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ON THE BEHAVIORAL FOUNDATIONS OF THE LAW OF SUPPLY AND DEMAND: HUMAN CONVERGENCE AND ROBOT RANDOMNESS

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SOCIAL SCIENCE WORKING PAPER 1079

December 1999

On the Behavioral Foundations of the Law of Supply and Demand: Human Convergence and Robot Randomness*

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In a seminal series of papers[1], Gode and Sunder[1993,b,1996] have explored the relationship between limited rationality, market institutions and the general equilibration of markets to the competitive equilibrium. Their fundamental discovery is that within the classical double auction market institution only the weakest elements of rationality need to be present for markets to exhibit high allocative efficiency and price convergence. While Gode and Sunder place more emphasis on allocative efficiency[2] than on price convergence, the apparent price convergence increases the agreement between their simulation results and observed price convergence in single isolated periods of double auction markets with humans[3]. Their 'Zero Intelligence' [ZI] agents, are governed

* The financial support of the National Science Foundation, the Caltech Laboratory for Experimental Economics and Political Science, the Hong Kong SAR University Grants Committee and the HKUST Center for Experimental Business Research is gratefully acknowledged. We would like to thank Tim Cason, Leonard Cheng, S. H. Chew, Jim Cox, John Dickhaut, Dan Friedman , Steve Gjerstad, John Ledyard, Charles Noussair, Jason Shachat, and Shyam Sunder for their comments.

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[1] Gode and Sunder[1993] introduce the ZI algorithm, with the theme of their paper being that random behavior subject to market and individual budget constraints can yield efficient outcomes – the only trader rationality that is required is the ability to abide by budget constraints. Gode and Sunder[1993b, 1996] use the ZI algorithm to test various institutional rules and budget constraints within the double auction framework to determine which rules are most responsible for market efficiency. Gode and Sunder[1997] use the ZI algorithm to test the implications of non-binding price-ceilings in markets.

[2] The efficiency emphasis is clear from their titles: 'What makes markets allocationally *efficient*?', 'Allocative efficiency. of markets with zero-intelligence traders', 'Lower bounds for efficiency of surplus extraction…'

[3] There are some differences among authors as to the definition of convergence. Gode and Sunder [1993; p.29] associate convergence with the final price in the market being closer to the predictions of initial supply and demand that early prices: 'By the end of a period, the price series in budget constrained ZI trader markets converges to the equilibrium level almost as precisely as the price series from human trader markets does.' Convergence can have a more restricted meaning when learning is possible over a series of repeated market periods – for example Smith[1962] discovered all the trades in a period may occur at the CE price given sufficient repetition of that period. Gjerstad and Dickhaut [1998; ft. 5] use this stricter notion in evaluating their model: 'We argue that prices in a stable market environment converge, if, after several periods, the mean deviation of all trades from equilibrium is small.' The ZI robots can not exhibit this type of convergence because they do not learn from previous periods – instead repeating similar stochastic behavior at the start of each new period. Other definitions of convergence, related to time series

by completely random choice and constrained only by a budget constraint, are coordinated by market forces to the competitive equilibrium. The results are closely related to the results by Becker[1962] that the budget constraint alone, in the presence of randomly behaving agents, assures that demand curves will be downward sloping.

Such results stimulate natural questions about the foundations of economics and the most fundamental laws of supply and demand. Is no more intelligence necessary for the aggregate operation of these laws than is present in agents whose individual behavior is limited only by their budget constraint? Is nothing else implied by the consistency or the price convergence observed in economic experiments? Is only the randomness of individual behavior responsible for the law of supply and demand or are deeper principles of behavior in operation? The experiments reported in this paper are designed to explore these questions.

While the questions above are motivated by the work of Gode and Sunder, they are not the questions that Gode and Sunder posed. Our questions are different. Gode and Sunder were keenly aware of the relationship between market institutions and individual rationality and their experiments exposed that relationship. Within their environment market dynamics tend to follow a path that leads trading prices to the competitive equilibrium price. This particular type of dynamics was first postulated by Alfred Marshall and later incorporated into theories of the dynamic behavior of the double auction (e.g. Easley and Ledyard [1993]). This property receives even greater emphasis in Cason and Friedman[1993]. The sequence of trades along the Marshallian path is a particular pairing of traders such that the last trade is necessarily at the equilibrium. It is easy to see that the nature of the Marshallian dynamic, the nontatonnement Marshallian path, is operating the Gode and Sunder framework. Since this dynamic is operating, convergence to the competitive equilibrium and the predictions of the law of supply and demand necessarily follow.

The issue of path has certainly not been lost to economic theory. Indeed, Walras invented the concept of tatonnement to illustrate how convergence to equilibrium could occur without the intervening influences of a trading path. The fear was that disequilibrium trades could change the equilibrium. Similarly, early experiments (Chamberlin[4][1948] and Smith[5][1962]) were clearly concerned about the possibility that trades, especially trades involving extramarginal units, could change the equilibrium by shifting the intersection of what they call the "moving" supply and demand. Chamberlin[1948] argued that one consequence of such a moving (or *instantaneous*)

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within and across periods have been based on the Ashenfelter-El Gamal model developed in Noussair, Plott, and Riezmann [1996].

[4] Chamberlin, p.102, suggests "Information during the market as to the equilibrium price would help establish a trend in that direction, but information as to actual prices may do the opposite, in so far as they are divergent from equilibrium and are falsely interpreted to be near it."

[5] Smith, footnote 6, points out that 'Whenever a buyer and seller make a contract and drop out of the market, the demand and supply schedules are shifted to the left in a manner depending upon the buyer's and seller's positions in the schedules. Hence, the supply and demand functions continually alter as the trading process occur… This means that the intra-trading period schedules are not independent of the transactions taking place.'

model is that predictions of price based on initial demand and supply would be impossible without also having knowledge of the trading path. Smith[1962], while aware of the moving model, discovered that initial supply and demand is sufficient to characterize the equilibrium market prices that eventually occur. Because his (and later) experiments involve a series of market periods where the initial supply and demand conditions are repeated, the market can be thought of as “learning” or involving “price discovery” over time, with the trading path adjusting to one which is more and more consistent with the single price equilibrium predictions of initial supply and demand. In contrast, Gode and Sunder, through numerical simulations, suggest that the predictions of initial supply and demand might be accurate only because of the nature of probable trading paths induced by random behavior within the rules of market institutions.

In summary, the conjunction of Gode and Sunder together with the Ledyard and Easley model of the convergence path suggest that the accuracy of the demand and supply model, as observed first by Smith[1962], is due primarily to the tendency for the Marshallian path to emerge. Clearly the Marshallian path is sufficient for convergence. The question posed here is whether or not it is also necessary. The experiments reported in this paper explore markets in which path in the traditional sense plays no role even though the system is non-tatonnement, and asks if demand and supply still have predictive power.

The approach taken here is to design an experimental environment which works against the Marshallian dynamics. We will ask on the one hand if markets populated by humans converge to the competitive equilibrium predicted by the law of supply and demand within the new environment. On the other hand, we will ask if robots with limited intelligence will exhibit the convergence process. The answer to the first question will be yes. Humans will converge. The answer to the second question will be no. The low intelligence robots will not converge. When the path for convergence identified by Gode and Sunder is not operative the law of supply and demand will be observed. Thus, humans bring some crucial feature to the convergence process that is not present in the Gode and Sunder ZI robots. Of course, what elements of human behavior are necessary for this convergence process is an open question.

In posing these questions we will invent a new framework and environment for the study of markets. In addition we will explore several different representations of the law of supply and demand in this new environment.

The remainder of this paper is organized as follows. Section 2 will introduce some notions of Marshallian path dynamics and its importance for price convergence in previous double auction experiments. These ideas apply regardless of whether a market is populated by ZI robots or humans. In section 3, we will construct an environment which works against the operation of the Marshallian path, if indeed a Marshallian path can even be defined in this environment. In section 4, we will explore models that might be appropriate for the new environment. Section 5 describes the procedures used in the laboratory experiments and ZI simulations. Section 6 reports the results of laboratory experiments involving humans and computational experiments involving the ZI robots. Section 7 reports conclusions.

II. MARSHALLIAN PRICE DYNAMICS

The Marshallian Path

The *Marshallian path* is simply a sequence of trades from left to right along the supply and demand curves. For example, in Figure 1 the Marshallian path theory predicts the following sequence of trades: (trade 1) buyer with value 140/seller with cost 30, (trade 2) buyer with value 125/seller with cost 35, (trade 3) buyer with value 110/seller with cost 40, (trade 4) buyer with value 95/seller with cost 45, (trade 5) buyer with value 80/seller with cost 50, (trade 6) buyer with value 65/seller with cost 55. No further trades are possible because the next buyer has a value [50] less than the seller's cost [60]. Trade prices can vary anywhere between a buyer's unit value and the seller's unit cost. Thus, the initial possible range of prices is quite wide [30-140], but the possible range of prices is forced closer to the equilibrium as trading progresses with the final trade [55-65] constrained to be near the competitive equilibrium [55<P<60].

