A model of plea bargaining with asymmetric information is presented. The prosecutor’s private information is the strength of the case; the defendant’s is his guilt or innocence. In equilibrium, some cases are dismissed because they are too likely to involve an innocent defendant. In the remaining cases, the prosecutor’s sentence offer reveals the strength of the case. A particular restriction on prosecutorial discretion is shown to be welfare-enhancing for some parameter configurations.

Under current practice, prosecutors have essentially unlimited discretion to dismiss a case or to negotiate a guilty plea to a lesser crime, thereby guaranteeing a lighter sentence. Much controversy surrounds the exercise of prosecutorial discretion. While it is acknowledged that guilty pleas save resources which would otherwise be devoted to trials, one major concern of opponents to plea bargaining is that the prosecutor is in an unfairly strong bargaining position. One source of bargaining strength for the prosecutor is simply the fact that the defendant is required to deal with him, and cannot "shop" for a better deal.

Another asymmetry between the bargaining positions of the prosecutor and the defendant stems from the fact that the prosecutor typically has better information about the strength of the case than does the defendant. The prosecution has presumably interviewed witnesses (including the defendant) and gathered evidence. In the modal criminal case, the time and investigative resources available to the defense are a fraction of those available to the prosecution. Thus although in principle the defense may have equal access to evidence and witnesses, in practice it relies heavily on summary data from the prosecution. It seems likely that in this case there will remain some uncertainty on the part of the defense regarding the strength of the prosecution’s case. Similarly, despite the evidence which the prosecution may possess, it seems likely that there will remain some uncertainty on the part of the prosecution regarding the defendant’s factual guilt.

In this paper, a model of the plea-bargaining process in the presence of asymmetric information is developed. While the issue of

---

*Departments of Economics and Management Science, University of Iowa, Iowa City, IA 52242. I would like to thank Kim Border, Richard Craswell, Ted Groves, Barry Nalebuff, Herman Quirmbach, Eric Rasmusen, and two anonymous referees for helpful comments and discussion. The financial support of the National Science Foundation, grant no. SES-8710578, the Alfred P. Sloan Foundation, and the Graduate School of Business of the University of Chicago are gratefully acknowledged.

†Plea bargaining is a prominent feature of the criminal justice system. According to Alschuler (1981, p. 652), "it is commonly estimated that 90% of all criminal convictions are the result of guilty pleas." Moreover, a large fraction of the cases are simply dismissed; based upon a sample of 1,382 felony arrests in New York City in 1971, the Vera Institute of Justice (1977, p. 7) estimates that "43% of the felony arrests were disposed of by dismissal."

‡In a related paper, Gene Grossman and Michael Katz (1983, p. 752) assume the probability of conviction differs for guilty and innocent defendants, and that these probabilities are common knowledge, arguing that the prosecutor is constitutionally required to provide to the defendant all of the state’s evidence against him, as well as a summary of what is necessary for conviction (Brady v. Maryland, 1963). By contrast, Albert Alschuler (1968, p. 66) claims that "the discovery privileges of the defense are highly restricted, and even the limited right of discovery that the law affords may be frustrated until plea negotiations are concluded."

Jennifer Reinganum and Louis Wilde (1986) examined the problem of the settlement and litigation of civil suits using a signaling model. Because this paper also concerns legal bargaining with private information, it is important to distinguish this paper from the previous one. In that paper, the plaintiff had private information about the extent of damages suffered, and made a settlement demand which was either accepted or re-
the desirability of prosecutorial discretion is not resolved in complete generality, two discretionary regimes are compared. In the first, the prosecutor has discretion to offer an arbitrary sentence in exchange for a plea of guilty, with a sentence of length zero interpreted as a dismissal. In the second, plea bargaining is still permitted, but all defendants who are charged with the same crime must be offered the same sentence. It is shown that, depending upon other features of the criminal justice system and upon the preferences of society, either of these regimes may be preferred to the other. In particular, it is possible that unlimited discretion is disadvantageous for the prosecution, since it carries with it the requirement of sequential rationality.

William Landes (1971) offered the first economics-based analysis of plea bargaining. Under the assumption that the prosecutorial (and social) objective is the maximization of the sum of expected sentences subject to a resource constraint, Landes argues (among other things) that the likelihood of disposition by negotiated plea should be higher the smaller is the sentence if convicted at trial and the greater is the resource cost to the defendant of trial versus negotiated plea. Empirical analyses conducted separately at the U.S. district court level and at the county court level supported the latter implication, but yielded support for the former implication only at the U.S. district court level. William Rhodes (1976) modified the Landes model, which is based upon individual choice, to apply directly to aggregate data which are more readily available. Using data from U.S. district courts and Minnesota county courts, he found that the ratio of guilty pleas to trials is negatively correlated with the severity of the sentence offered in plea negotiations, and that increases in the defendant's resource cost of trial increase the ratio of negotiated pleas to trials. David Weimer (1978) tested a variant of the Landes model on individual case data from the Alameda County (California) Superior Court. He found that the plea-bargain sentence offer is an increasing function of the expected trial sentence (i.e., the product of the probability of conviction and the anticipated sentence upon conviction at trial), and the probability that a trial will be demanded is a decreasing function of the difference between the anticipated sentence upon conviction at trial and the sentence offered in plea negotiations. Brian Forst and Kathleen Brosi (1977) examine yet another variant of Landes' model, in which they use the length of time the prosecution carries a case as a measure of prosecutorial effort. Using data from District of Columbia courts, they find that the length of time the prosecutor will carry a case is strongly positively related to the strength of the evidence against the defendant, and mildly positively related to the seriousness of the offense (as measured by, for example, the maximum sentence applicable upon conviction at trial). While these empirical findings are all consistent with the implications of the model developed below, none of these analyses represents an appropriate structural test of this model. Thus these findings can be taken as at best weak supporting evidence.

One difficulty with Landes' theory is that the implications he tests do not strictly follow from the theory, since the actual sentence offered in a plea bargain is indeterminate (there exists a range of mutually acceptable sentence offers) if the defendant is risk averse. Moreover, in this case, the model...
always predicts disposition by negotiated plea; no trials should occur. In addition, the analysis presumes that all defendants are guilty, since the objective is to maximize the sum of expected sentences. Grossman and Katz (1983) admit the possibility of innocent defendants and, using an objective function which incorporates the social disutility of punishing the innocent, they provide an alternative justification of plea bargaining on the basis of its potential roles as an insurance device and as a screening device: the optimal sentence offer can induce the guilty and the innocent to self-select, so that the guilty choose negotiated pleas, while the innocent choose trial.

My analysis does not examine the desirability of the institution of plea bargaining per se; rather, it takes the existence of plea bargaining as given, and discusses the extent to which prosecutorial discretion within that framework might be desirable. It is similar to that of Grossman and Katz in its treatment of the prosecutor’s (and society’s) objective function. Because private information is one-sided in their model (only the defendant knows whether he is guilty or innocent), a single offer is made to all defendants; thus selective dismissals are not possible. In my model, both parties have private information (which is correlated); with unrestricted discretion, offers can be individualized and cases can be selectively dismissed. However, the regime with restricted discretion is quite similar to their model because a single sentence is offered to all defendants.

In Section I, the basic model is described and one sequential equilibrium is presented. I argue that this is an appropriate sequential equilibrium for the game with complete discretion by appealing to the work of Jeffrey Banks and Joel Sobel (1987) on “(universally) divine” sequential equilibrium and that of In-Koo Cho and David Kreps (1987) on “intuitive” sequential equilibria. In Section II, the alternative game with restricted discretion is described and its equilibrium behavior is characterized. Section III compares the welfare of the prosecutor (and, by assumption, that of society) under the two regimes, and describes circumstances under which each regime may be preferred to the other. Section IV summarizes, discusses two respects in which the basic model can be generalized, and outlines potential avenues for future research. All proofs are confined to the Appendix.

