Long-lived inverse chirp signals from core collapse in massive scalar-tensor gravity

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This letter considers stellar core collapse in massive scalar–tensor theories of gravity. The presence of a mass term for the scalar field allows for dramatic increases in the radiated gravitational wave signal. There are several potential smoking gun signatures of a departure from general relativity associated with this process. These signatures could show up within existing LIGO–Virgo searches.

Introduction – General relativity (GR) has successfully passed numerous tests [1, 2] and, in the words of [3], “occupies a well-earned place next to the standard model as one of the two pillars of modern physics”. And yet, the enigmatic nature of dark energy and dark matter evoked in the explanation of cosmological and astrophysical observations [4], as well as theoretical considerations regarding the renormalization of the theory in a quantum theory sense, indicate that GR may ultimately need modifications in the low and/or high-energy regime [5].

Tests of GR have so far been almost exclusively limited to relatively weak fields. But the recent breakthrough detection of gravitational waves (GWs) by LIGO [6] has opened a new observational channel towards strong-field gravity, and tests of Einstein’s theory are a key goal of dynamic strong-field systems in massive ST theory through exploring GW generation in core collapse. The most promising range of the scalar field mass μ for generating strong scalarization and satisfying existing binary pulsar constraints has been identified as 10−15 eV ≤ μ ≤ 10−9 eV in [16]. In view of the LIGO sensitivity window 10 Hz ≤ f ≤ 105 Hz we study in this work spontaneously scalarized signals in the range 10−15 eV ≤ μ ≤ 10−13 eV and how the resulting signatures may be searched for using existing LIGO-Virgo search-strategies or, through absence of signals, used to rule out massive ST theories.

Formalism – The starting point of our formulation is the generic action for a scalar-tensor theory of gravity that (i) involves a single scalar field non-minimally coupled to the metric, (ii) obeys the covariance principle, (iii) results in field equations of at most second differential order, and (iv) satisfies the weak equivalence principle. In the Einstein frame, the action can be written in the form (using natural units G = c = 1) [5, 10]

\[ S = \frac{1}{16\pi} \int dx^4 \sqrt{-\tilde{g}} \left( \tilde{R} - 2\tilde{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 4V(\varphi) \right) + S_m[\tilde{\psi}_m, \tilde{g}_{\mu\nu}/F(\varphi)], \]

(1)

where ϕ is the scalar field, V(ϕ) the potential, and \( \tilde{R} \) and \( \tilde{g} \) are the Ricci scalar and determinant constructed from the conformal metric \( \tilde{g}_{\mu\nu} \). Matter fields, collectively denoted by \( \psi_m \), couple to the physical or Jordan-Fierz metric \( \tilde{g}_{\mu\nu} = g_{\mu\nu}/F(\varphi) \) and the physical energy momentum tensor is \( T^{\mu\nu} = (2g)^{-1/2} \delta S_m/\delta g_{\mu\nu} \), assumed here to describe a perfect fluid with baryon density \( \rho \), pressure \( P \), internal energy \( \epsilon \), enthalpy \( H \) and 4-velocity \( u^a \),

\[ T_{\alpha\beta} = \rho H u_\alpha u_\beta + P g_{\alpha\beta}, \quad H = 1 + \epsilon + P/\rho. \]

(2)
The equations of motion are given by
\[ \bar{\alpha}_{\alpha\beta} = 2 \partial_\alpha \varphi \partial_\beta \varphi - g_{\alpha\beta} \partial_\mu \varphi \partial_\mu \varphi + 8\pi T_{\alpha\beta} - 2V g_{\alpha\beta}, \]
\[ \nabla^\mu \nabla_\mu \varphi = 2 \rho (F_{\varphi} / F) T + V_{\varphi}, \]
\[ \nabla_\mu T^{\alpha\mu} = -\frac{1}{2} \frac{F_{\varphi}}{F} \bar{\alpha}^{\alpha\beta} \partial_\mu \varphi, \quad \nabla_\mu (\rho u^\mu) = 0, \quad (3) \]
where the conformal energy momentum tensor is \( \bar{T}_{\alpha\beta} = T_{\alpha\beta} / F, \) \( \nabla \) is the covariant derivative constructed from \( \bar{g}_{\mu\nu}, \) the subscript \( \varphi \) denotes \( d/d\varphi \) and the last equation arises from conservation of the matter current density in the physical frame.