In order for the Marshallian path to have empirical support it must be modified by some notion of randomness. The sequence of trading is generally very noisy in comparison to any exact ordering of trading partners.

As a practical model, aspects of the Marshallian path correspond well with stylized facts observed in both Sunder's ZI simulations and in laboratory data with human traders: (1) highly profitable trades tend to occur before less profitable trades [see Cason and Friedman ,1993, p. 277]; (2) variance in prices tends to decrease as trading progresses [Smith,1962]; (3) trading price for the last unit is near the CE price [generally observed]; (4) the total units traded is equal to the CE quantity [approximately observed]; (5) the final bid and ask in a period are close to the extramarginal redemption values and costs [Jamison and Plott, 1997].

Review of ZI Robot Behavior (The Gode and Sunder Phenomena)

A brief replication of the Gode and Sunder results will help motivate the questions we pose. The potential that trading can occur *noisily* along a Marshallian path, while receiving little formal attention[6], is essentially underlying price convergence in the ZI robot demonstrations. The purpose of this section is to briefly review the results on ZI robots and to show the robots' tendency to trade noisily along the Marshallian path.

In Gode and Sunder[1992], ZI robots submit random bids and asks drawn from a uniform distribution with support equal to an agent's budget constraint. A buyer's bids are distributed U[0,v], where v is the buyer's value for a unit. A seller's ask is distributed

[6] Gjerstad and Shachat[1996; p.14], and John Ledyard, in correspondence, has made the observation that the ZI algorithm and Marshallian path are really special cases of the "B-process" described for a general exchange economy in Hurwicz, Radner, and Reiter[1975a,b]. In the B-process, agents continually choose points in their upper-contour sets and suggest trades until competitive equilibrium is reached. The ZI algorithm differs only by not allowing resale.

$U[c,H]$ where c is the seller's cost for a unit and H is an upper limit of trading. With the ZI robots, a pre-defined upper limit to trading is necessary as a uniform distribution over the seller's actual budget constraint, $U[c, \infty)$, would be ill-defined. Typically, H is set at least as high as the highest buyer's value.

Under the standard double auction rules, a trade occurs when a new bid is made that is greater than a pre-existing ask, or when a new ask is made that is less than a pre-existing bid. The trading price is equal to that of the pre-existing bid/ask, whose acceptance is triggered automatically by the new entry.

Two important features of ZI robot trading immediately follow: (1) ZI's continue trading until gains from trade are exhausted, (2) while trades are random, the probability that the high value buyer trades with the low cost seller is higher than any other pairing.

An example of a trading sequence from ZI robots is shown in Figure 2. In this example, the supply and demand curves of remaining traders are shown before and after each trade. Individual bids and asks of the ZI robots are not shown. In this example we see that the initial trades are far from equilibrium but the final trade occurs at a price (55) near the competitive equilibrium ($55<P<60$).

Like the Marshallian path theory, early transaction prices can be far from the equilibrium while later transaction prices are generally forced closer to the equilibrium by the absence of high surplus buyers and sellers. There is a possibility that early trades can involve extramarginal buyers or sellers, and so ZI trading need not be 100% efficient as would be predicted under the Marshallian path. Still, extramarginal trades are not frequent. ZI trading is a kind of noisy traversal of a Marshallian path in the following sense: although the buyers and sellers are picked more or less at random, at each moment in time the current high value buyer and current low cost seller have the highest probability of trading.

The consistency of such patterns lead Gode and Sunder to the conclusion that markets populated by ZI traders converge to competitive equilibrium. By repeating the example of figure 2, we can examine the basis for these claims.

Figure 3 shows the price distributions of initial and final trades obtained in 1000 ZI trading periods using the environment of Figure 1. Here we can clearly see that the initial trades are rather widely distributed in price compared to the final trades. The final trades, while not exactly clustered around the equilibrium of [55-60], are much closer to the equilibrium than the initial trades. Thus, there is an appearance of convergence.

If convergence of market prices to competitive equilibrium is due simply to the existence of a Marshallian path, then it might be possible to construct an environment where a classical Marshallian path does not exist in a classical sense. Constructing such an environment is the primary topic of our next section.

III. CONTINUOUSLY REFRESHED SUPPLY AND DEMAND ENVIRONMENT

The purpose of this section is to describe the parameters used in the experiments of section V. In particular, we seek to construct a new kind of supply/demand environment together with the supporting methodology necessary to conduct laboratory research.

Continuously refreshing the supply and demand parameters leads to an environment where the supply and demand curves do not shrink back to the left as trading progresses. Of course, the classical Marshallian path is removed as a process for price convergence.

Previous Methodology

A bit of explanation about the history of our methodology is useful. In Chamberlin[1948] and subsequent early research[7], buyers had cards which told them the value of purchasing a single unit. The card could only be used only once. Similarly sellers had cards, usable only for sales of a single unit, which explained a unit's cost. Later experiments (Plott and Smith, 1978) expanded the continuous auction methodology to multiple units. Giving buyers and sellers sheets of paper on which redemption values and costs were listed facilitated accounting. These sheets are typically called *redemption value sheets* for buyers and *cost sheets* for sellers. The cards/sheets are private information. Profits of a buyer or seller are simply the difference between the price obtained in the market and the value or cost on the sheet.

The arbitrage of redemption values/costs against the market is the source of agents' profits in traditional experiments. The experimenter set the redemption values and costs privately and observed the resulting, public, market behavior. That part of the methodology is retained.

The new methodology involves 2 elements: a Private Markets methodology and a Refreshing methodology. The Private Markets involve a move of the redemption value and cost sheets onto the computer in the fom of a private market, where they can then be continuously refreshed in a particular way.

Private Markets Methodology

In our experiments the arbitrage opportunity takes the form of public and private markets instead of a market and cost/value sheets. Each participant sat at a PC running a specialized market program prepared in Java. The program divided the participant's screen into two sections – a *private* market and a *public* market. The public market was public in the sense that it provides a means for a participant to post offers to buy or sell that can be seen and acted upon by other participants. The private markets, however, are different for each participant.

The private markets contain the equivalent of redemption value and cost cards or sheets. That is, the concept of a "private market" is one in which the participant receives offers from the experimenter. In the private markets, participants could not make counteroffers or negotiate in the private markets in any way. The offers from the experimenter are

[7] see Plott[1982] or Smith[1982] for a review of the results and methodologies in use from the 1960s-early 1980s.

"private" in the sense that they can only be seen by or executed by a particular participant. The private markets serve as an electronic replacement for the redemption value and cost sheets used in traditional experiments, but the function is essentially the same.

The division of subjects into roles as buyers or sellers is operationalized with private markets in the same way that it would be with value and cost sheets. For instance, a Buyer receives buy offers from the experimenter in the private market in our new experiments, just as a buyer would receive buy offers in the form of redemption values from the experimenter in a traditional experiment. A Seller receives sell offers from the experimenter in the private market, just as a seller would receive sell offers in the form of cost sheets from the experimenter in a traditional experiment.

Offers placed in the private market by the experimenter expired after two minutes, but participants were also informed that new offers could appear on the screen at any time. Thus, if no new orders were distributed, the environment would have been similar to a traditional environment with two minute periods.

The public market showed buy and sell offers from the other participants. Participants could negotiate with each other in the public market under price improvement rules standard to most double auction experiments: buy offers must go up, sell offers must go down – and after a trade any remaining offers are cleared from the market.

Continuously Refreshed Redemption Values and Costs

When a unit was traded in a private market by exercising a private market offer, or when the two minutes expired, the offer was immediately recycled to another participant. For example, if buyer #3 used a private market offer (a redemption value) from the experimenter, this same offer would immediately be made to the *next* buyer (e.g., buyer #4). Similarly, offers to sell (costs) were recycled to the next seller. Subjects had no knowledge at all about this refreshing[8]. Subjects knew only that new orders could appear in their private markets at any time.

Refreshing the private offers in this way keeps the instantaneous supply and demand curves constant at every moment in time. If an offer is used or expires, it does not vanish from the pool of supply and demand. Instead, it is recycled to someone else. Thus, the opportunities of gains from trade are never exhausted. The market demand and supply functions as represented by redemption values and costs are always constant - independent of the patterns of trade.

In practical terms, the experimenter has a finite cash budget and finite time to conduct the experiments. Eventually the experiment must be terminated. To avoid any end-of-

[8] Implications of revealing this information are unclear since any Pareto improvements in decisions would involve both coordination and public goods problems. Our purpose was to study markets in which the set of redemption values and costs in a market are unchanged by trading without becoming involved with behavior that might be motivated by attempts to optimize payoffs by coordination over time.

experiment effect, participants were given only a general idea of how long the experiment would last (e.g., 2-3 hours).