I. Unrestricted Prosecutorial Discretion

In plea bargaining, the key element need not be the guilt or innocence of the defendant (for the prosecution and the court may be unable to systematically determine the truth), but rather the strength of the case. In other words, the case may appear very strong, despite the fact that the defendant is innocent; similarly, the case against a guilty defendant may be very weak.

Let \( t = g, i \) denote the two types of defendants: guilty and innocent, respectively. The defendant’s type is assumed to be known only to the defendant. Assume that the strength of an arbitrary case is represented by the probability \( \pi \in [0, 1] \) that the defendant will be found guilty at trial. The strength of the case is a summary statistic of the extent and quality of the evidence which is available to the prosecution, including verifiable information obtained from the defendant (for example, an alibi, the names of potential witnesses for the defense). Thus a defendant with \( \pi = .7 \) is either guilty and lucky to have a 30 percent chance of acquittal or innocent and unlucky to have a 70 percent chance of conviction. I assume that the probability of conviction is private information possessed by the prosecutor. However, the defendant’s type and the prosecutor’s case are not unrelated; in particular, guilt and evidence are assumed to be jointly distributed. Let \( G(\pi, t) \) denote the joint distribution of \( \pi \) and \( t \); that is, \( G(\pi, t) = \text{Prob} \{ \text{case is of strength } \leq \pi \text{ and defendant is of type } t \} \). The expression which will be relevant to the prosecutor’s decision is denoted \( f(\pi) \) and represents the probability that, given the strength of the case, the defendant actually is guilty: \( f(\pi) = \text{Prob} \{ \text{defendant is guilty} | \text{case is of strength } \leq \pi \} = \frac{dG(\pi, g)}{dG(\pi, g) + dG(\pi, i)} \), where \( dG(\pi, t) \) denotes the density with respect to \( \pi \). Notice that \( f(\pi) \) need not be the identity function; \( \pi \) and \( f(\pi) \) can differ because different
standards of evidence apply in court; for example, the prosecutor may impute a higher likelihood of guilt than conviction if in court guilt must be proved beyond a reasonable doubt. The expression which will be relevant to the defendant's decision problem is denoted \( \Phi(\pi|t) \) and represents the conditional distribution of \( \pi \) given \( t \): \( \Phi(\pi|t) = \text{Prob} \{ \text{case is of strength } \leq \pi | \text{defendant is of type } t \} = G(\pi, t)/G(1, t) \).

Although arrest itself may convey some information about the strength of the prosecutor's case (for example, if arrest requires that the evidence exceed a certain threshold value), we suppress this possibility for the moment in the interest of expositional simplicity. Similarly, one could model the discovery process as generating for the defendant a signal which is imperfectly correlated with the actual strength of the prosecution’s case. These two potential information sources are discussed further in Section IV. For the present, I assume that the arrest process is essentially random, with \( q \) denoting the proportion of guilty among those arrested; that is, \( q = G(1, g) \). The distribution function \( G(\pi, t) \) is assumed to be common knowledge.

**ASSUMPTION 1:** \( f'(\pi) > 0 \) for all \( \pi \in [0, 1] \); that is, the better the case against the defendant, the greater is the likelihood of guilt.

Define \( E_g(\pi|\delta) \) to be the defendant of type \( t \)'s expectation of \( \pi \), given that \( \pi \) belongs to the set \( \delta \subseteq [0, 1] \).

**ASSUMPTION 2:** \( E_g(\pi|\delta) \geq E_i(\pi|\delta) \) for all \( \delta \); that is, the distribution of \( (\pi, t) \) is such that, conditional on \( \pi \in \delta \), a guilty defendant faces a stronger case (in expectation) than does an innocent defendant.\(^3\)

\(^3\)In the Appendix, it is shown that the distribution
\[ G(\pi, i) = \eta[1 - e^{-h_i \pi}]/[1 - e^{-h_i}] \text{ and} \]
\[ G(\pi, g) = (1 - \eta)[1 - e^{-h_g \pi}]/[1 - e^{-h_g}], \]
with \( h_i > h_g \), satisfies Assumptions 1 and 2 for subsets \( \delta \) of the form \([a, b]\).

Grossman and Katz assumed that guilty defendants are more likely to be convicted than are innocent defendants; Assumption 2 is simply a conditional form of this assumption.

Suppose \( s \) is the sentence offered in a plea bargain, and \( x \) is the sentence anticipated upon conviction at trial; \( s \) and \( x \) are non-negative and represented in terms of utility. The sentence upon conviction \( x \) is taken as exogenous by the prosecutor and defendant alike. Although in practice the prosecutor may be able to influence \( x \) through variations in the charges brought, I assume that sentencing following conviction at trial is fundamentally a judicial decision, which is correctly anticipated by both parties. One consequence of Assumption 2 is that, conditional on \( \pi \in \delta \), a guilty defendant expects a greater punishment from trial—because he is (on average) more likely to be convicted —than his innocent counterpart. Let \( k \) denote the disutility of trial for the defendant. I have assumed that guilty and innocent defendants suffer equally in the event of trial and if given the same sentence. Grossman and Katz also assume that guilty and innocent defendants have identical utility functions. Thus my model is a natural generalization of theirs to the case in which evidence may be of variable strength. A plausible alternative assumption would be that innocent defendants suffer more in the event of trial and if given the same sentence than do guilty defendants (due to the injustice of their punishment). This possibility is considered in Section IV.

Events are assumed to proceed in the following order. First, the defendant and the prosecutor observe their private information. The prosecutor, on the basis of his private information, makes a sentence offer in exchange for a plea of guilty. Then the defendant, on the basis of his private information and whatever inference he draws from the prosecutor's offer, either accepts or rejects the offer. If he accepts the offer, both parties collect the associated payoffs. If he rejects the offer, the case must go to trial, and both parties suffer a disutility associated with trial. If the defendant is acquitted at trial, neither party receives any further payoff. Finally, if
Given this sequence of moves, a strategy for the defendant of type \( t \) is a function \( p_t(s) \) specifying the probability that the defendant rejects a sentence offer of \( s \). We can write the expected utility to a type-\( t \) defendant who is offered the sentence \( s \) and rejects it with probability \( \rho \) as

\[
DU_t(s, \rho; \delta(s)) = -\rho \left( E_t(\pi|\delta(s))x + k \right) - (1-\rho)s,
\]

where \( \delta(s) \) describes the defendant’s beliefs upon observing \( s \); that is, \( \delta(s) \) is the set of prosecutor types \( \pi \) which the defendant believes would offer \( s \); note that this is not subscripted, because there is no reason for different defendants to have different conjectures about this set. However, the expectation is subscripted because the innocent and the guilty may assign different distributions over the set \( \delta(s) \) because \( (\pi, t) \) are jointly distributed. However, if the plea bargain \( s \) reveals \( \pi \), then the guilty and the innocent will have identical (degenerate) expectations about \( \pi \) given an offer of \( s \). In this case, they might as well use the same strategy \( p(s) \).

This “symmetry” assumption is formalized below in Assumption 3. In the sequel I will examine only equilibria which are symmetric in this sense, but I have been unable to rule out the possibility that other “asymmetric” equilibria may exist. When the sentence offer does not reveal \( \pi \), then the two types of defendants may use different strategies: \( p_t(s) \), for \( t = g, i \).

**ASSUMPTION 3:** When both defendant types are indifferent about accepting or rejecting a sentence offer \( s \), they use the same strategy \( p(s) \).

The objective function of the prosecutor is assumed to coincide with that of society at large, and involves three goals: appropriate punishment of the guilty, avoidance of punishment of the innocent and the conservation of resources spent on trials. The first two of these three goals are made explicit in the following excerpt from the Supreme Court opinion in Berger v. United States (quoted by Bruce Jackson, 1984, p. 143).

The United States Attorney is the representative not of an ordinary party to a controversy but of a sovereignty whose obligation to govern impartially is as compelling as its obligation to govern at all; and whose interest, therefore, in a criminal prosecution is not that it shall win a case, but that justice shall be done. As such, he is in a peculiar and very definite sense the servant of the law, the twofold aim of which is that guilt shall not escape or innocence suffer. [Emphasis added.]