Henceforth, we assume spherical symmetry, writing
\[ ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = -F^2 dt^2 + F^2 dr^2 + r^2 d\Omega^2, \quad (4) \]
where \( \alpha = \alpha(t, r), X = X(t, r) \) and we also define for convenience \( \Phi = \ln(\sqrt{F} \alpha) \) and the gravitational mass \( m = r^2 [1 - (F X^2)^{-1}] / 2. \) In spherical symmetry, the 4-velocity in the Jordan frame is
\[ u^\mu = (1 - v^2)^{-1/2} [\alpha^{-1}, v X^{-1}, 0, 0], \quad (5) \]
where the velocity field \( v \) as well as the other matter variables \( \rho, P, \epsilon \) and \( H \) are also functions of \( (t, r) \). High-resolution shock capturing requires a flux conservative formulation of the matter equations which is achieved by (cf. [23]) changing from variables \( (\rho, v, H) \) to
\[ D = \frac{\rho X F^{-3/2}}{\sqrt{1 - v^2}}, \quad S' = \frac{\rho H v F^{-2}}{(1 - v^2)}, \quad \tau = \frac{S'}{v} - \frac{P}{F^2} - D. \quad (6) \]
Finally, we introduce \( \eta = X^{-1} \partial_t \varphi \) and \( \psi = \alpha^{-1} \partial_t \varphi. \) The resulting system of equations is identical to Eqs. (2.21), (2.22), (2.27), (2.28), (2.33)-(2.39) in [23] except for the following additional potential terms (bracketed numbers denote right-hand-sides in Ref. [23])
\[ \partial_r \Phi = [2.21] - r F X^2 V, \]
\[ \partial_r m = [2.22] + r^2 V, \]
\[ \partial_r \psi = [2.28] - \alpha F V_{\varphi}, \]
\[ s_{S'} = [2.38] - r \alpha F X^2 \{ (S' v - \tau - D + F^{-2} P) \}. \quad (7) \]
where \( s_{S'} \) is the source term in the evolution of \( S'. \) All other equations in the above list remain unaltered.

We have implemented these equations by adding the potential terms to the GR1D code originally developed in [24] and extended to massless ST theory in [23]. As in [23], we use a phenomenological hybrid equation of state (EOS) \( P = P_e + P_{th}, \epsilon = \epsilon_e + \epsilon_{th} \) with the cold part
\[ \rho \leq \rho_{\text{max}}: \quad P_e = K_1 \rho^{\Gamma_1}, \quad \epsilon_e = \frac{K_1}{\Gamma_1 - 1} \rho^{\Gamma_1 - 1}, \]
\[ \rho > \rho_{\text{max}}: \quad P_e = K_2 \rho^{\Gamma_2}, \quad \epsilon_e = \frac{K_2}{\Gamma_2 - 1} \rho^{\Gamma_2 - 1} + E_3, \quad (8) \]
where \( \rho_{\text{max}} = 2 \times 10^{14} \text{g cm}^{-3}, K_1 = 4.9345 \times 10^{14} \text{[cgs]}, K_2 \) and \( E_3 \) follow from continuity; \( \epsilon_{\text{th}} \) measures the departure of the evolved internal energy \( \epsilon \) from the cold contribution and generates a thermal pressure component \( P_{th} = (\Gamma_{th} - 1) \rho \epsilon_{th}. \) We thus have three parameters to specify the EOS. As in [23], we consider \( \Gamma_1 = \{1.28, 1.3, 1.32\} \) for the subnuclear, \( \Gamma_2 = \{2.5, 3\} \) for the supernuclear EOS and \( \Gamma_{th} = \{1.35, 1.5\} \) for the thermal part describing a mixture of relativistic and non-relativistic gases. For the conformal factor, we use the quadratic Taylor expansion commonly employed in the literature [11, 25] and the potential endows the scalar field with a mass \( \mu, \)
\[ F = \exp(-2 \alpha_0 \varphi - \beta_0 \varphi^2), \quad V = h^{-2} \mu^2 \varphi^2 / 2. \quad (9) \]
The discretization, grid and boundary treatment are identical to those described in detail in Sec. 3 of [23].