Experimental Parameters

Table 1 and Figures 4-6 show the offers put into the private markets in each of the three experiments that were run. The Buy offers from the experimenter create an induced demand curve and the sell offers create an induced supply curve, allowing the calculation of competitive equilibrium prices in the usual way. Each experiment consists of continuous trading under two or three different economic environments[9]. Unlike most experiments where a trading period has a start and end announced by the experimenter whereby subjects might be signalled that the environment is about to change, changes in environment in our experiment are unannounced. Because of these differences we will refer to *trading intervals* rather than trading periods.

Notice that many of the environments have apparently identical competitive equilibria but different slopes in the induced supply and demand curves. We chose the values shown in an attempt to separate various interpretations of the law of supply and demand. This will be more fully explained after covering models of the operations of the law are detailed in the next section.

IV. MODELS IN THE NEW ENVIRONMENT

The purpose of this section is to describe some qualitative models about how markets populated by either ZI robots or Human traders might behave given the continuously refreshed supply and demand environments of Section III. As pointed out by Cason and Friedman[1993], Easley and Ledyard[1993] and others, there is no fully accepted model of double auction market dynamics. As no fully worked out and acceptable models exist, what follows must by necessity be quite rough. The discussion here will attempt to answer 3 questions: (1) What concept of convergence has been used? (2) What are some models of price convergence and non-convergence applicable to the continuously refreshed experimental environments? (3) What are the hypothesized relationships between markets populated by ZI robots vs. markets populated by Humans?

Price Convergence: Definition

Typically in double auction data one observes a trend of prices, usually moving towards some notion of competitive equilibrium. This trend may be described as "price convergence", but across the literature there is no universally adopted test for determining when this convergence is or is not taking place. For purposes of making definitions operational this convergence will be characterized by three properties, which are given below:

<u>Properties of Price Convergence (</u>relative to some theoretical equilibrium price P_{eq}<u>)</u>

1. Initial prices are further from the equilibrium than final prices.
2. Variance of prices decreases over time
3. If a parameter change moves P_{eq}, the prices move towards the new equilibrium.

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[9] Changes in private market offers from one environment to another are done without stopping the trading or alerting the subjects, but are distributed in a continuous fashion to create a smoother "transition region".

These properties of convergence requires a non-stochastic equilibrium concept, i.e. a single price or range of prices. The next section explores what equilibrium concepts are appropriate to the experimental environments under consideration.

Competitive Equilibrium Models

Three equilibrium concepts can be identified with the literature. Two of these are appropriate to environments with continuously refreshed supply and demand. We will call these models the *Traditional Supply and Demand, the Instantaneous Competitive Equilibrium* (I-CE) and *Velocity Adjusted Competitive Equilibrium* (V-CE) models[10]. Different supply and demand curves are used in the two models, as stated below:

T-CE Model. *Traditional Supply and Demand* – the supply and demand curves are computed from the redemption values and costs that existed at the beginning of a period. Clearly the concept of a period is needed. Since periods do not exist in the environment we study, this model is listed only for completeness.

I-CE Model. *Instantaneous Supply and Demand* – The instantaneous supply and demand curves are computed from the private market orders that exist in the market at an instant. In a traditional environment these curves change after each trade. In an overlapping generations environment the I-CE has been developed and studied by Aliprantis and Plott (1992). In a continuously refreshed environment the instantaneous supply and demand curves are stationary. The intersection of instantaneous supply and demand curves determines the competitive equilibrium.

V-CE Model. *Velocity-adjusted (ex-post) Supply and Demand* – the supply and demand curves are adjusted (ex-post) to take account of the number of times a particular supply or demand unit appeared in the private market of some participant (velocity).[11] The intersection of adjusted supply and adjusted demand determines the equilibrium. This model can not be determined ex-ante, because the velocities are only known ex-post.

In the environments we study, the fundamental predictive difference in the two models is that the V-CE model could be sensitive to the *slopes* of the private market offers whereas the I-CE model is only concerned with the intersection. If units circulate at different speeds, the shape of the V-CE model curves will change and the equilibrium of the model could be changed as a result.

Consider two environments that have the same I-CE. Suppose one environment has a steeper slope in the buyers private offers than the other environment. Intervals 1 and 3

[10] Smith[1962] makes a distinction between the "initial" supply and demand as the basis of equilibria as opposed to instantaneous supply and demand. His primary reason for using the initial supply and demand is that it would be too cumbersome to recalculate the instantaneous supply and demand after each trade. He suggests that the differences between initial and instantaneous equilibria are likely to be small when trading is efficient. However, his concern indicates that the relevant equilibrium is by no means obvious.

[11] The idea of a flow or a rate of demanders and suppliers appearing in a market is frequently found in the classical literature. For example Marshall makes an attempt to deal with it by postulating a long run analysis as opposed to a short run analysis.

defined above have this property (as do Intervals 2 and 4) with Interval 3 having the steeper slope in buyers' induced values. Suppose further that the steeper buyers' surpluses in Interval 3 induces differences in trading velocities among the buyers in Interval 3 vis a vis the buyers in Interval 1. Ex-ante, one might expect trading velocity to increase with increasing profit, which would pull equilibrium prices higher in Interval 3.

Non-Convergence Models

Given the properties of price convergence above, many stationary stochastic models of prices will be a non-convergence model. Three models are considered:

IID Random – Prices are independent, identically distributed random draws from some stationary random distribution.

Martingale – Prices drift with constant variance per unit trade. Differences in prices from one trade to the next are normally distributed with mean 0 and finite, constant variance.

Other - Prices do not converge, but are not IID random or Martingale in nature.

ZI Behavior under Continuously Refreshed Supply and Demand

From our previous discussion of the operation of the ZI robots in section 2, it would appear that there is no mathematical mechanism for price convergence in the ZI-populated markets when demand and supply are continuously refreshed. The absence of a Marshallian path means that a squeeze between willingness to accept and willingness to pay never occurs. Instead of drying up, demand and supply is continuously replenished.

Under the ZI robot algorithm, the trading price P_{ZI} can be thought of as a random variable whose distribution is dependent upon the instantaneous supply and demand curves and whose support is limited to the range of possible voluntary trades. In previous, traditional market experiments the instantaneous supply and demand curves are shrinking in such a way that the support of P_{ZI} shrinks and prices appear to converge.

Thus, with continuously refreshed supply and demand, the instantaneous supply and demand curves are held constant, and so P_{ZI} must give independent and identical (IID) distributed random draws. There can be no price-convergence in ZI-populated markets in an environment with continuously refreshed supply and demand, according to this model.

The primary issue in our experiments is whether or not we will see equilibration in markets populated by humans.

V. EXPERIMENTAL PROCEDURES

Details regarding the special methodologies and parameters used in these experiments can be found in Section II.

Laboratory Experiments

Participants in the experiments were Caltech undergraduates recruited via an announcement on a web-based bulletin board. Each experiment involved 16 participants and lasted 2-3 hours. Trading was continuous, with 1 or 2 unannounced parameter changes and no announcements as to when the experiment would be terminated. Participants received cash payments in proportion to trading profits.[12]

Software for the experiments was written in Java and ran inside a Netscape browser. Even though our software was web-capable, the experiments were conducted in the standard, controlled manner – with groups of participants assembling at the laboratory to listen to instructions, ask questions, and take part in the experiment.

Each participant's screen was divided into two sections, with the private market on the left side and the public market on the right side. Each market displayed the current buy and sell orders against a grid of prices. Entering buy and sell offers into the public market was accomplished by using the mouse to click at the relevant price. The large 21" computer screens used in the laboratory made precision pricing of orders easy. The interface was very efficient and resulted in a much higher trading volume than could be expected with MUDA or similar double auction software.

Examples of the screens and participant instructions are provided in Appendix A.

ZI Simulations

A Monte Carlo study of trades using the ZI robot algorithm was performed for comparison to the human-subject experiments. The software was written in version 5 of the PERL language, which provides a built-in random number generator. Since most random number generators are, in fact, deterministic and since some are not sufficiently 'random', the generator was tested. This pre-simulation testing of the random number generator revealed no flaws either in distribution or serial correlation (independence of draws).

The ZI algorithm was run on a Linux-based workstation for several hours. Data were obtained for approximately 1.5 million trades[13] for each of the 6 experimental environments. In addition, we ran 1,000 period replications of environment 1 for the non-refreshed, standard supply and demand case. This allowed a comparison with previous ZI results in order to check procedures and to generate Figures 2 and 3 used in our review of ZI trading behavior given in Section 2.

VI. EXPERIMENTAL RESULTS

Figures 7 to 9 provide, for each of the three experiments, a side by side comparison of the predictions of the instantaneous CE model (left pane),the trading prices observed in the experiment (center pane), and the velocity-corrected CE model calculated ex-post (right pane).

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[12] These payments were on average, roughly US$30 per participant.

[13] Of course, since there is no natural stopping point in this environment, we could have kept going or terminated the programs earlier. This size does allow us to generate distributions for statistics such as finite-sample price means and variance.