Though the last goal—that of conserving resources—is more prosaic, it is clearly of some social concern.

If \( c \) is the social cost of trial (again in terms of utility), one reasonable utility function for the prosecutor is the difference (in expected value terms) between the social valuation of the disutility of the guilty and the social valuation of the disutility of the innocent, less the social disutility of court costs. Therefore assume that society values disutility \( d > 0 \) to a guilty defendant at \( \gamma d \) and disutility \( d \) to an innocent defendant at \( -\lambda d \), where \( \gamma, \lambda > 0 \). A strategy for the prosecutor is a function \( s(\pi) \) specifying the sentence offered when the case is of strength \( \pi \). Then expected prosecutor utility from a case of strength \( \pi \) when a sentence of \( s \) is
offered can be written\(^5\)\(^6\)
\[
\text{PU}(\pi, s; p_g(s), p_i(s)) = f(\pi)\left\{ p_g(s)[-c + \gamma(\pi x + k)] + [1 - p_g(s)]\gamma s \right\} + (1 - f(\pi)) \times \left\{ p_i(s)[-c - \lambda(\pi x + k)] - [1 - p_i(s)]\lambda s \right\}.
\]

To interpret equation (2), assume that the prosecutor has observed a case of strength \(\pi\) and offered a sentence of \(s\). With probability \(f(\pi)\) the defendant is actually guilty, in which case he will reject the offer with probability \(p_g(s)\), forcing the case to trial, where the prosecutor suffers the expected utility of punishing a guilty defendant \(\gamma(\pi x + k)\). With probability \(1 - p_g(s)\) the defendant accepts the offer, yielding prosecutor utility of \(\gamma s\). With probability \(1 - f(\pi)\), the defendant is innocent, in which case he will reject the offer with probability \(p_i(s)\), forcing the case to trial,

\[
\text{PU}(\pi, s; p(s)) = p(s)[-c + a(\pi)(\pi x + k)] + [1 - p(s)]a(\pi)s,
\]

where \(a(\pi) = f(\pi)\gamma - (1 - f(\pi))\lambda\). The expression \(a(\pi)\) represents the expected net social utility of an additional unit of disutility imposed upon a defendant against whom there is a case of strength \(\pi\), where the expectation arises from the fact that the defendant’s guilt is unverifiable. Notice that \(a'(\pi) = f'(\pi)\gamma + (1 - f(\pi))\lambda > 0\). Define \(\pi_0\) such that \(a(\pi_0) = 0\). If \(\pi_0 \in (0, 1)\), it will be unique, and hereafter we assume its existence and interiority. Also define \(A(s) = \int a((s - k)/x)ds\); note that \(A(\cdot)\) is an increasing function for \(s > \pi_0 x + k\).

For a general definition of sequential equilibrium, see David Kreps and Robert Wilson (1982); for our purposes, the following definition will suffice.

**Definition 1:** A sequential equilibrium consists of beliefs \(\delta^*(\cdot)\) and strategies \((p_g^*(\cdot), p_i^*(\cdot), s^*(\cdot))\) such that
- \(p^*_t(s)\) maximizes \(DU_t(s, \rho; \delta^*(s))\), for \(t = g, i;\)
- \(s^*(\pi)\) maximizes \(\text{PU}(\pi, s'; p^*_g(s), p^*_i(s));\) and
- \(\delta^*(s) \subseteq [0, 1]\) for all \(s\), and \(\delta^*(s) = \{\pi | s = s^*(\pi)\}\) whenever this set is nonempty.

That is, the equilibrium strategy of each defendant type maximizes that defendant’s expected utility, given the beliefs. The prosecutor’s strategy maximizes the prosecutor’s expected utility, given the anticipated responses of the two types of defendants. Finally, the defendants’ beliefs are always con-
fined to the set of prosecutor types which are known to be possible, and these beliefs are correct for equilibrium sentence offers.

**PROPOSITION 1:** A sequential equilibrium for this model is for the prosecutor to offer \( s^* = 0 \) (i.e., dismiss the case), if \( \pi < \pi_0 \); otherwise offer \( s^* = \pi x + k \). Let \( s = \pi_0 x + k \), and \( \hat{s} = x + k \). Then the defendant (whether guilty or innocent) rejects the offer \( s \) with probability \( p^*(s) = 1 \) if \( s > \hat{s} \), with probability \( p^*(s) = 1 - \exp\{[A(s) - A(\hat{s})]/c\} \) for \( s \in [\hat{s}, \hat{s}] \), and with probability \( p^*(s) = 0 \) if \( s < \hat{s} \). Finally, the defendants’ beliefs are \( \delta^*(s) = 1 \) for \( s > \hat{s} \), \( \delta^*(s) = (s - k)/x \) for \( s \in [\hat{s}, \hat{s}] \), \( \delta^*(s) = \pi_0 \) for \( s \in (0, \hat{s}) \), and \( \delta^*(s) = [0, \pi_0) \) for \( s = 0 \).

Notice that sufficiently weak cases—those with \( \pi < \pi_0 \)—are dismissed. How weak is “sufficiently weak” depends only upon the form of the inference function \( f(\pi) \) and upon \( \lambda / (\gamma + \lambda) \), and not on the resource cost of trials. That is, a case is never dismissed on account of the resource costs of pursuing it; this decision is based only on the merits of the case. If equal consideration is given to punishing the guilty and avoiding punishing the innocent (i.e., \( \lambda = \gamma \)), then to be dismissed a case must leave the prosecutor believing innocence is more likely than guilt \( (f(\pi) < 1/2) \). If society is more concerned with avoiding the punishment of the innocent than with ensuring the punishment of the guilty (i.e., \( \lambda > \gamma \)), then the prosecutor will dismiss some cases in which guilt is more likely than innocence. The result that the prosecution will dismiss cases in which there is sufficient doubt regarding the defendant’s guilt is consistent with anecdotal evidence. Alschuler (1981, p. 708) remarks that “…when a prosecutor does entertain serious doubts concerning a defendant’s factual guilt, he is likely to decline to prosecute…” and Charles Silberman (1980, p. 367) concludes that “Most prosecutors believe that they should not press charges unless they are convinced of the defendant’s guilt.”

Formally, the equilibrium consists of two portions. One involves complete pooling for prosecutor types \( \pi < \pi_0 \) while the other involves complete separation for types \( \pi \geq \pi_0 \).

Under the assumption that the sentence offer reveals the strength of the case both types of defendant use the same strategy \( p(s) \), the uniqueness proof of Reinganum and Wilde (1986) can be adapted to show that the separating portion of the equilibrium is unique. If this symmetry assumption is relaxed, I have been unable to rule out the possibility of a separating equilibrium in which the two defendant types use different equilibrium strategies.\(^7\)

Signaling models typically possess a multiplicity of equilibria, some of which depend upon the specification of out-of-equilibrium beliefs. In view of the potential sensitivity of equilibrium behavior to out-of-equilibrium beliefs, it is worthwhile examining alternatives to those beliefs specified in Proposition 1. In the Appendix, it is shown that one sufficient condition for the equilibrium strategies specified in Proposition 1 to be supported by any out-of-equilibrium beliefs is the parametric restriction \( \pi_0 \leq c/\lambda x \). Alternatively, if \( \delta^*(s) \) is a singleton for \( s \in (0, \hat{s}) \), then any such beliefs will support the equilibrium behavior specified in Proposition 1. Thus under plausible circumstances this equilibrium behavior is “(universally) divine” (Banks and Sobel, 1987) and “intuitive” (Cho and Kreps, 1987). Indeed, it will survive any equilibrium refinement which relies only on belief restrictions. Although this does not