**Code tests** – In order to test the extended code, we have repeated the convergence analysis displayed in Fig. 3 of [23] but now using a massive scalar field with \( \mu = 10^{-12} \text{eV} \) and \( \alpha_0 = 10^{-4} \) and \( \beta_0 = -20. \) We observe the same convergence between first and second order.

As a further test, we have evolved the 12 \( M_{\odot} \) zero-age-main-sequence progenitor WH12 of the catalog of realistic pre-SN models [26] for the same \( \mu, \alpha_0 \) and \( \beta_0, \) employing a uniform grid with \( \Delta r \) inside \( r = 40 \text{ km} \) and logarithmically increasing grid spacing up to the outer boundary at \( 1.8 \times 10^5 \text{ km}. \) Convergence of \( r \varphi \) extracted at \( r_{ex} = 3 \times 10^9 \text{ cm} \) is tested with three different resolutions \( \Delta r_1 = 250 \text{ m}, \Delta r_2 = 125 \text{ m}, \Delta r = 62.5 \text{ m} \) in the interior and a total number of \( N_1 = 5000, N_2 = 10000, N_3 = 20000 \) grid points, respectively, so that the differences between high, medium and low resolution are expected to scale with \( Q_1 = 2 \) for first and \( Q_2 = 4 \) for second-order convergence. This expectation is borne out by Fig. 1 where we study the convergence of the strong peak signal generated at core bounce at \( t = r_{ex} \approx 38 \text{ ms} \) which dominates all our wave signals. The good agreement between the solid and dotted curves demonstrates convergence close to second order and implies a discretization error of about \( 6 \% \) \( (3 \%) \) for coarse (medium) resolution. Henceforth, we use \( \Delta r = 166 \text{ m} \) and extend the outer grid to \( 9 \times 10^5 \text{ km} \) while keeping the resolution in the extraction zone unchanged.

**Simulations** – All simulations presented here start with the WH12 model with initially vanishing scalar field. The evolution is then characterized by six parameters: the above mentioned EOS parameters \( \Gamma_1, \Gamma_2 \) and \( \Gamma_{th} \) as well as mass \( \mu \) of the scalar field and \( \alpha_0, \beta_0 \) in the conformal function which we vary in the ranges
\[ 0 \leq \mu \leq 10^{-13} \text{eV}, \quad 10^{-4} \leq \alpha_0 \leq 1, \quad -25 \leq \beta_0 \leq -5. \quad (10) \]
Our observations in these simulations are summarized as follows. (i) The collapse dynamics are similar to the scenario displayed in the left panels of Fig. 4 in [23]. As conjectured therein, the baryonic matter strongly affects the scalar radiation but itself is less sensitive to the scalar field. (ii) For sufficiently negative \( \beta_0 \) the scalar field reaches amplitudes of order unity, independent of
the collapsing stars in massive ST theory (as compared with the more strongly constrained massless case) and the resulting substantially larger GW signals are one of the key results of this work. Translating this increase into improved observational signatures for GW detectors, however, requires careful consideration of the signal’s dispersion as it propagates from source to detector; this is the subject of the remainder of this letter.