In the center pane, the two solid lines show the instantaneous CE at various points in the experiment[14]. Two symbols are used for the trades. Diamonds represent 'buys' or acceptances of a sell offer by a buyer, and pluses represent 'sells' or acceptances of a buy offer by a seller.

Table 2 provides mean prices and overall price variance for each experiment. Averages and variances are reported by experiment, by interval, and by every 100 trades within an interval. For completeness, transition intervals are also identified and reported but do not play a role in the analysis presented here. The table shows that within each interval, price variance tends to decrease as trades occur.

The most striking feature of figures 7-9 is that prices appear to converge. These tendencies are summarized as Result 1.

<u>Result 1</u>: Price convergence can occur in human-populated markets within continuously refreshed supply and demand environments.
<u>Support:</u> Each of the three convergence properties of section 4 must be shown in operation.
1. Initial prices are further from the equilibrium than final prices. From figure 7 we see that for interval 1, initial prices are near 100 and are further from either the V-CE or I-CE than the final prices which are near 60-65. Table 2 also supports this same observation as the first 100 trades of interval 1 have an average price of 81.2 while the final 93 trades have an average price of 63.4. Similarly, for interval 2 in the latter part of figure 7, prices move toward the equilibrium from below. Interval 3 prices start high and move towards v-ce/I-ce equilibria -- from table 2 we see that the first 100 trades have an average price of 85.9, and the final 66 trades have an average price of 75.9. Interval 4, in the latter half of figure 8, shows convergence from below. In Figure 9, Intervals 5 and 7 show some evidence of convergence from above while Interval 6 shows convergence from below.
2. Variance of prices decreases over time. While some notion of variance can be seen from the dispersion of points in the figures, Table 2 is more reliable, with the transaction price variance tabulated for groups of 100 trades. For Interval 1, the variance of the first 100 trades is 85.5, followed by 26.3, 21.5, 18.1, 22.3, 8.9, 9.6 for each successive 100 trades, and finally, for the last 93 trades, the transaction price variance 6.5. Over Interval 1 the variance decreases by a factor of 13:1, and with 2 minor exceptions, (18.1→22.3, 8.9→9.6), the variance is strictly decreasing with time. Similar patterns are seen in Interval 3 (decrease from 55.0 to 8.7), Interval 4 (decrease from 167 to 77), Interval 5 (decrease from 97.1 to 9.3), and Interval 6 (decrease from 245.1 to 5.7).
3. If a parameter change moves P_{eq}, the prices move towards the new equilibrium. Intervals 2, 4, 6, and 7 involved a parameter shift from a previous trading interval. In each transition, Table 2 shows a sudden increase in transaction price variance, while the Figures show the price moving toward the new equilibrium•

[14] Note that the dashed lines also track the I-CE in the region where one period's private values are being changed to another, such as the transition region of trades 793-820 that separates period 1 from period 2 in experiment 1.

The next result addresses the question of which model is more accurate. The Instantaneous-CE (I-CE) model seems to predict the trend of prices, though not the exact price to which prices converge. While we can not say that prices converge to the instantaneous CE model, a convergence phenomena does appear to be present as the prices do track changes in the parameters in a similar manner to the instantaneous CE.

The right panes of Figures 7-9 provide the alternative model – the Velocity Adjusted Competitive Equilibrium (V-CE) model - that better approximates a convergence process. The visual impression is stated below as our next result.

Result 2: The Velocity Adjusted Competitive Equilibrium (V-CE) Model best corresponds to the observed pattern of prices.
Support: The models defined in Section 4 are considered for completeness. Non-convergence models can be rejected as Result 1 shows that price variance appears to be decreasing. The I-CE model, based on Instantaneous supply and demand, and the V-CE model, based on a velocity adjustment calculated ex-post, can be compared in Figures 7-9. In particular, note that the I-CE model predicts identical outcomes for intervals 1 and 3 [$55 \leq P \leq 60$], intervals 2 and 4 [$180 \leq P \leq 185$], and intervals 5 and 7 [$80 \leq P \leq 85$]. From Table 2 we see that Intervals 2 and 4 and intervals 5 and 7 have almost identical final price levels, but in interval 3 prices are observed to converge to a much higher level [75.9 for the last 100 trades] than in interval 1 [63.4 for the last 100 trades]. Looking to Figures 7 and 8 for Intervals 1 and 3, we see that both Intervals 1 and 3 fit the V-CE model very well. Thus, the V-CE model accounts for a difference in observed prices that the I-CE model did not capture•

Turning to examination of markets populated by the ZI robots, Figure 10 shows the price sequence of 1,000 ZI trades with the environment 1 continuously refreshed environment. Price variance is constant and prices are independent draws from a complicated random distribution. We are left with result 3.

Result 3: Price Convergence does not occur in markets populated by ZI traders with a continuously refreshed supply and demand environment.
Support: The simulation produced exactly what the model predicts: trades have IID random prices, and price variances do not decrease over time but instead remain constant. The constant variances that were observed are reported in Table 3, and are generally quite high in comparison with even the initial variance in the markets populated by humans. Since prices are not decreasing in variance, by definition price convergence does not occur•

A summary of the ZI trading data is reported in Table 3. Results 4 and 5 will compare and contrast the markets populated by the ZI robots and markets populated by the humans. While there are a number of differences, there also will remain a puzzling similarity: although price convergence does not occur in the markets populated by ZI robots, the mean prices correspond well to V-CE equilibrium prices.

In the continuously refreshed case, markets populated by ZI robots exhibit neither price convergence nor pricing distributions similar to human markets. It is possible that there are significant differences between the behavior of markets populated by ZI-robots and markets populated by humans under conditions of ordinary supply and demand as well. The differences may be harder to detect in practice, but Fig. 10 and Fig. 11 suggest that they might exist.[15]
A comparison of the distribution of prices between the markets populated by the ZI traders and the markets populated by human traders is shown as Figure 12. In these graphs, the "Observed" data are from the humans and the "ZI" data is from the robot simulations.

Each row of the graph provides this data for one of our three experiments, with the data for each interval shown sequentially in panes from left to right.

Result 4: In environments with continuously refreshed supply and demand, markets populated by Humans differ from markets populated by ZI robots as follows:

(a) transaction prices in markets populated by humans tend to converge whereas in markets populated by ZI robots transaction prices do not converge

(b) the distribution of transaction prices is more tightly peaked in markets populated by Humans and does not exhibit the stepped artifacts of ZI Robot pricing distributions.

(c) peaks of the observed distribution of transaction prices are in different locations in the two types of markets

Support: (a) is merely a restatement of Results 1 and 3. (b) The difference in tightness of the transaction price distribution can be clearly seen in any of the trading intervals in Figure 12. For example, Intervals 1, 2, and 3, given in the 3 larger panes of Figure 12, the density for markets populated by humans to peak at about 10-12%/franc. Given the same instantaneous supply and demand parameters, the transaction price distribution for markets populated by ZI-robots have peak densities of 1-2%/franc. (c) the difference in location of the peak of transaction price distribution can be seen in any of the trading intervals in Figure 12 with the exception of interval 1. For example, in Interval 2 the peak (or mode) of the distribution occurs at a price of about 170 for markets populated by humans, but at a price of about 190 for markets popualted by ZI•

[15] Figure 3 shows the price distribution for 1000 repetitions of a interval 1 ZI market with non-refreshed supply and demand. Figure 11 shows the price distribution from 1.5 million trades from a interval 1 ZI market with continuously refreshed supply and demand. In Figure 3, the upper pane shows the distribution of prices for all trades in 1000 repetitions of the standard environment. The second pane shows the distribution of prices for the final trades in these 1000 repetitions. Notice that while the variance decreases at the final trade of each interval, the distribution is somewhat skewed and has a large mass in the 40s -- well below the CE of $55<P<60$. Thus, convergence in the standard environment is not necessarily to the CE although the decrease in variance suggests some sort of convergence is occurring. Because the distribution in Fig. 11 is even flatter than that of the overall trades in Figure 3, and is peaked above the CE, it is clearthat price convergence does not occur with ZI robots when there is continuously refreshed supply and demand even though it does occur to a limited extent in the non-refreshed environment.

As can be seen, the mean transaction prices of ZI robots in environments 1, 3, and 5 are far from the instantaneous CE implied by the parameters. While environments 2, 4, and 6 show mean prices near the I-CE, the variance is constant and quite high as a proportion of price. In contrast, the mean transaction prices correspond well with the V-CE calculated for the robot trading. Given the lack of price convergence for the ZI robot markets, the existence of a correspondence between the mean prices and the V-CE is a bit puzzling.

The similarities between the markets populated by humans or ZI robots, related to the predictive abilities of the V-CE model in both cases is stated below as Result 5.