\(^7\)However, some likely candidates can be ruled out. For instance, a candidate in which the innocent defendant rejects \( s \) when indifferent while a guilty defendant randomizes would be characterized by \( \delta(\pi) = \pi x + k \) for \( \pi \in [0,1] \). Thus \( \hat{s} = k \) and \( \hat{s} = x + k \). Strategies for the defendants: \( \bar{\pi}(s) = \bar{\pi}(s) = 1 \) for \( s > \hat{s} \), \( \bar{\pi}(s) = 1 - \exp\{y(s - \hat{s})/c\} \) for \( s \in [\hat{s}, \hat{s}] \) and \( \bar{\pi}(s) = \bar{\pi}(s) = 0 \) for \( s \in [0, \hat{s}] \) are supported by the beliefs \( \delta(s) = (s - k)/x \) for \( s \in [\hat{s}, \hat{s}] \) and for arbitrary out-of-equilibrium beliefs \( \delta(s) \subseteq [0,1] \). Although \( \delta(\pi) \) provides a stationary point of \( \mu(\pi; \bar{\pi}(\pi), \bar{\pi}(\pi)) \), it does not provide a maximum for \( \pi \) sufficiently near 0. To see this, note that \( \mu(\pi, \delta(\pi); \bar{\pi}(\delta(\pi))) = a(\pi) \delta(\pi) - f(\pi) \bar{\pi}(\delta(\pi)) + (1 - f(\pi))c \), while \( \mu(\pi_0, \bar{\pi}(0), \bar{\pi}(0)) = 0 \). Evaluating both expressions at \( \pi = 0 \) implies that \( \mu(0, \bar{\pi}(0), \bar{\pi}(0)) > \mu(0, \delta(0); \bar{\pi}(\delta(0)), \bar{\pi}(\delta(0))) \), if and only if \( 0 > a(0) \delta - [1 - f(\pi)]c \). Since \( a(0) < 0 \), the inequality holds. Thus prosecutors with sufficiently weak cases (i.e., \( \pi \) sufficiently near 0) would prefer \( s = 0 \) to \( \bar{s}(\pi) \).
imply that this is the unique such equilibrium, it does suggest that it is a highly plausible one which deserves consideration and close inspection.

Another expression of interest is the equilibrium probability of trial as a function of the strength of the case. This is the composition of the equilibrium probability of rejection and the equilibrium sentence offer:

\[ \hat{p}(\pi) = p^*(s^*(\pi)). \]

(4) \[ \hat{p}(\pi) = 1 - \exp \left\{ \left[ A(s) - A(s^*(\pi)) \right]/c \right\}. \]

PROPOSITION 2: (a) For \( s \in (\tilde{s}, \bar{s}) \), the equilibrium probability of rejection \( p^*(s) \) increases with an increase in the sentence \( s \) offered in exchange for a guilty plea, and the social weight \( \gamma \) given to the guilty defendant’s disutility; \( p^*(s) \) decreases with an increase in the social cost of trial \( c \), the defendant’s disutility of trial \( k \), the sentence anticipated upon conviction at trial \( x \), and the social weight \( \lambda \) given to the innocent defendant’s disutility.

(b) For \( \pi \in [\tilde{\pi}, 1] \), the equilibrium sentence \( s^*(\pi) \) offered in exchange for a plea of guilty increases with an increase in the strength of the case \( \pi \), the sentence anticipated upon conviction at trial \( x \), and the defendant’s disutility of trial \( k \). It is unaffected by the parameters \( c, \gamma, \) and \( \lambda \).

(c) For \( \pi \in (\tilde{\pi}, 1] \), the equilibrium probability of trial \( \hat{p}(\pi) \) increases with the strength of the case \( \pi \), and the social weight \( \gamma \) given to the guilty defendant’s disutility; \( \hat{p}(\pi) \) decreases with an increase in the social cost of trial \( c \), and the social weight \( \lambda \) given to the innocent defendant’s disutility. The effects of \( k \) and \( x \) upon \( \hat{p}(\pi) \) are indeterminate because there are two conflicting effects; the direct effect is to reduce the probability that a given sentence offer \( s \) is rejected, but the indirect effect is to raise the equilibrium sentence offer \( s^* \), which in turn increases the likelihood of rejection.

An important implication of equilibrium is that when a case is not dismissed, the likelihood that is will be resolved by a guilty plea is greater the weaker is the case. This is one plausible interpretation of Alschuler’s (1968, p. 60) statement that “…the greatest pressures to plead guilty are brought to bear on defendants who may be innocent. The universal rule is that the sentence differential between guilty-plea and trial defendants increases in direct proportion to the likelihood of acquittal,” where the “sentence differential” is defined as the difference between the sentence upon conviction \( x \) and the realized sentence \( s^*(\pi) \). The reader should note that I quote Alschuler’s statement in order to invoke its empirical claim, without necessarily endorsing his terminology or his interpretation of plea bargaining as the use of pressure tactics. As I remarked in the introductory discussion, the comparative statics implications of this model are also consistent with the empirical findings of Forst and Brosi, 1977; Landes, 1971; Rhodes, 1976; and Weimer, 1978.

II. Restricted Prosecutorial Discretion

The model of Section I involved considerable discretion upon the part of the prosecutor. It has been argued that such discretion is undesirable because it gives rise to horizontal inequities: defendants charged with the same crime and subject to the same penalties upon conviction are offered different sentences in plea negotiations. As Alschuler (1968, p. 60) puts it, “the practice of bargaining hardest when the case is weakest leads to grossly disparate treatment for identical offenders” (where by “bargaining hardest” he means offering the greatest sentence differential between trial and negotiated plea). Such “inequities” are eliminated if prosecutors are constrained to offer the same plea bargain to all such defendants. Economists and some legal scholars (for example, Thomas Church, 1979) are likely to reject this notion of horizontal inequity by arguing that defendants who face a different probability of conviction are indeed “different” defendants, or by invoking an alternative notion of ex ante horizontal equity. While this controversy is lively and interesting, in Section III, I examine an alternative rationale for such a constraint; I ask whether it can improve the prosecutor’s (and society’s)
In order to answer this question, it is first necessary to characterize equilibrium behavior in a regime of restricted discretion. When the prosecutor is required to offer the same sentence to every defendant who is charged with the same crime, independent of the strength of the prosecution's case, he must make a "pooling" offer. This can result in self-selection by the guilty and innocent defendants, because they have different ex ante expected values of $\pi$. Let $E_i$ denote the defendant of type $i$'s prior expectation over $\pi$:

$$E_i = \int_0^1 \pi d\Phi(\pi|i).$$

By Assumption 2, $E_g \geq E_i$; the guilty defendant expects a greater likelihood of conviction than does the innocent defendant. If a pooled offer $s$ is made, it is the expectation $E_i$ which governs the defendant's decision.

$$\text{DU}_i(s, \rho; [0,1]) = -\rho(E_i x + k) - (1 - \rho)s.$$

The innocent defendant rejects $s$ if and only if $s > s_0 = E_i x + k$, while the guilty defendant rejects $s$ if and only if $s > s^0 = E_g x + k$. Thus any offer $s \in (s_0, s^0)$ will be rejected by the innocent and accepted by the guilty.

The prosecutor must determine an optimal sentence offer, given the anticipated behavior of the defendant types. If an offer of $s \in (s_0, s^0]$ is made, it is accepted by the guilty and rejected by the innocent; thus the best such offer is $s = s^0$, which yields ex ante expected prosecutor utility

$$U_1 = q\gamma s^0 - (1 - q)[c + \lambda(E_i x + k)].$$

Any offer $s > s^0$ is rejected by all defendants, yielding ex ante expected prosecutor utility

$$U_2 = -c + q\gamma (E_g x + k) - (1 - q)\lambda(E_i x + k).$$

Finally, an offer $s \leq s_0$ is accepted by all defendants, yielding ex ante expected prosecutor utility

$$U_3(s) = [q\gamma - (1 - q)\lambda]s.$$

When $q\gamma - (1 - q)\lambda > 0$, the optimal such offer is $s = s_0$; when $q\gamma - (1 - q)\lambda < 0$, the optimal such offer is $s = 0$ (i.e., dismiss all cases). When $q\gamma - (1 - q)\lambda = 0$, any offer is $[0, s_0]$ is optimal. A comparison of equations (5), (6), and (7) yields the following characterization of equilibrium with restricted discretion.

