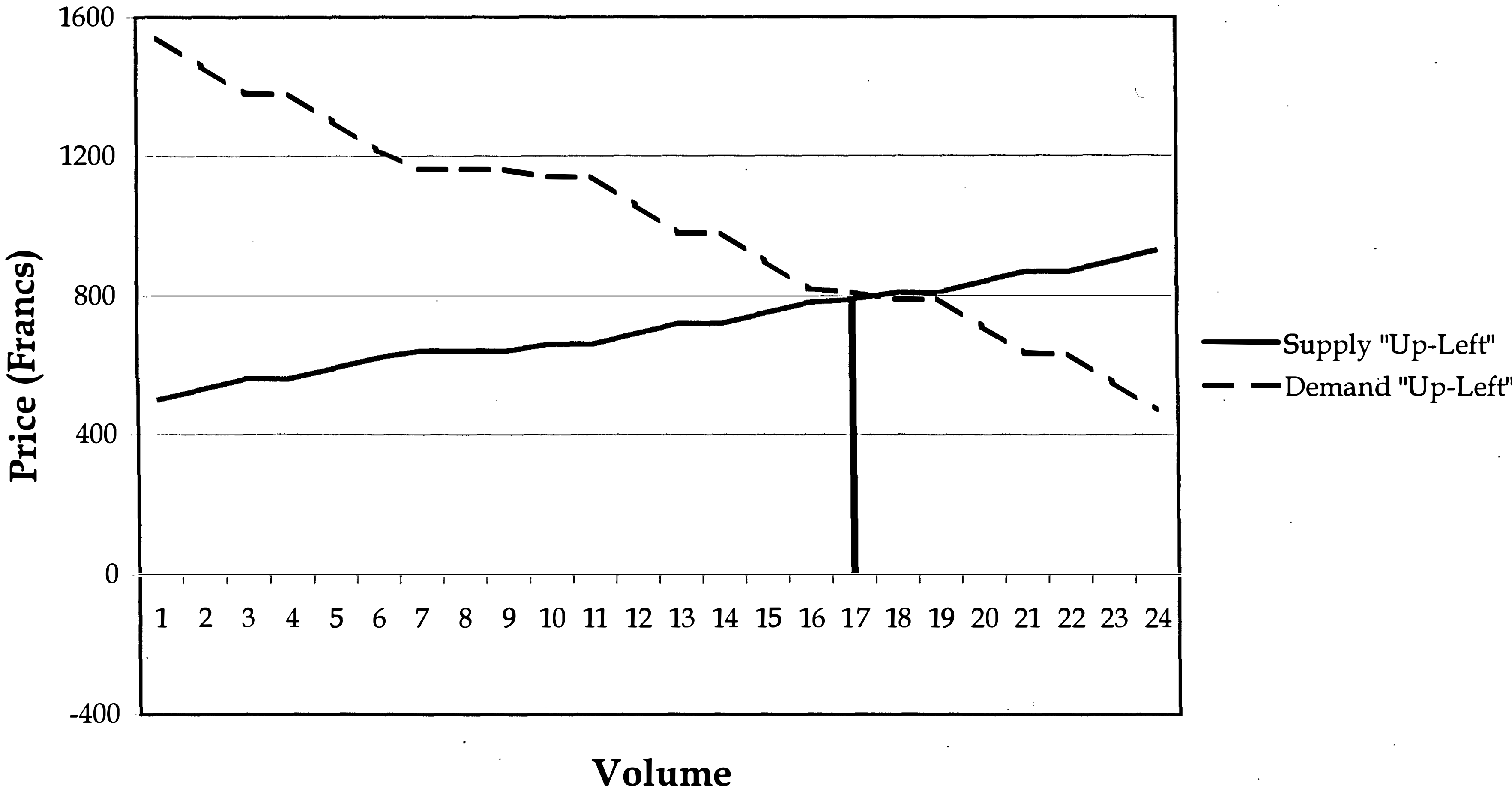


Figure 4

Behaviors in Session Feb 22
Converge to the pure-strategy Nash Equilibrium "Down-Right"