Result 5: In environments with continuously refreshed supply and demand, the V-CE model predicts mean transaction prices in markets populated by Humans as well as markets populated by ZI robots.
Support: From Result 3 we know the V-CE model is appropriate to mean prices generated in human markets. It remains to be shown that V-CE corresponds to mean prices observed in the robot simulations. Table 3 shows that the V-CE model also applies to the mean prices exhibited by the robots. In Environement 1, the I-CE price range is 55<P<60, the V-CE price is 80, and the actual mean is 78.3. Similarly, in Environment 2, V-CE (175) compares more favorably to mean price (176.82) than the I-CE (180<P<185). While in each trading environment the mean transaction price of ZI robots corresponded more closely to the V-CE price than to the I-CE price, the biggest differences between V-CE and I-CE are seen in Environment 3 (I-CE (55,60), V-CE 90, mean[P]=95.04) and Environment 5 (I-CE (80,85), V-CE 110, mean[P]=118.16•

VI. CONCLUDING REMARKS

The principle result is that the Marshallian path does not account for the observed accuracy of the law of supply and demand as a price discovery process in markets populated by humans. An immediate corollary is that some aspect of rationality in addition to that imposed by the budget constraint and trading institutions is operating. While the Marshallian path can be called the "cause" of price convergence in markets populated by ZI robots it is not the "cause" of price convergence in markets populated by humans. While it might be helpful in some environments, it is not essential.

The continuously refreshed environment offers a new paradigm for studying the type of market adjustments that the concept of tatonnement was designed to study. Thus the environment holds the potential for theory development and testing. In this context the volume adjusted competitive model suggests itself as a step towards an improved theory for how markets adjust. The flows or speeds with which demand and supply units are received in the market are natural parameters with which to adjust the classical models.

The experiments also help to focus on the fact that supply and demand equilibria correspond more to some form of time aggregation (where velocity can be important) than to an instantaneous model (where velocity is not important). More work is clearly needed to understand the dynamics of price convergence in markets. The V-CE model is in some ways too simple and unsatisfying. There are many alternatives. One could consider models where there is a fixed time window into the past, or where various moving averages of instantaneous demands are taken. Various forms of differential equations might also be considered.

The experiments help define the context in which the use of robots might be most valuable. Humans are known to be more complex than ZI robots in their information processing capabilities. ZI robots are designed *not* to learn, imitate, adapt, or otherwise react in a strategic manner. It is this simplicity of ZI robots that is appealing in conducting simulations, as otherwise the simulations would depend on a number of initial parameters (beliefs, adaptation speeds, length of memory for use in histories, etc.) that are conveniently absent. However, this simplicity means that the ZI behavior will not robustly predict the behavior of humans in all possible market environments, as human behavior involves the complexities that are purposefully omitted from ZI robots. The results reported here demonstrate that the path of convergence requires more intelligence thanthe Gode and Sunder ZI framework postulates.

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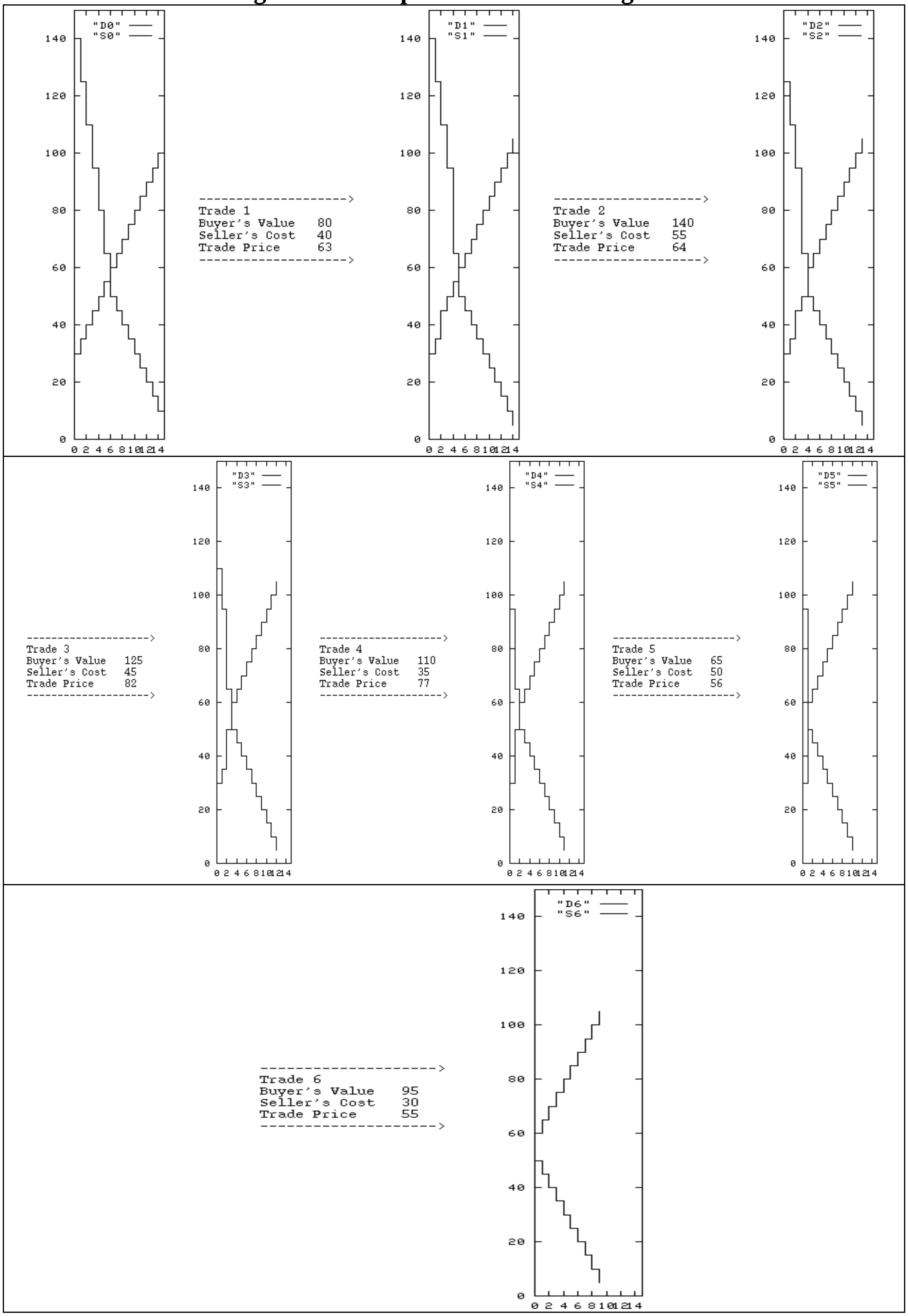
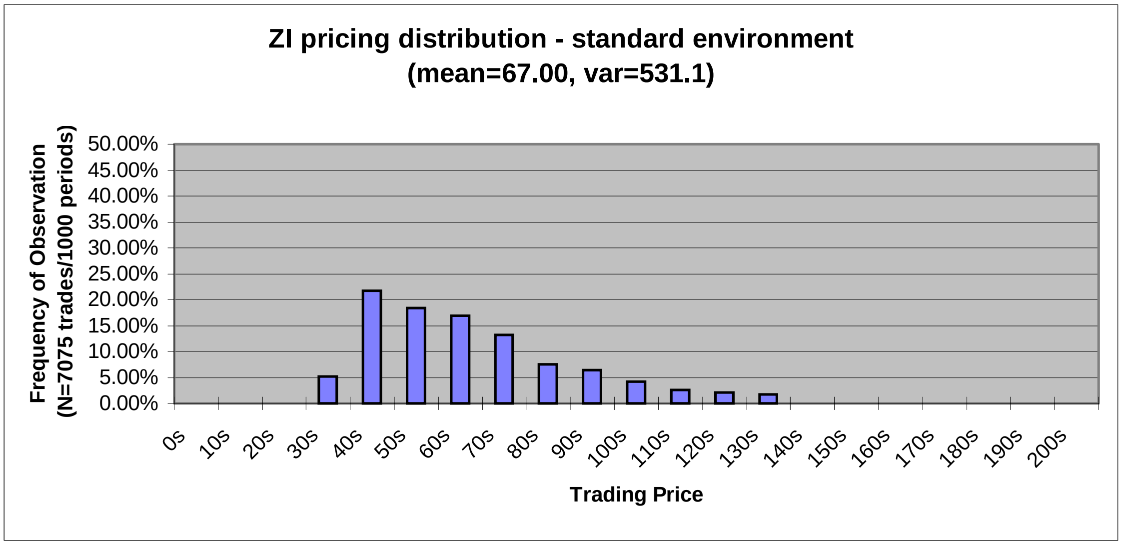
Figure 2: Example ZI Robot Trading Period