**PROPOSITION 3:** (A) Suppose that $\lambda x(E_g - E_i) \leq c$. Define three intervals:

- $I_{1A} = [0, \lambda/(\lambda + \gamma)];$
- $I_{2A} = [\lambda/(\lambda + \gamma), c/(\gamma x(E_g - E_i) + c)];$
- $I_{3A} = [c/(\gamma x(E_g - E_i) + c), 1].$

The optimal sentence offer with restricted discretion is $s = 0$ for $q \in I_{1A}$; that is, all cases are dismissed. The optimal offer is $s = s_0$ for $q \in I_{2A}$; that is, all cases are settled by negotiated plea at a sentence of $s_0$. Finally, the optimal sentence offer is $s = s^0$ for $q \in I_{3A}$; that is, guilty defendants plead guilty in exchange for a sentence of $s^0$, while innocent defendants go to trial. Expected ex ante prosecutor utility with restricted discretion is given by

$$EPU' = \begin{cases} 0 & q \in I_{1A}, \\ [q\gamma - (1 - q)\lambda](E_i x + k) & q \in I_{2A}, \\ q\gamma(E_g x + k) - (1 - q) \times [c + \lambda(E_i x + k)] & q \in I_{3A}. \end{cases}$$

(B) Suppose $\lambda x(E_g - E_i) \geq c$. Define two intervals: $[I_{1B} = [0, (c + \lambda(E_i x + k))]$
\[(c + \lambda(E, x + k) + \gamma(E_g x + k)); \text{ and}\]

\[
I_{2B} = \left[ (c + \lambda(E, x + k)) / (c + \lambda(E, x + k) + \gamma(E_g x + k)), 1 \right].
\]

The optimal sentence offer with restricted discretion is \(s = 0\) for \(q \in I_{1B}\); that is, all cases are dismissed. The optimal offer is \(s^0\) for \(q \in I_{2B}\); that is, guilty defendants plead guilty in exchange for a sentence of \(s^0\), while innocent defendants go to trial. Ex ante expected prosecutor utility with restricted discretion is given by

\[
EPU^* = \begin{cases} 
0 & q \in I_{1B}, \\
qy(E_g x + k) - (1-q) & q \in I_{2B},
\end{cases}
\]

It is interesting to note that when the prosecutor types separate (that is, the offer is a revealing one), the defendant types pool, but when the prosecutor types pool, the defendant types may separate (i.e., the guilty plead at \(s^0\) and the innocent go to trial). The screening outcome is much the same as that of Grossman and Katz (1983), and suffers from the same disturbing feature; the prosecutor knows that each case he takes to trial is against an innocent defendant.

III. Characterization of the Preferred Regime

In this section, we wish to compare equilibrium ex ante expected prosecutor utility in the regimes of unrestricted and restricted discretion described in Sections I and II, respectively. A convenient characterization of the circumstances under which discretion is preferred can be made in terms of the exogenous expression \(q\).

Define

\[
PU^*(\pi) = PU(\pi, s^*(\pi); p^*(s^*(\pi))).
\]

By the envelope theorem,

\[
PU^*(\pi) = \partial PU(\pi, s(\pi); p^*(s^*(\pi))) / \partial \pi = a'(\pi)(\pi x + k) + p^*(s^*(\pi))a(\pi) x > 0
\]

for \(\pi \geq \pi_0\). Thus \(PU^*(\pi) > 0\) for \(\pi > \pi_0\), while \(PU^*(\pi) = 0\) for \(\pi \leq \pi_0\). The density function for \(\pi\) can be written \(q d\Phi(\pi|g) + (1-q) d\Phi(\pi|i)\). Thus the prosecutor’s equilibrium ex ante expected utility with discretion can be written

\[
EPU^* = \int_{0}^{1} PU^*(\pi) q d\Phi(\pi|g) + \int_{0}^{1} PU^*(\pi)(1-q) d\Phi(\pi|i).
\]

Note that \(EPU^* > 0\) since \(PU^*(\pi) > 0\) for \(\pi > \pi_0\) and \(PU^*(\pi) = 0\) for \(\pi \leq \pi_0\). The expression \(EPU^*\) can be compared with ex ante expected prosecutor utility under restricted discretion \(EPU^r\) to obtain the following result.

**PROPOSITION 4:** For given values of the parameters \(E, E_g, \gamma, \lambda, c, k, \text{ and } x\), there exists a unique \(q_0 \in (0, 1)\) such that unrestricted discretion is preferred for \(q < q_0\) and restricted discretion is preferred for \(q > q_0\).

Intuitively, when the arrest process does not do a good job of screening out the innocent (i.e., \(q < q_0\)), then discretion at the prosecution stage is preferred. On the other hand, when the prosecutor can be confident that most defendants are guilty, the restriction to a uniform offer is preferred. Many factors influence the comparison between discretionary regimes. Recall that in the regime with restricted discretion, all defendants accept sentence offers at or below \(s_0 = E, x + k\), while only guilty defendants accept sentence offers in the range \((s_0, s^0]\), where \(s^0 = E_g x + k\). With a sufficiently high proportion of innocent defendants the prosecutor optimally dismisses all cases; an offer in \((0, s_0]\) is too costly in terms of punishing the innocent, while an offer in \((s_0, s^0]\) is too costly in terms of the trials generated by innocent defendants’ rejection of the plea offer. In the regime with unrestricted discretion the prosecutor can selectively dismiss cases while retaining the ability to impose penalties upon defendants who are likely to be guilty. Thus discretion is preferred. With a sufficiently high proportion of guilty defendants, the optimal sentence offer with re-
restricted discretion is $s^0$; since innocent defendants reject this offer, it is never imposed upon them. Since there are relatively few innocent defendants, the costs generated by their trials and occasional erroneous convictions are not great. With unrestricted discretion, there is always a fraction of cases involving guilty defendants which goes to trial; these trials generate court costs and occasional erroneous acquittals. Thus restricted discretion is preferred.

It is straightforward to determine which discretionary regime is preferred by each type of defendant by comparing $\text{EDU}_r^* = -\int_{\pi_0}^1 (\pi x + k) d\Phi(\pi | t)$ with

$$
\text{EDU}_r^* = \begin{cases} 0 & q \in I_{1A} \text{ or } q \in I_{1B}, \\
-\int_0^1 (\pi x + k) d\Phi(\pi | t) & q \in I_{2A}, \\
-\int_0^1 (\pi x + k) d\Phi(\pi | t) & q \in I_{3A} \text{ or } q \in I_{2B},
\end{cases}
$$

for $t = g, i$, where the sets $I_{jk}$ are those defined in Proposition 3. For $q \in I_{1A}$ or $I_{1B}$, both innocent and guilty defendants prefer unrestricted discretion; for $q \in I_{2A}$ or $I_{2B}$, both prefer restricted discretion; finally, for $q \in I_{3A}$, either both prefer unrestricted discretion (for $\pi_0$ large) or innocent defendants prefer unrestricted discretion while guilty defendants prefer restricted discretion (for $\pi_0$ small).

These results can be combined with those of Proposition 4 to show that each of the following preference patterns arises from some configuration of parameters: (1) both types of defendants prefer unrestricted discretion, while the prosecutor prefers restricted discretion; (2) both types of defendants prefer restricted discretion while the prosecutor prefers unrestricted discretion; (3) all parties unanimously prefer unrestricted discretion; (4) innocent defendants and the prosecutor prefer unrestricted discretion while guilty defendants prefer restricted discretion; and (5) guilty defendants and the prosecutor prefer restricted discretion while innocent defendants prefer unrestricted discretion. For most parameter values, there will be some disagreement among the interested parties regarding the preferred discretionary regime, so the existence of controversy is not surprising. When unanimity does occur, it favors the regime of unrestricted discretion.