**Wave extraction and propagation** – At large distances from the source, the dynamics of the scalar field are well approximated by the flat-space equation,

\[ \partial^2_t \varphi - \nabla^2 \varphi + h^{-2} \mu^2 \varphi = 0. \]  

(11)

Plane–wave solutions propagate with phase and group velocities \( v_{g/p} = [1 - (\omega^2 / \sigma^2)]^{1/2} \) for angular frequencies above \( \omega_s \equiv \mu / h \), but are exponentially damped for lower frequencies. In spherical symmetry, Eq. (11) reduces to a 1D wave equation for \( \sigma \equiv r \varphi \).

In the massless case (\( \mu = 0 \)) the general solution for \( \sigma \) is the sum of an ingoing and an outgoing pulse propagating at the speed of light. This makes interpreting the output of core collapse simulations particularly simple; one extracts the scalar field at some extraction radius sufficiently large that the dynamics are governed by Eq. (11), \( \sigma(t; r_{\text{ex}}) \), and after imposing outgoing boundary conditions the signal at \( r > r_{\text{ex}} \) is \( \sigma(t; r) = \sigma(t - (r - r_{\text{ex}}); r_{\text{ex}}) \).

In the massive case, the situation is complicated by the dispersive nature of wave propagation. However, an analytic solution for the field at large radii can still be written down, albeit in the frequency domain:

\[ \tilde{\sigma}(\omega; r) \equiv \int dt \sigma(t; r) e^{i \omega t}. \]

The boundary conditions need to be modified for the massive case; frequencies \( |\omega| > \omega_s \) propagate and we continue to impose the outgoing condition for these, however frequencies \( |\omega| < \omega_s \) are exponential and we impose that these modes decay with radius. These conditions determine the Fourier transform of the signal at large radii in terms of the signal on the extraction sphere (note the \( \omega \) ranges in Eq. (12)),

\[ \tilde{\sigma}(\omega; r) = \tilde{\sigma}(\omega; r_{\text{ex}}) \begin{cases} e^{-i \sqrt{\omega^2 - \omega_s^2}(r-r_{\text{ex}})} & \text{for } \omega < -\omega_s \\ e^{+i \sqrt{\omega^2 - \omega_s^2}(r-r_{\text{ex}})} & \text{for } \omega > -\omega_s \end{cases}. \]  

(12)

Note that the power spectrum, \( |\tilde{\sigma}(\omega; r)|^2 \), is unchanged during propagation except for the exponential suppression of frequencies \( \omega < \omega_s \).

As signals propagate, they spread out in time, but the frequency content above the critical frequency \( \omega_s \) remains unchanged. Consequently, the number of wave cycles in the signal increases with propagation distance. In the limit of large distances (relevant for LIGO observations of galactic supernovae) the signals are highly oscillatory, i.e., the phase varies much more rapidly than the frequency, and the inverse Fourier transform of Eq. (12) may be evaluated in the stationary phase approximation (SPA [27]). At each instant the signal is quasi-monochromatic with frequency

\[ \Omega(t) = \omega_s / \sqrt{1 - [(r - r_{\text{ex}})/t]^2} \quad \text{for } t > r - r_{\text{ex}}. \]  

(13)
This time–frequency structure sounds like an inverse chirp, with high frequencies arriving before low ones. The origin of this structure can be understood by noting that each frequency component arrives after the travel time of the associated group velocity. Using the SPA the time domain signal is given by

\[ \sigma(t, r) = A(t, r) \cos(\phi(t, r)), \]