Efficiency = Total Trading Surplus / Maximum Possible Surplus = 360/360 = 100%

Figure 3: Price Distributions within Markets populated by ZI traders



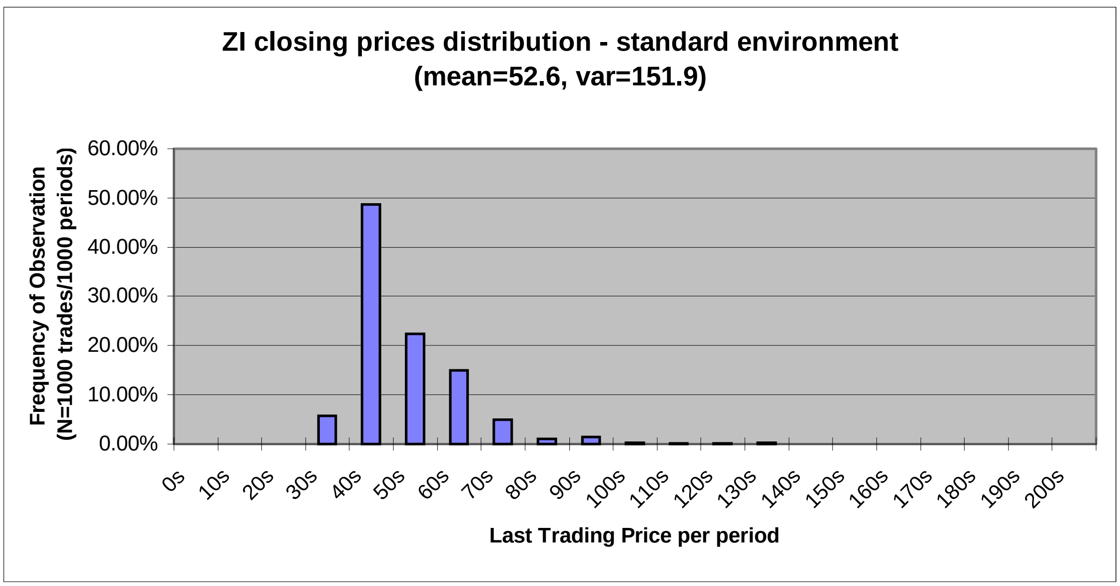
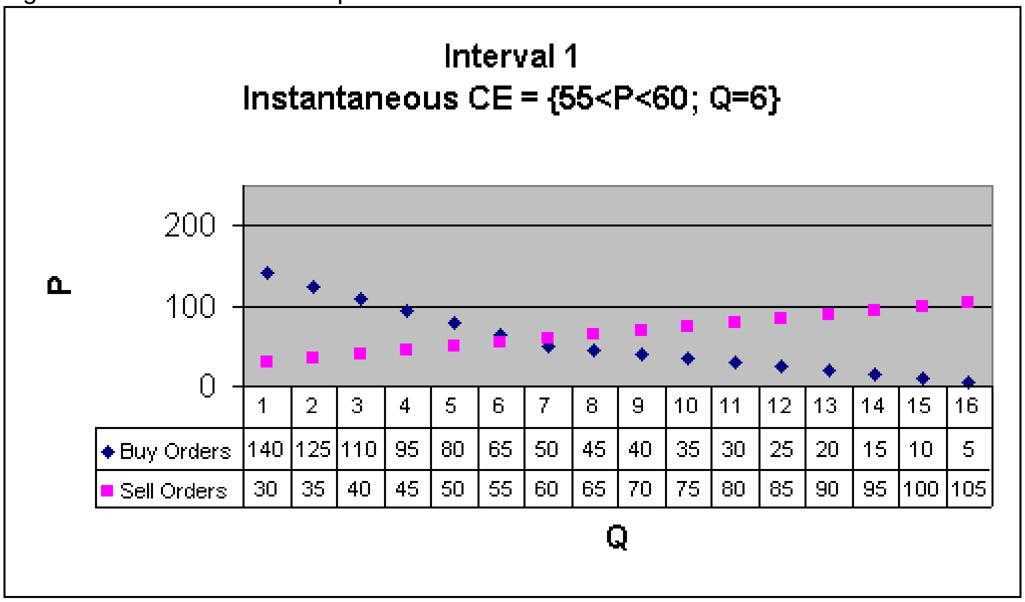
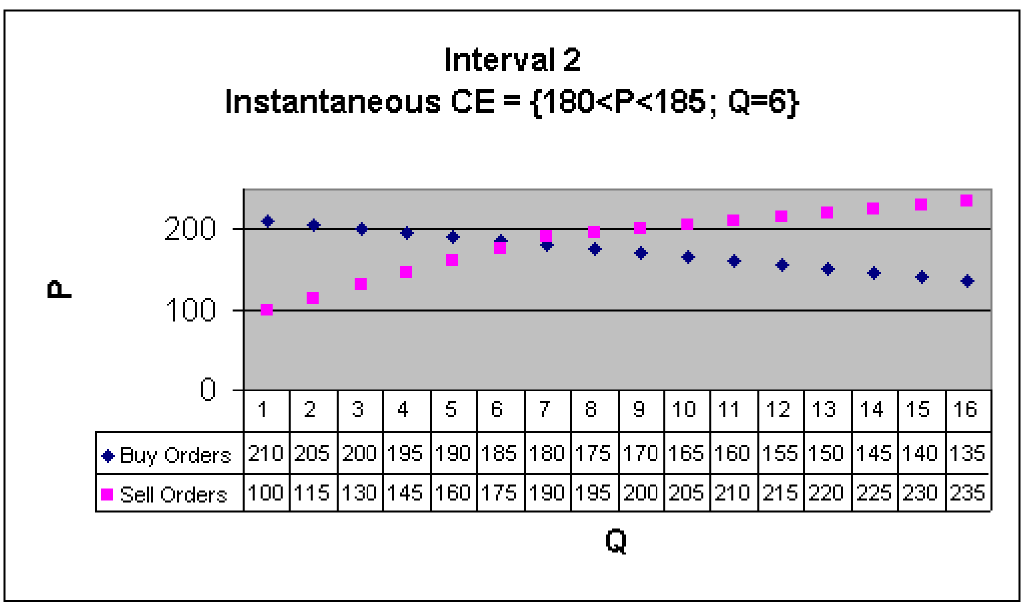


Figure 4 - Parameters for Experiment 1

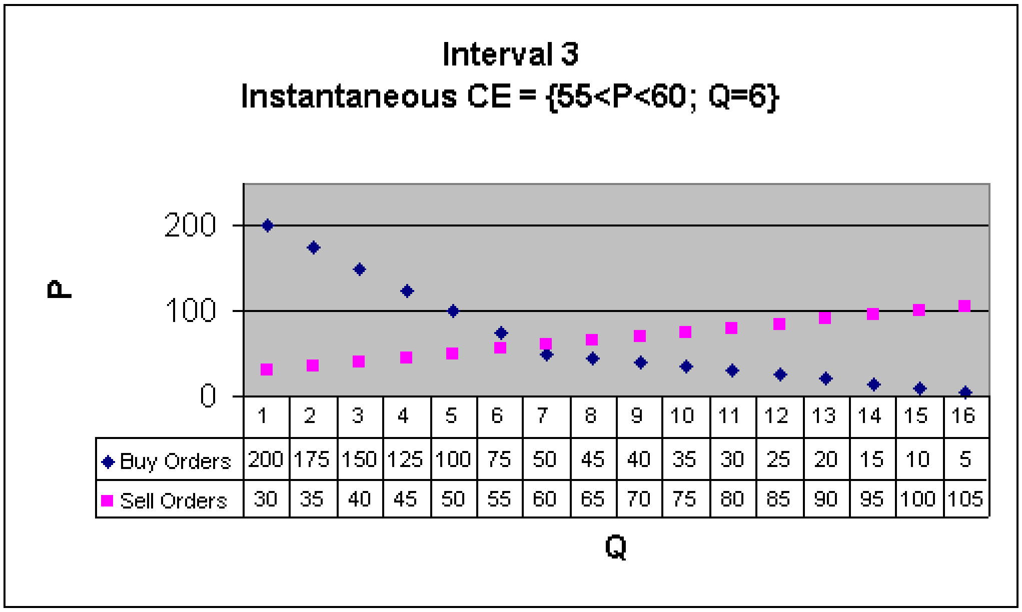


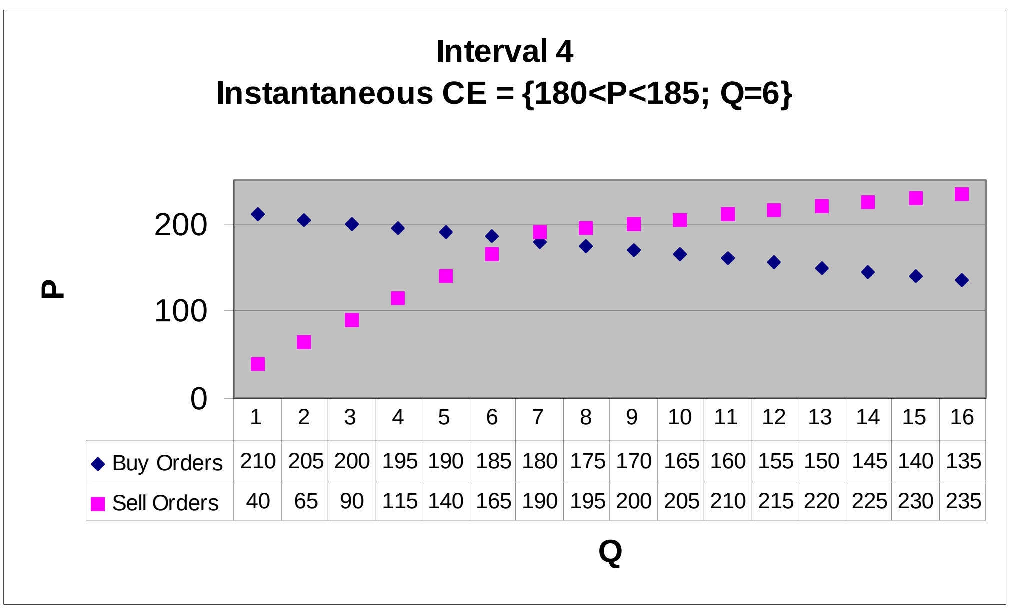
Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Buy Orders	140	125	110	95	80	65	50	45	40	35	30	25	20	15	10	5
Sell Orders	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105