Period 1

	L	R
U	**3**	**5**
D	**2**	**4**

Period 2

	L	R
U	**0**	**9**
D	**1**	**5**

Period 3

	L	R
U	**1**	**2**
D	**0**	**8**

Period 4

	L	R
U	**0**	**6**
D	**0**	**11**

Period 5

	L	R
U	**0**	**1**
D	**0**	**12**

Period 6

	L	R
U	**0**	**0**
D	**0**	**11**

Period 7

	L	R
U	**0**	**1**
D	**0**	**14**

Period 8

	L	R
U	**0**	**1**
D	**0**	**14**

Period 9

	L	R
U	**0**	**0**
D	**0**	**17**

Period 10

	L	R
U	**0**	**0**
D	**0**	**17**

Period 11

	L	R
U	**0**	**0**
D	**0**	**15**

Period 12

	L	R
U	**0**	**0**
D	**0**	**16**

Period 13

	L	R
U	**0**	**0**
D	**0**	**16**

Period 14

	L	R
U	**0**	**0**
D	**0**	**16**

Figure 5

Prices and Behaviors Converge to the RE-Consistent Values (400, "Down-Right") on Feb 22

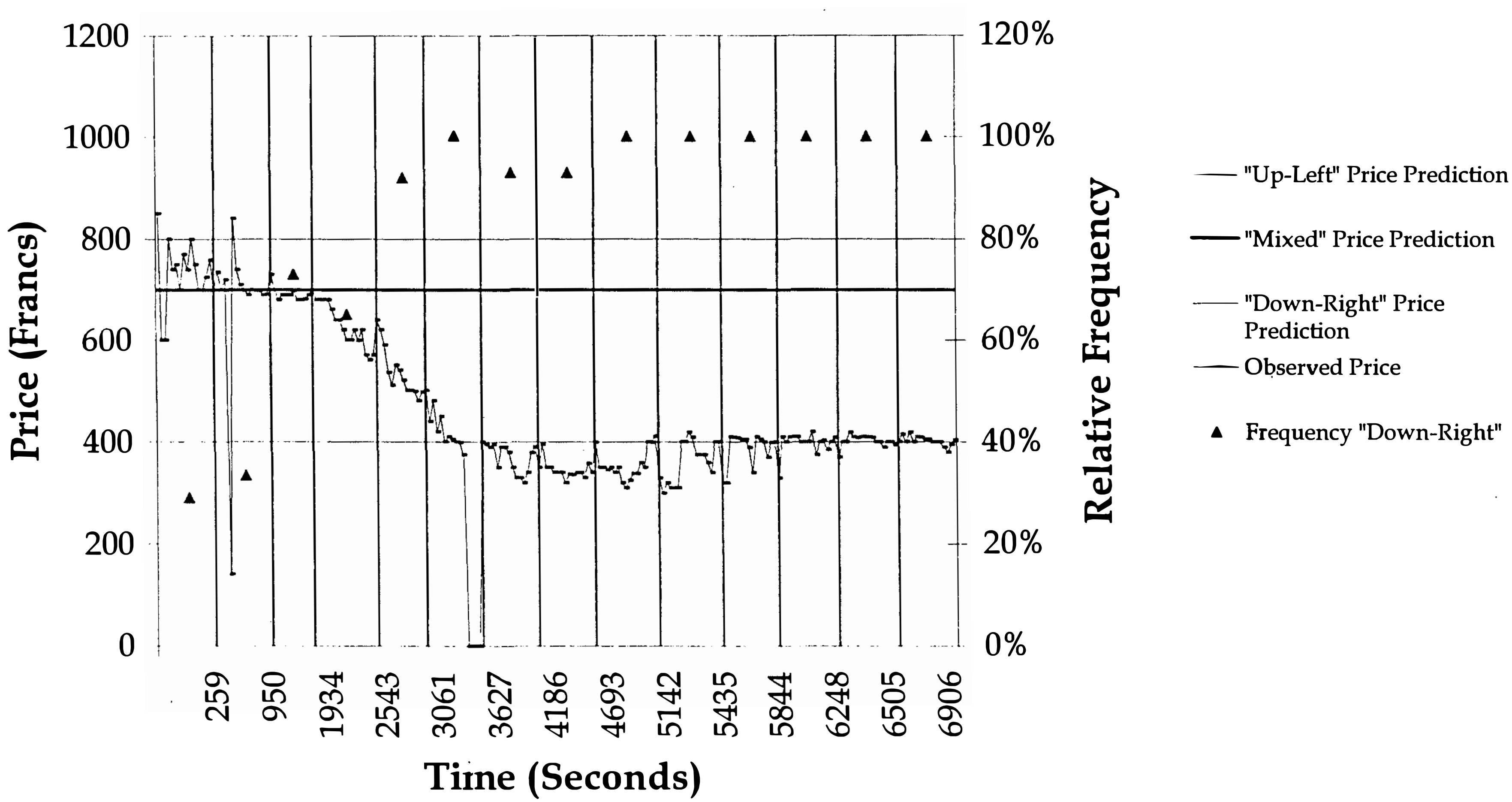


Figure 6

Prices and Behaviors Converge to the RE-Consistent Values (800, "Up-Left") on Nov 13

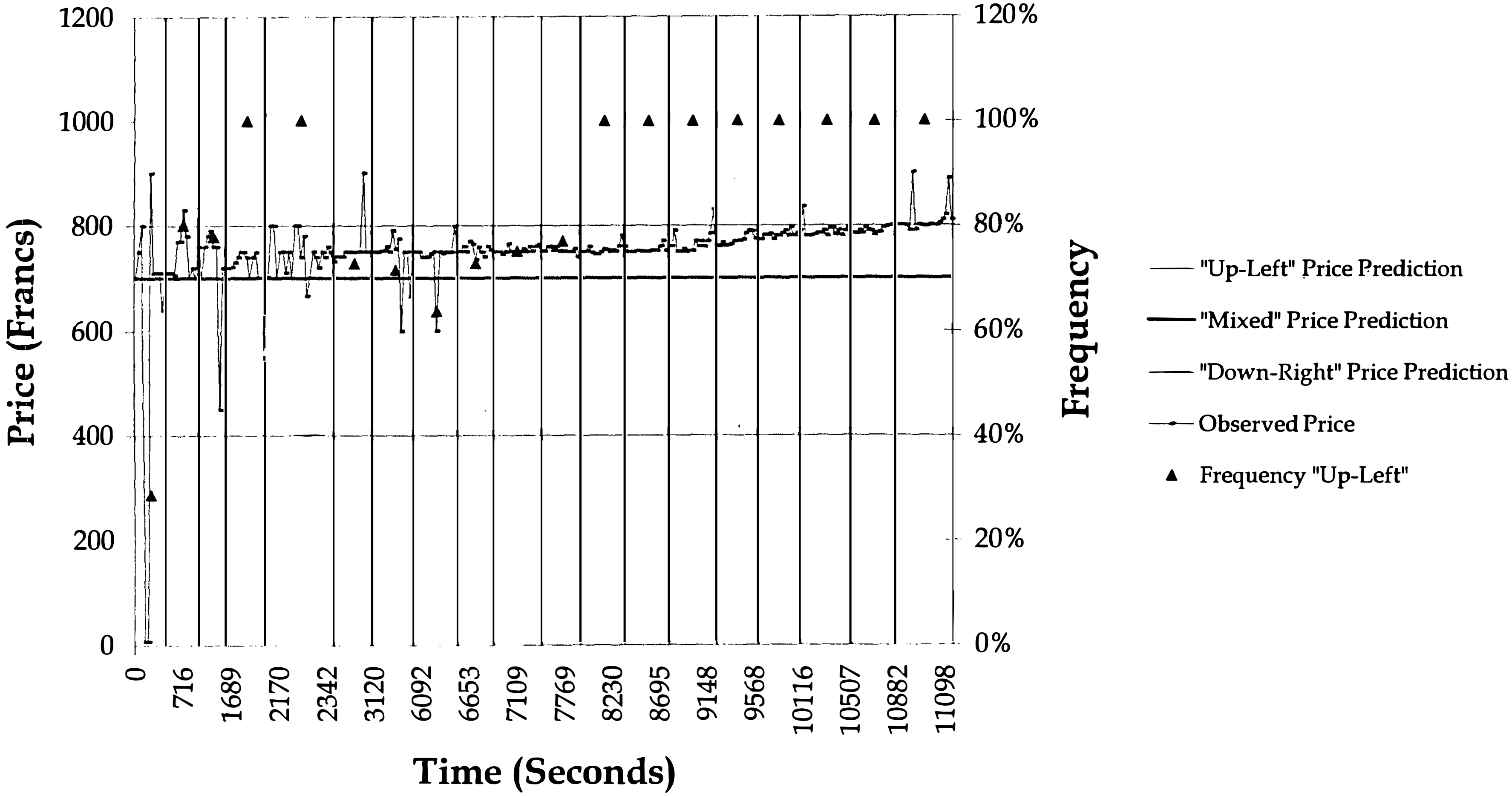


Figure 7

Prices and the Sum of Frequencies (p +q) Converge to Joint Rational-Expectations Equilbria