IV. Conclusions

The essential features of plea-bargaining equilibrium which emerged from this analysis are that sufficiently weak cases are dismissed, where this sufficiency does not depend upon the resource cost of trial but upon the social costs and benefits of punishing the innocent and the guilty, respectively; that defendants against whom a sufficiently strong case exists are offered a sentence (in exchange for a plea of guilty) which increases with the likelihood of conviction at trial and the defendant’s anticipated disutility of trial and conviction; and finally, the defendants are more likely to reject higher sentence offers, so that the likelihood of trial is an increasing function of the strength of the case.

A uniform-offer restriction upon prosecutorial discretion was found to improve ex ante expected welfare when the proportion of guilty among those arrested is sufficiently high. Typically the prosecutor prefers the disposition of cases involving guilty defendants under restricted discretion, and prefers the disposition of cases involving innocent defendants under unrestricted discretion (see equation (A11) in the Appendix). When the proportion of guilty among those arrested is sufficiently high, the benefits from restricted discretion outweigh the costs.

Thus far I have assumed that arrest was a totally random process. However, arrest itself may convey information about the prosecutor’s case if arrest requires that the evidence exceed a certain standard. For instance, if arrest requires that the case be of strength at least $\pi_{a0}$, then the defendant of type $t$ can infer that the relevant density function for $\pi$ is $d\Phi(\pi | t)/(1 - \Phi(\pi_{a0} | t))$. In this case, the proportion of guilty among those arrested is

$$q(\pi_a) = \frac{\int_{\pi_{a0}}^1 dG(\pi, g)}{\int_{\pi_{a0}}^1 dG(\pi, g) + \int_{\pi_{a0}}^1 dG(\pi, i)}.$$
It can be shown that, for the truncated exponential distribution mentioned in Section II and examined in the Appendix, \( q'(\pi_x) > 0 \); that is, a higher arrest standard increases the proportion of guilty among those arrested. Although their statements need to be modified (in the obvious ways), Propositions 1–4 also hold with this specification of the model. This suggests that arrest standards are important determinants of the institutional forms of related aspects of the criminal justice system. One effect of low arrest standards is to make prosecutorial discretion more attractive since low arrest standards generate a higher fraction of cases likely to involve innocents, cases which must be weeded out at the prosecution stage. On the other hand, high arrest standards make restricted discretion more attractive. Of course, the arrest standard also has a direct effect upon ex ante expected social utility, since it determines the number of cases as well as the fraction likely to involve a guilty defendant. The determination of the optimal arrest standard and the associated discretionary regime is beyond the scope of this paper, but some of the relevant tradeoffs have been identified. A more difficult task is to incorporate the discovery process. One way to do this is to assume that the defendant receives a signal which is (imperfectly) correlated with the actual strength of the case. If the pros- ecutor also observes this signal, then this is basically an exercise in updating priors. Both parties simply use their posterior distributions in their decision making. If the signal is private information for the defendant, matters could become considerably more complicated.

One might plausibly argue that innocent defendants are likely to suffer more than guilty defendants in the same circumstances because their punishment is undeserved. One natural way to model this would be to assume that a sentence yielding disutility \( d \) to a guilty defendant yields disutility \( ad \) to an innocent defendant, where \( a > 1 \). This “injustice” multiplier should be present whether the sentence is imposed upon conviction at trial or upon the acceptance of a plea bargain. Since the disutility of trial is also unjustly imposed upon innocent defendants,
of this paper imply that stronger cases are more likely to go to trial (in equilibrium) even absent such a divergence between the preferences of society and the prosecutor.

Some conclusions about the likely effects of divergent objectives can be obtained by assuming that the prosecutor’s objective has the same form as that of society, but that he misperceives the parameters. For instance, if he overestimates the social value of punishment, he will pursue cases he should have dismissed, and the likelihood of trial in each case will be too low. On the other hand, if the prosecutor overestimates the social resource cost of trial, the likelihood of trial in each case will be too high. While these implications are suggestive, a more thorough investigation of alternative prosecutorial objective functions is necessary in order to make any inferences about objectives on the basis of observed behavior.

APPENDIX

Example 1: Let

\[ G(\pi, i) = \eta[1 - e^{-h\pi}]/[1 - e^{-h\pi}] \]

and

\[ G(\pi, g) = (1 - \eta)[1 - e^{-h\pi}]/[1 - e^{-h\pi}] \]

where \( h_i > h_g \) and \( \pi \in [0, 1] \). We wish to verify that Assumptions 1 and 2 (for \( \delta \) of the form \([a,b]\)) are satisfied.

ASSUMPTION 1: \( f'(\pi) > 0 \) for \( \pi \in [0,1] \). The probability of guilt conditional upon \( \pi \) is \( f(\pi) = dG(\pi, g)/dG(\pi, g) + dG(\pi, i) \). Differentiating and collecting terms implies that \( f'(\pi) > 0 \) if and only if

\[ dG(\pi, i)/d^2G(\pi, i) > dG(\pi, g)/d^2G(\pi, g) \]

For our example, this inequality becomes \(-1/h_i > -1/h_g \) or \( h_i > h_g \).

ASSUMPTION 2: \( \delta \) of the form \([a,b]\). \( E_1(\pi[a,b]) = \int_a^b \pi d\Phi(\pi|r)/[\Phi(h|r) - \Phi(a|r)] \]

\[ = a + \int_a^b \{[\Phi(h|r) - \Phi(\pi|r)]/\Phi(h|r) - \Phi(a|r)] \} d\pi. \]

For our example, \( [\Phi(h|r) - \Phi(\pi|r)]/\Phi(h|r) - \Phi(a|r)] = [e^{-h\pi} - e^{-h\pi}]/[e^{-h\pi} - e^{-h\pi}] \). Since \( w(h) = [e^{-h\pi} - e^{-h\pi}]/[e^{-h\pi} - e^{-h\pi}] \) is nonincreasing in \( h \) for \( \pi \in [a,b] \), \( h_i > h_g \) implies \( w(h_i) < w(h_g) \) for \( \pi \in [a,b] \).

Thus \( E_1(\pi[a,b]) = E_1(\pi[a,b]) \).

Proof of Proposition 1. Given \( \delta^*(s) \), is \( p^*(s) \) optimal? For \( s > \delta^*(s) = 1 \) and \( DU_i = -\rho(x + k) - (1 - \rho)s \), so \( p^*(s) = 1 \) is optimal. For \( s \in [\delta^*(s), 1] \), \( DU_i = -\rho(s - k) + (1 - \rho)s \), so any \( p \) works. Thus \( p^*(s) = 1 - \exp\{[A(s) - A(s)]/c\} \) is optimal for \( s \in [\delta^*, 1] \). For \( s \in (0, \delta^*(s)) \), \( p^*(s) = \pi_0 \) and \( DU_i = -\rho(\pi_0 x + k) - (1 - \rho)s \), so \( p^*(s) = 0 \) is optimal for \( s \in (0, \delta^*(s)) \).

Finally, for \( s = 0 \), \( \delta^*(s) = [0, \pi_0] \). Since \( E_i(\pi([0, \pi_0]) x + k) - (1 - \rho)s \) implies that \( p^*(s) = 0 \) is optimal.

Given \( p^*(s) \), is \( s^*(\pi) \) optimal? Since \( s^*(\pi) \) is the same for both defendant types, we can use the prosecutor’s payoff as described in equation (3). For \( \pi < \pi_0 \), \( PU(\pi, s; p^*(s)) < 0 \) for all positive values of \( s \) while \( PU(\pi, 0; p^*(0)) = 0 \), so \( s^*(\pi) = 0 \) is optimal for \( \pi < \pi_0 \).

For \( \pi \geq \pi_0 \), \( PU(\pi, s; p^*(s)) > PU(\pi, s; p^*(s)) \) for \( s > \delta^*(s) \) and \( PU(\pi, s; p^*(s)) > PU(\pi, s; p^*(s)) \) for \( s < \delta^*(s) \) (with equality only at \( \pi = \pi_0 \)). So \( s^*(\pi) \in [\delta^*(s), 1] \) for all \( \pi \). Differentiating \( PU(\pi, s; p^*(s)) \) with respect to \( s \) and equating to zero yields

\[ dPU/ds = \exp\{[A(s) - A(s)]/c\} \times \{[A'(s)/c] - c + a(\pi)(\pi x + k - s) \} \]

\[ + a(\pi) = 0. \]

Upon noting that \( \exp(\cdot) \) is never zero, and that \( A'(s) = a((s - k)/x) \), this simplifies to

\[ a(\pi)(\pi x + k - s) = [a((s - k)/x) - a(\pi)]. \]

Since \( a(\cdot) \) is an increasing function, whenever the left-hand side is positive, the right-hand side is negative, and vice versa. Thus the only solution is \( s^* = \pi x + k \) and \( s^*(\pi) = (\pi x + k)/x \).