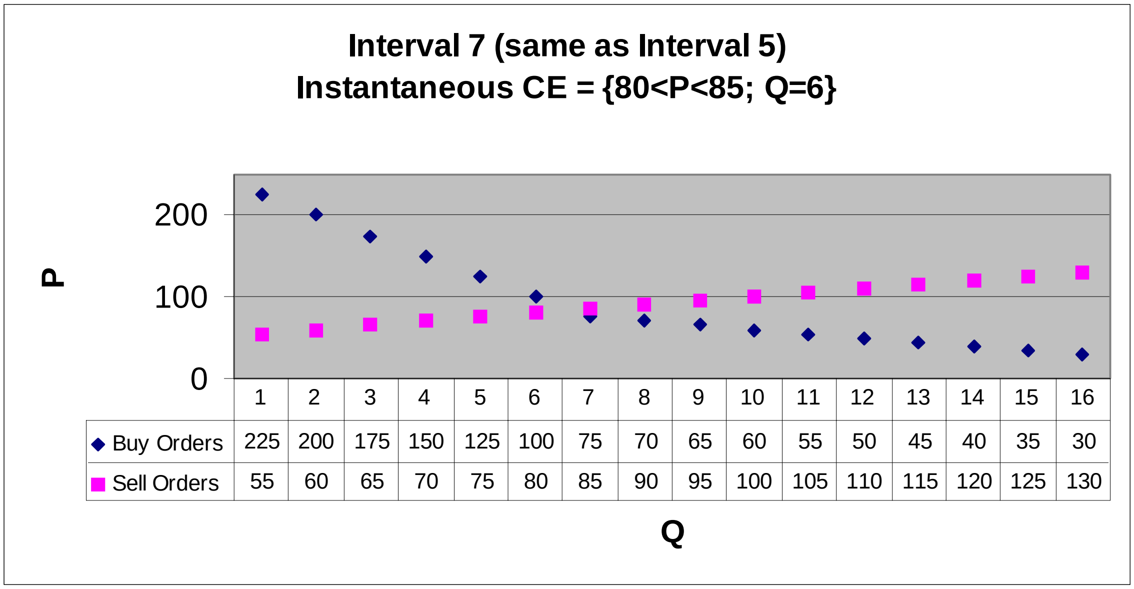


Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Buy Orders	210	205	200	195	190	185	180	175	170	165	160	155	150	145	140	135
Sell Orders	100	115	130	145	160	175	190	195	200	205	210	215	220	225	230	235

Figure 5 - Parameters for Experiment 2







	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Buy Orders	225	200	175	150	125	100	75	70	65	60	55	50	45	40	35	30
Sell Orders	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130

Figure 10: Highest Bids, Lowest Asks, and Transaction Prices [ZI Robots, Environment 1]

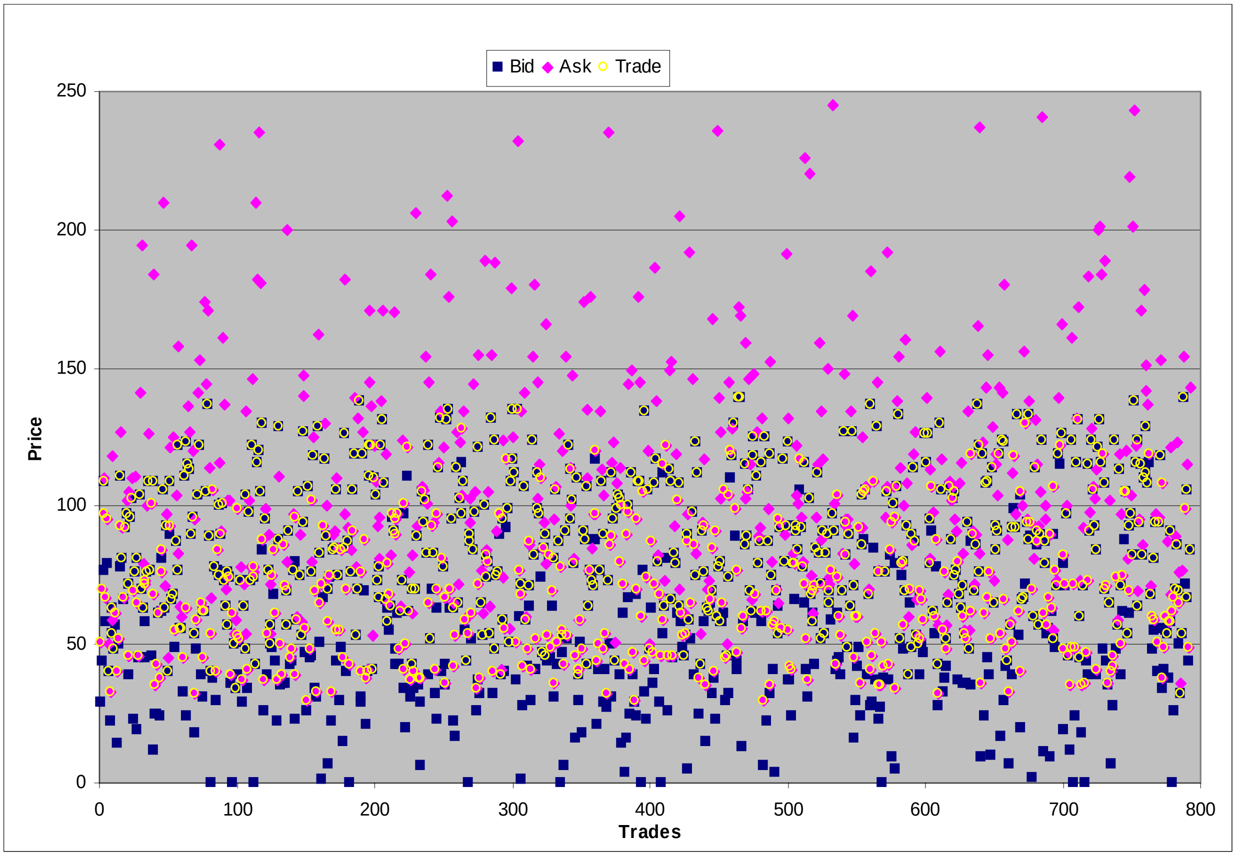


Figure 11: ZI Price Distribution with Continuously Refreshed Supply/Demand

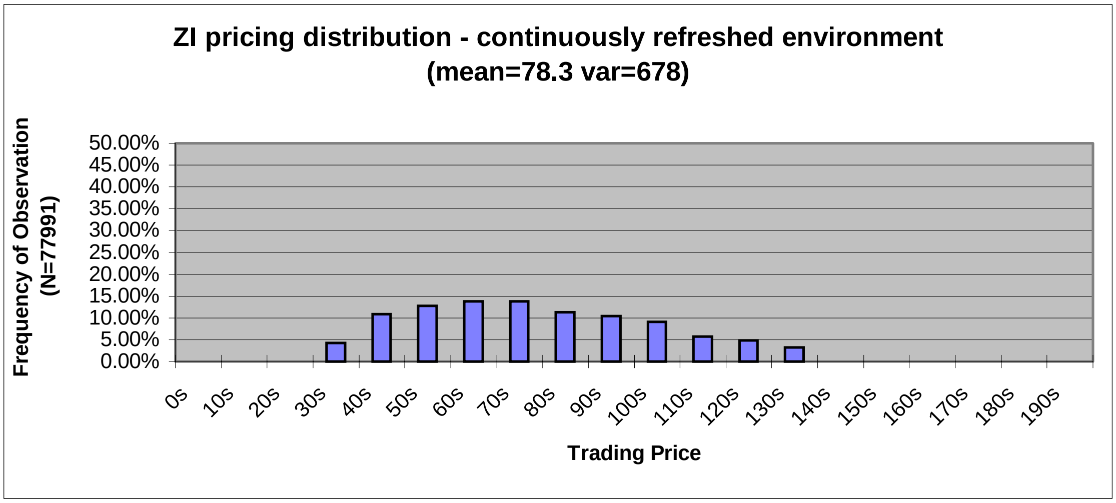


Figure 12: Comparison of Price Distributions: ZI markets vs. Human markets

[Observed = Humans]