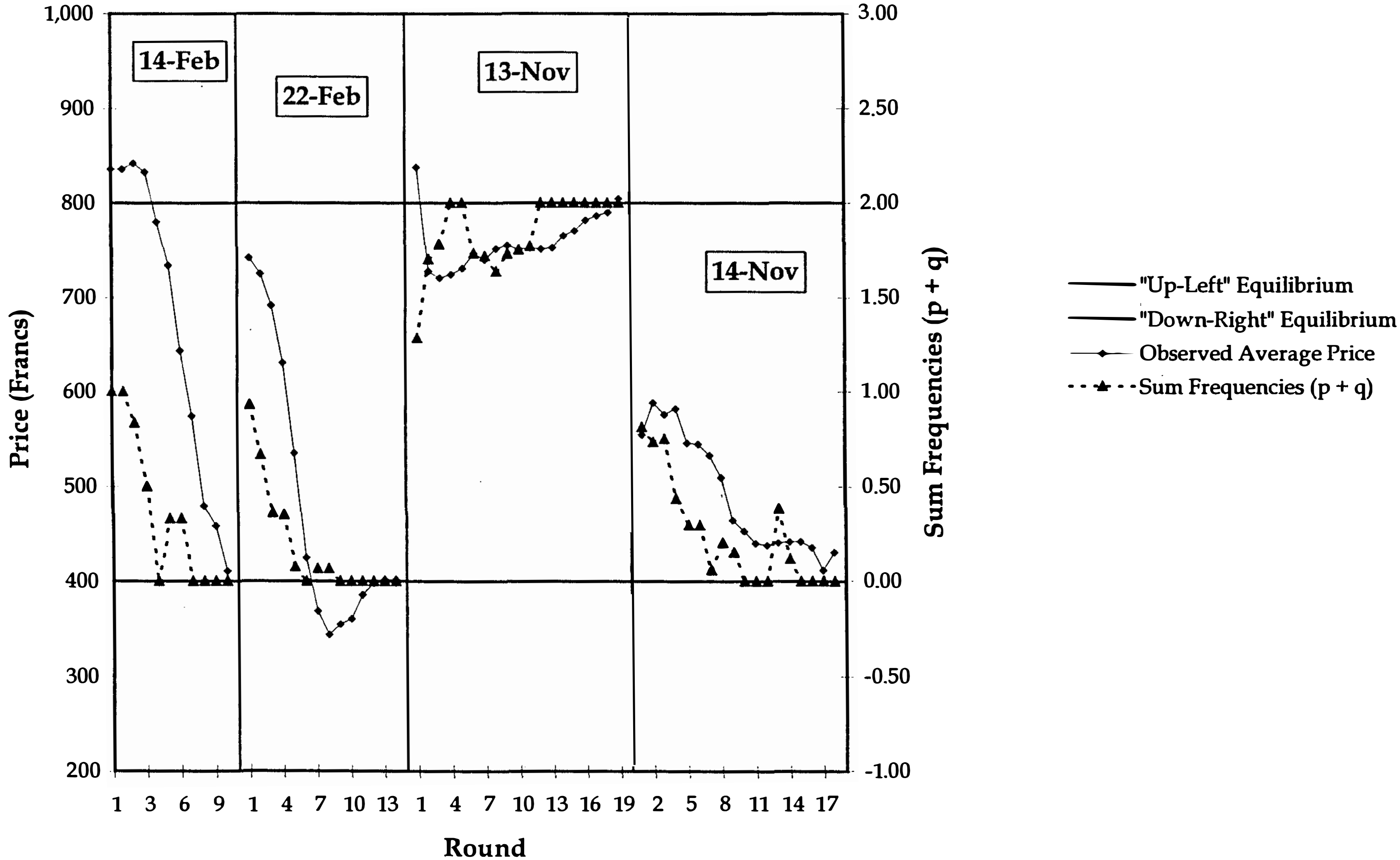
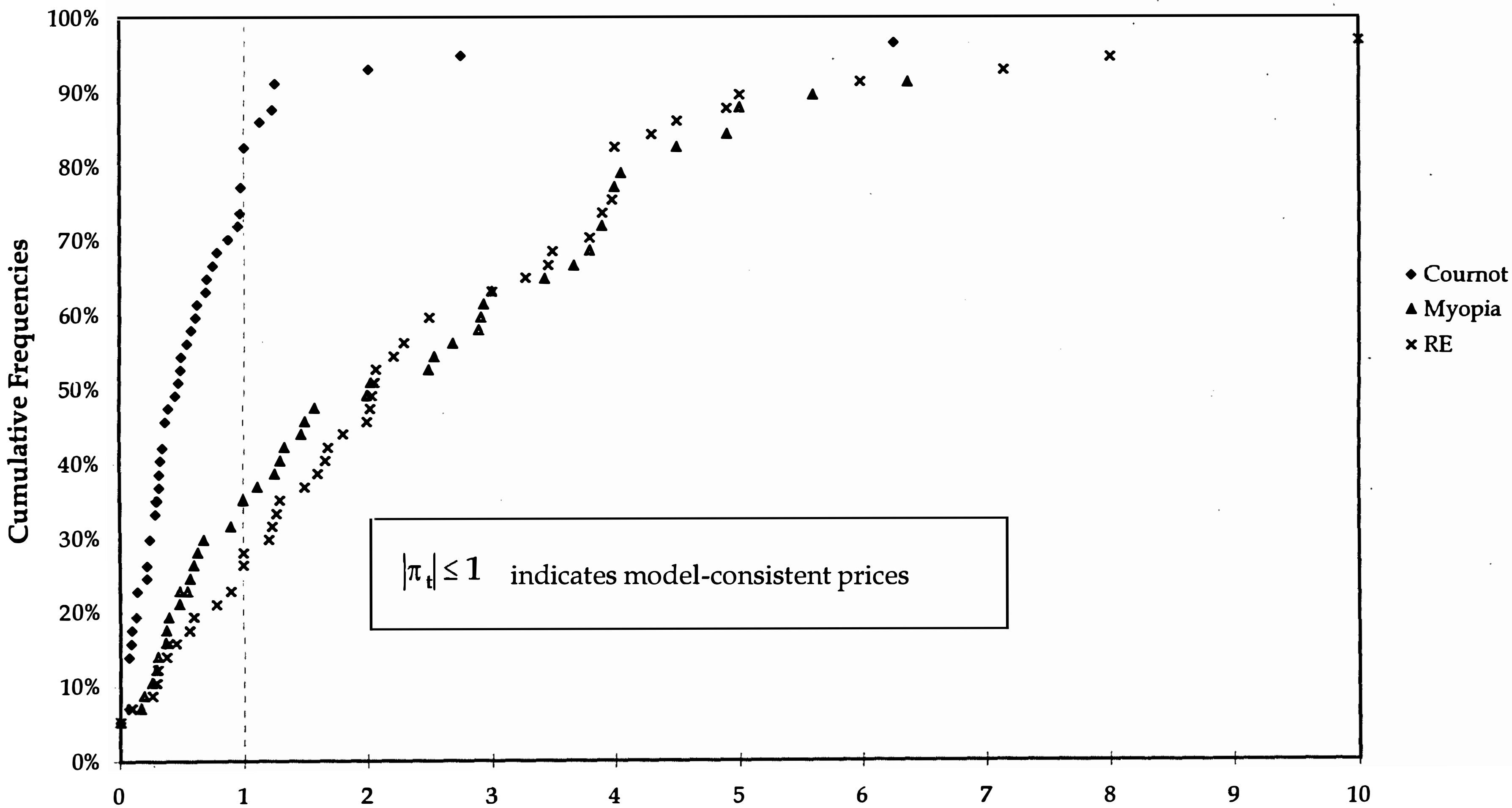


Figure 8

The Cournot process characterizes the evolution of prices with more success than the Myopia and RE processes



$|\pi_t| = \left| \frac{P_t - \hat{P}_t}{(\overline{P}_t - \underline{P}_t)/2} \right|$ **= a measure of deviation of prices each round from model-consistent prices**

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