Therefore, for \( s^* \), \( s^*(\pi) = (\pi x + k)/x \).

Proposition 1 to be robust to out-of-equilibrium beliefs.

Alternative Out-of-Equilibrium Beliefs. I claim that a sufficient condition for the equilibrium strategies of Proposition 1 to be robust to out-of-equilibrium beliefs is: \( \pi_0 \leq c/\lambda x \). To see this, recall that since \( E_1(\pi(\delta(s))) \geq E_1(\pi(\delta(s))) \) for all \( \delta(s) \) (by Assumption 2), if an innocent defendant strictly prefers to accept \( s \), then so does a guilty defendant. Thus (assuming identical behavior when both defendant types are indifferent), only three types of asymmetric behavior can arise: (1) \( g \) accepts \( s \) and \( i \) rejects \( s \); (2) \( g \) randomizes due to indifference and \( i \) rejects \( s \); and (3) \( g \) accepts \( s \) and \( i \) randomizes due to indifference.

Consider first the case of identical behavior. For \( s \neq 0 \), \( \delta^*(s) \subset [0,1] \). Moreover, \( \delta^*(\pi) = \pi \) for \( \pi \in (\pi_0, 1] \) and \( \delta^*(0) = (0, \pi_0) \), which is exactly the set of cases dismissed by the prosecutor (i.e., offered \( s^*(\pi) = 0 \)).

\[ PU(\pi, s; p^*(s)) = p(s)\left[ - c + a(\pi)(\pi x + k) \right] \]

\[ + [1 - p(s)]a(\pi) \]
for some \( p(s) \in [0,1] \). Note that any \( s > \delta \) is optimally rejected by both defendant types regardless of their beliefs. Let \( \text{PU}^*(\pi) = \text{PU}(\pi, s^*(\pi); p^*(s^*(\pi))) \). Since 
\[
\text{PU}^*(\pi) = a(\pi)(\pi x + k) - p^*(s^*(\pi))c > a(\pi)(\pi x + k) - c
\]
for all \( \pi \geq \pi_0 \), no such \( \pi \) would offer \( s > \delta \). Similarly, \( \text{PU}^*(\pi) > 0 \) for \( \pi < \pi_0 \), so no such \( \pi \) would offer \( s > \delta \).

For \( s \in (0, \delta) \), when the same behavior \( p(s) \) arises, \( \text{PU}(\pi, s; p(s)) < 0 = \text{PU}^*(\pi) \) for \( \pi < \pi_0 \). For \( \pi \geq \pi_0 \), notice that
\[
\text{PU}(\pi, s; p(s)) \leq p(s)[-c + a(\pi)(\pi x + k)] + [1 - p(s)]a(\pi)(\pi_0 x + k).
\]
But
\[
\text{PU}^*(\pi) = a(\pi)(\pi x + k) - p^*(s^*(\pi))c > a(\pi)(\pi x + k) - c
\]
and \( \text{PU}^*(\pi) \geq \text{PU}(\pi, s; p^*(s^*(\pi))) = a(\pi)(\pi_0 x + k) \) for all \( \pi \geq \pi_0 \). Thus no \( \pi \geq \pi_0 \) would prefer \( s \in (0, \delta) \) to it.

Recall that whenever \( s^*(s) \) is a singleton, both defendant types have identical expectations and are assumed to behave in an identical manner; thus the equilibrium behavior of Proposition 1 is robust to any out-of-equilibrium beliefs which consist only of singletons.

Next consider asymmetric behavior of type (1): \( g \) accepts \( s \) and \( i \) rejects \( s \), for \( s \in (0, \delta) \). Then
\[
\text{PU}(\pi, s; p(s), 1) = f(\pi)\{p(s)[-c + \gamma(\pi x + k)] + [1 - p(s)]\gamma s\} + [1 - f(\pi)][-c + \lambda(\pi x + k)] + [1 - p(s)\lambda s].
\]
For \( \pi \geq \pi_0 \), \( \text{PU}(\pi, s; 0, 1) = \text{PU}(\pi, s; 0, 1) - p^*(s) \leq \text{PU}^*(\pi) \). For \( \pi < \pi_0 \), a sufficient (but not necessary) condition for \( \text{PU}(\pi, s; 0, 1) > 0 = \text{PU}^*(\pi) \) is \( \pi_0 \leq c/\lambda x \).

Next consider type (2) asymmetric behavior; \( g \) randomizes using \( p_\gamma(s) \) and \( i \) rejects \( s \), for \( s \in (0, \delta) \). Then
\[
\text{PU}(\pi, s; p_\gamma(s), 1) = f(\pi)[p_\gamma(s)[-c + \gamma(\pi x + k)] + [1 - p_\gamma(s)]\gamma s] + [1 - f(\pi)][-c + \lambda(\pi x + k)] = p_\gamma(s)[a(\pi)(\pi x + k) - c] + [1 - p_\gamma(s)]\text{PU}(\pi, s; 0, 1),
\]
where \( \text{PU}(\pi, s; 0, 1) \) is given above. It was argued above that for all \( \pi, \text{PU}^*(\pi) > a(\pi)(\pi x + k) - c \) and \( \text{PU}^*(\pi) > \text{PU}(\pi, s; 0, 1) \). Thus \( \text{PU}^*(\pi) > \text{PU}(\pi, s; p_\gamma(s), 1) \) for all \( \pi \).

Finally, consider type (3) asymmetric behavior: \( g \) accepts \( s \) and \( i \) randomizes using \( p_i(s) \), for \( s \in (0, \delta) \). Then
\[
\text{PU}(\pi, s; 0, p_i(s)) = f(\pi)[p_i(s)[-c + \lambda(\pi x + k)] + [1 - p_i(s)]\lambda s] = [1 - p_i(s)]a(\pi)s + p_i(s)\text{PU}(\pi, s; 0, 1).
\]

Previous arguments imply that \( \text{PU}^*(\pi) > a(\pi)s \) and \( \text{PU}^*(\pi) > \text{PU}(\pi, s; 0, 1) \) for all \( \pi \). Thus \( \text{PU}^*(\pi) > \text{PU}(\pi, s; 0, p_i(s)) \). The parametric restriction \( \pi_0 < c/\lambda x \) was used only to show that \( \text{PU}^*(\pi) \) was sufficient to ensure this could be substituted.

**Proof of Proposition 2.** (a) Recall that \( p^*(s) = 1 - \exp\{[A(s) - A(s)]/c\} \). Note that \( A(\cdot) \) also depends upon the parameters \( k, x, \gamma, \lambda \); \( A(s; k, x, \gamma, \lambda) = f(a(s - k)/x)ds \), where \( a(\cdot) = f(\cdot)\gamma - (1 - f(\cdot))\lambda \). It follows immediately that \( p^*(s) > 0 \) and \( \partial p^*(s)/\partial c < 0 \) for \( s \in (0, \delta) \). Differentiation of \( p^*(s) \) with respect to any other parameter \( m \) yields the following formula:
\[
\partial p^*(s)/\partial m = -(1/c)\exp\{[A(s) - A(s)]/c\} \times [A'(s)(ds/dm) + \partial A(s)/\partial m - \partial A(s)/\partial m].
\]
Since
\[
A'(s) = a(\pi_0) = 0,
\]
\( \text{sgn} \partial p^*(s)/\partial m = \text{sgn} \partial A(s)/\partial m - \partial A(s)/\partial m \)
\( = \text{sgn} \partial^2 A(s)/\partial m ds \)
\( = \text{sgn} \partial a(s - k)/x/\partial m. \)

The assertions of Proposition 2 (a) then follow from the facts that
\[
\partial a((s - k)/x)/\partial k = a'((s - k)/x)(-1/x) < 0,
\]
\( \partial a((s - k)/x)/\partial x = a'((s - k)/x)(-1/x^2)(s - k) < 0, \)
\( \partial a((s - k)/x)/\partial \gamma = f((s - k)/x) > 0 \) and \( \partial a((s - k)/x)/\partial \lambda = -(1 - f((s - k)/x)) < 0. \)

(b) Recall that for \( \pi \in [\pi_0, 1], s^* = \pi x + k \); the assertions follow immediately.