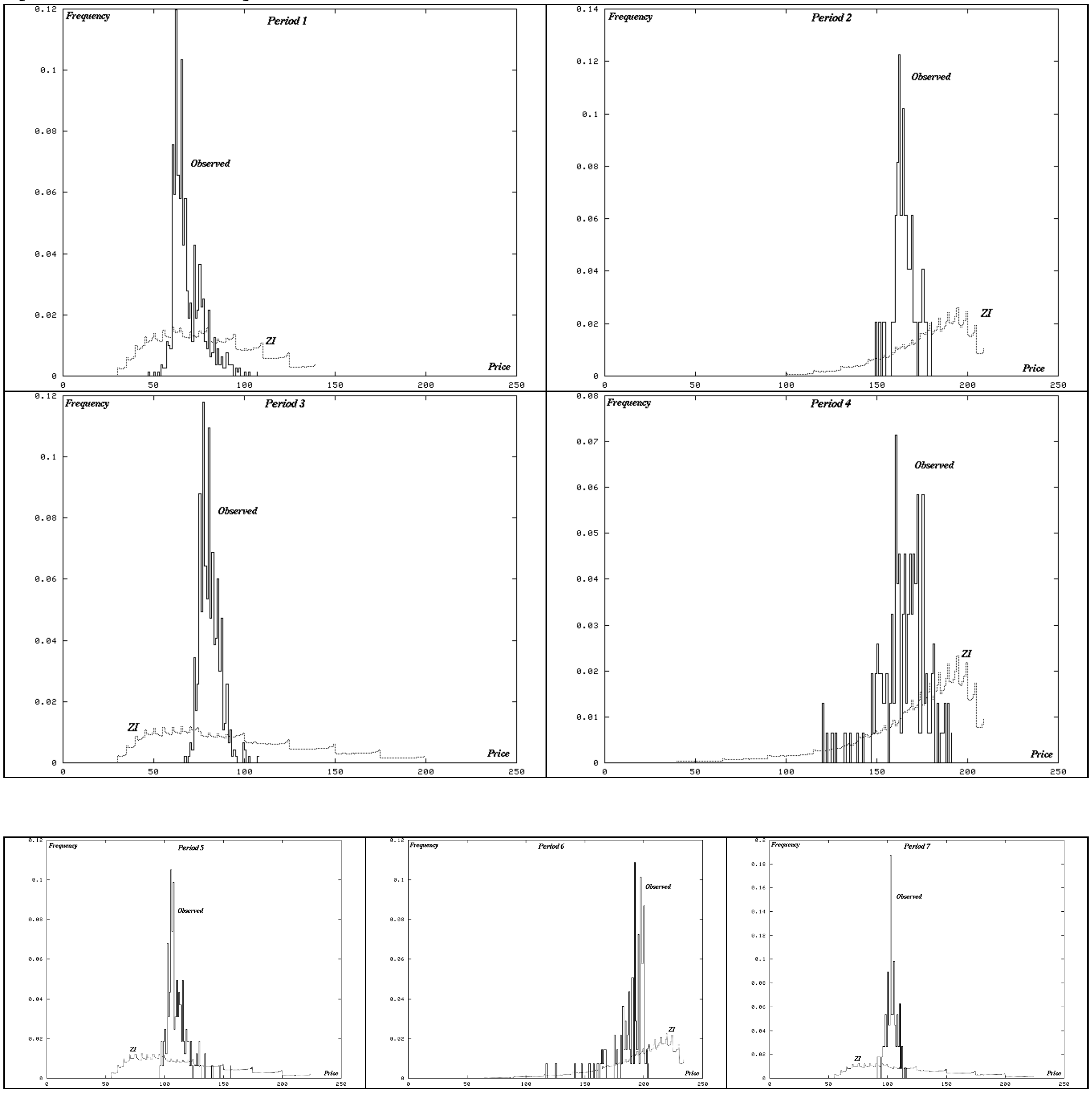


Table 1: Supply and Demand parameters maintained in the experiments

Experiment	Interval/ Environment	Private Market Orders	Instantaneous Competitive Equilibrium
1	1/1 (tr#1-793) 2/2 (tr#820-end)	Buy (140,125,110,95,80,65,50,45,40,35,30,25,20,15, 10, 5) Sell (30, 35, 40,45,50,55,60,65,70,75,80,85,90,95,100,105) Buy (210,205,200,195,190,185,180,175,170,165,160,155,150,145,140,135) Sell (100,115,130,145,160,175,190,195,200,205,210,215,220,225,230,235)	$55 \leq P \leq 60$ $180 \leq P \leq 185$ (see Figure 4)
2	3/3 (tr#1-466) 4/4 (tr#504-end)	Buy (200,175,150,125,100,75,50,45,40,35,30,25,20,15, 10, 5) Sell (30, 35, 40, 45, 50,55,60,65,70,75,80,85,90,95,100,105) Buy (210,205,200,195,190,185,180,175,170,165,160,155,150,145,140,135) Sell (40, 65, 90,115,140,165,190,195,200,205,210,215,220,225,230,235)	$55 \leq P \leq 60$ $180 \leq P \leq 185$ (see Figure 5)
3	5/5 (tr#1-162) 6/6 (tr#176-313) 7/5 (tr#348-end)	Buy (225,200,175,150,125,100, 75, 70,65, 60, 55, 50, 45, 40, 35, 30) Sell (55, 60, 65, 70, 75, 80, 85, 90,95,100,105,110,115,120,125,130) Buy (235,230,225,220,215,210,205,200,195,190,185,180,175,170,165,160) Sell (65, 90,115,140,165,190,215, 220,225,230,235,240,245,250,255,260) Buy (225,200,175,150,125,100, 75, 70,65, 60, 55, 50, 45, 40, 35, 30) Sell (55, 60, 65, 70, 75, 80, 85, 90,95,100,105,110,115,120,125,130) Note: same as Period 5	$80 \leq P \leq 85$ $205 \leq P \leq 210$ $80 \leq P \leq 85$ (see Figure 6)

Table 2: Averages and Variances of Prices Observed in Experiments [Humans]

Experiment/Interval/Trades	Number of Transactions	Mean [Transaction Price]	Var [Transaction Price]
Experiment 1	868	74.8	637.6
Interval 1: trades 1-793	793	68.0	68.8
Detail			
1-100	100	81.2	85.5
101-200	100	75.8	26.3
201-300	100	69.8	21.5
301-400	100	64.8	18.1
401-500	100	63.7	22.3
501-600	100	63.2	8.9
601-700	100	61.9	9.6
701-793	93	63.4	6.5
Transition 12: trades 794-819	26	112.8	1036.4
Interval 2: trades 820-868	49	164.7	40.4
Experiment 2	657	100.9	1304.9
Interval 3: trades 1-466	466	80.5	32.5
Detail			
1-100	100	85.1	55.0
101-200	100	83.0	13.8
201-300	100	79.8	15.9
301-400	100	77.3	9.6
401-466	66	75.9	8.7
Transition 34: trades 467-503	37	94.2	176.3
Interval 4: trades 504-657	154	164.1	174.1
Detail			
504-603	100	159.5	167.3
604-657	54	172.5	77.7
Experiment 3	459	134.3	1491.6
Interval 5: trades 1-162	162	110.5	87.6
Detail			
1-100	100	114.3	97.1
101-162	62	104.2	9.3
Transition 56: trades 163-175	13	107.5	5.0
Interval 6: trades 176-313	138	187.6	216.0
Detail			
176-275	100	183.9	245.1
276-313	38	197.4	5.7
Transition 67: trades 314-347	34	145.4	868.0
Interval 7: trades 348-459	112	102.9	24.8

Table 3. Price Data from ZI Markets

ZI Parameters	Data Collected (trades)	Mean[P]	Var[P]	Instantaneous CE Ex-Ante	Velocity-Adjusted CE Ex-Post
S/D not refreshed Environment 1	7075 trades 1000 periods	67.0 (all trades) 52.6 (final)	531 152	55≤P≤60	---
S/D continuously refreshed Environment 1	1.56 million	78.31	675	55≤P≤60	80
Environment 2	1.41 million	176.82	486	180≤P≤185	175
Environment 3	1.77 million	95.04	1657	55≤P≤60	90
Environment 4	1.57 million	169.58	964	180≤P≤185	165
Environment 5	1.73 million	118.16	1650	80≤P≤85	110
Environment 6	1.57 million	193.47	1017	205≤P≤210	190