(c) For \( \tilde{p}(\pi) = 1 - \exp\{[A(s) - A(s(\pi))]\} \), the assertions \( \tilde{p}'(\pi) > 0 \) and \( \partial \tilde{p}(\pi)/\partial c < 0 \) are immediate for \( \pi \in [\pi_0, 1] \). Differentiating with respect to any other
The parameter $m$ yields

$$\frac{\partial \hat{p}(\sigma)}{\partial m} = -\frac{1}{c} \exp\left\{\left[ A(s) - A(s^*(\pi)) \right]/c \right\} \times \left\{ A'(s) \frac{ds}{dm} - A'(s^*) \frac{ds^*}{dm} + \frac{\partial A(s)}{\partial m} - \frac{\partial A(s^*)}{\partial m} \right\}. $$

Again, $A'(s) = a(\sigma) = 0$, and $\frac{ds^*}{dm} = 0$ for $m = \gamma$ and $m = \lambda$, so $\text{sgn} \left\{ \frac{\partial \hat{p}(\sigma)}{\partial m} \right\} = \text{sgn} \left\{ \frac{\partial^2 A(s)}{\partial m \partial \sigma} \right\} \left[ \frac{\partial A(s)}{\partial m} - \frac{\partial A(s^*)}{\partial m} \right]$. The claims that $\frac{\partial \hat{p}(\sigma)}{\partial \gamma} > 0$ and $\frac{\partial \hat{p}(\sigma)}{\partial \lambda} < 0$ follow from the facts that $\frac{\partial a(s - k)/x}{\partial \gamma} = f(s - k)/x > 0$ and $\frac{\partial a(s - k)/x}{\partial \lambda} = -(1 - f(s - k)/x) < 0$, respectively. For $m = k, x$ there are two conflicting effects: $\text{sgn} \left\{ \frac{\partial \hat{p}(\sigma)}{\partial m} \right\} > 0$, while $\text{sgn} \left\{ \frac{\partial A(s)}{\partial m} \right\} < 0$. Thus the signs of $\frac{\partial \hat{p}(\sigma)}{\partial k}$ and $\frac{\partial \hat{p}(\sigma)}{\partial x}$ are indeterminate at this level of generality.

Proof of Proposition 3. Substituting for $s^0$ in equation (5), it is clear that $U_1 \geq U_2$; it is never optimal to take all cases to trial. Comparing $U_1$ and $U_3$ gives the following four cases:

**Case 1:** For $q \geq \max\left\{ \lambda/(\gamma + \lambda), c/(\gamma(x_E - E_x) + c) \right\}$, $s = s^0$ is optimal; the payoff is given by $U_1$.

**Case 2:** For $q \in \left[ \lambda/(\gamma + \lambda), c/(\gamma(x_E - E_x) + c) \right]$, $s = s_0$ is optimal; the payoff is given by $U_2$.

**Case 3:** For $q \in \left[ c + \lambda(x_E + k)/(\gamma(x_E + k) + c), \lambda/(\gamma + \lambda) \right]$, $s = s^0$ is optimal; the payoff is given by $U_1$.

**Case 4:** For $q \leq \min\left\{ \lambda(x_E + k)/(\gamma(x_E + k) + c), \lambda/(\gamma + \lambda) \right\}$, $s = s^0$ is optimal; the payoff is given by $U_1$.

Actually, when the interval in Case 2 is nonempty, the interval in Case 3 is empty and vice versa. This allows the simplification in Proposition 3.

Proof of Proposition 4. Define the function $\Delta(q) = \text{EPU}^*(q) - \text{EPU}'(q)$, where $\text{EPU}^*(q)$ is given by equation (10) and $\text{EPU}'(q)$ is given by equations (8) and (9) in cases (A) and (B), respectively, of Proposition 3. Restricted discretion is preferred if and only if $\Delta(q) < 0$. We need to show that there exists a unique $q_0 \in (0, 1)$ such that $\Delta(q) > 0$ for $q < q_0$, and $\Delta(q) < 0$ for $q > q_0$. Note that $\text{EPU}^*(q)$ is a linear function of $q$, while $\text{EPU}'(q)$ is a continuous and piecewise linear function of $q$. Thus $\Delta(q)$ is also a continuous and piecewise linear function of $q$. Consider first case (B) of Proposition 3. $\Delta(q) = \text{EPU}^*(q) > 0$ for all $q \in I_{2B}$. For $q \in I_{2A}$, $\Delta(q)$ can be written

$$\Delta(q) = \int_0^1 \left[ \text{EPU}^*(\sigma) - \gamma(\pi_x + k) \right] q d\Phi(\sigma | q) + \int_0^1 \left[ \text{EPU}^*(\sigma) + c + \lambda(\pi_x + k) \right] (1 - q) d\Phi(\sigma | i).$$

Since $\text{EPU}^*(\sigma) = a(\sigma)(\pi_x + k) - \hat{p}(\sigma)c < \gamma(\pi_x + k)$ for $\sigma > \pi_0$, $\text{EPU}^*(\sigma) = 0 < \gamma(\pi_x + k)$ for $\sigma \leq \pi_0$, the first integrand is negative. Since $\text{EPU}^*(\sigma) \geq 0$ for all $\sigma$, the second integrand is positive. Thus $\Delta(q) < 0$ for $q \in I_{2B}$ and $\Delta(q) > 0$ for $q \in I_{2A}$. Recall that $\Delta(q) > 0$ for $q \in I_{1B}$ and $\Delta(q) < 0$ for $q > q_0$. This proves the claim for Case (B).

Next consider Case (A) of Proposition 3. For $q \in I_{1A}$, $\Delta(q) = \text{EPU}^*(q) > 0$. For $q \in I_{1A}$, $\Delta(q)$ can be written

$$\Delta(q) = \int_0^1 \text{EPU}^*(\sigma) q d\Phi(\sigma | q) - \int_0^1 \gamma(\pi_x + k) q d\Phi(\sigma | i) + \int_0^1 \left[ \text{EPU}^*(\sigma) + \lambda(\pi_x + k) \right] (1 - q) d\Phi(\sigma | i).$$

For $q \in I_{1A}$, $\Delta(q)$ is as above in equation (A11). For $q \in I_{1A}$, $\Delta(q)$ is a linear function with $\Delta(\lambda/(\lambda + \gamma)) > 0$. Thus $\Delta(q)$ either remains positive throughout $I_{1A}$ or there exists a unique $q_0 \in I_{1A}$ such that $\Delta(q) > 0$ for $q < q_0$, and $\Delta(q) < 0$ for $q > q_0$. If $\Delta(q)$ remains positive throughout $I_{1A}$, then since $\Delta(q)$ is continuous with $\Delta(1) = 0$ and $\Delta(q) < 0$ for $q \in I_{1A}$, there exists a unique $q_0 \in I_{1A}$ such that $\Delta(q) > 0$ for $q < q_0$, and $\Delta(q) < 0$ for $q > q_0$. If $\Delta(1) = 0$ for some $q_0 \in I_{1A}$, then since $\Delta(q) < 0$ for $q \in I_{1A}$, $\Delta(q)$ remains negative thereafter. In either case, the claim follows for Case (A).

REFERENCES


Easterbrook, Frank H., “Criminal Procedure as a Market System,” Journal of Legal Stud-