and the SPA frequency, \( \Omega(t) \), is given by Eq. (13). The Jordan frame metric perturbation is determined by the scalar field \( \varphi \) (the tensorial GW degrees of freedom vanish in spherical symmetry). Any GW detector, small compared to the GW wavelength \( \lambda = 2\pi / \omega \), measures the electric components of the Riemann tensor; \( R_{0i0j} \) [2]. In massless ST theory this 3-tensor is transverse to the GW wavevector, \( R_{0i0j} \propto \delta_{ij} - k_i k_j \), with strain amplitude \( h_B = 2\omega_0 \varphi \) (this is called a breathing mode). In massive ST theory there is an additional longitudinal mode, \( R_{0i0j} \propto k_ik_j \), with suppressed amplitude \( h_L = (\omega_0 / \omega)^2 h_B \). A GW interferometer responds identically (up to a sign) to both of these polarizations meaning they cannot be distinguished [2]; henceforth we refer to the overall measurable scalar signal with amplitude \( h_S = h_B - h_L = 2\alpha_0 [1 - (\omega_0 / \omega)^2] \varphi \). In practice this factor reduces the strain only by at most a few % at \( t \lesssim 10^8 \) s.

**LIGO observations** – GW signals from stellar collapse in ST theory may show up in several ways in existing LIGO–Virgo searches. In each case there is, in principle, a smoking gun which allows the signal to be distinguished from other types of sources. Here, it is argued that a new dedicated program to search for ST core collapse signals is not needed; however, the results of this work should be kept in mind in analyzing results from existing searches. Monochromatic searches – The highly dispersed signal (described by Eq. (14), see right–hand panels of Fig. 3) at large distances can last for many years and is nearly monochromatic on timescales of \( \lesssim 1 \) month.

These signals may be detected by existing monochromatic searches provided the data is analyzed in segments sufficiently small that the frequency can be approximated as constant in each segment (the frequency change from segment-to-segment allows the scalar mass, \( \mu \), to be measured). These signals may show up in all–sky searches, however greater sensitivities can be achieved via directed searches at known nearby supernovae (all–sky searches achieved sensitivities that constrain \( h \lesssim 9.7 \times 10^{-25} [28] \), whereas model–based, directed searches at a supernova remnant have achieved sensitivities of \( h \lesssim 2.3 \times 10^{-25} [29] \) at frequencies \( \sim 150 \) Hz). Methods to detect signals of any polarization content have recently been presented in [30]; note that interferometers are a factor 2 less sensitive to scalar than to tensor GWs. A directed search should begin within a few months to years of the supernova observation and may last for decades with sensitivity improving as time\(^{-1/2} \) (see the amplitude as a function of time in Fig. 3). In fact, the amplitude can remain at detectable levels for so long that directed searches aimed at historical nearby supernovae (e.g. SN1987A\(^{1} \)) may be worthwhile; a non-detection from such a search can place the most stringent constraints to date on certain regions of the massive ST parameter space, \( (\mu, \alpha_0, \beta_0) \).

In any monochromatic search there would be two smoking gun features indicating an origin of hyper-scalarized core–collapse in massive ST theory: the scalar polarization content, and the long signal duration with gradual frequency evolution according to Eq. (13). Our simulations suggest that the intrinsic amplitude of the scalar field is insensitive to \( \alpha_0, \beta_0 \) and \( \mu \) over wide parameter ranges. However, the GW strain scales linearly with the coupling; \( h \propto \alpha_0 \varphi \). Extrapolating the results in Fig. 3 suggests that if a supernova at 10 kpc were to be observed and followed up by a directed monochromatic search by aLIGO at design sensitivity, the coupling could be constrained to \( \alpha_0 \lesssim 3 \times 10^{-4} \) (assuming no signal was in fact observed) which compares favorably with the impressive Cassini bound in the massless case [15].

**Stochastic searches** – As shown above, stellar core collapse in massive ST theory can generate large amplitude signals, allowing them to be detected at greater distances. However, the signals propagate dispersively, spreading out in time and developing a sharp spectral cut–off at the frequency of the scalar mass. The long duration signals from distant sources can overlap to form a stochastic background of scalar GWs with a characteristic spectral shape around this frequency. A detailed analysis of this stochastic signal will be presented in [31].

**Burst searches** – If the scalar field is light (\( \mu \lesssim 10^{-20} \) eV) then signals originating within the galaxy will not be significantly dispersed (i.e. the spread in arrival times across the LIGO bandwidth, \( (10–10^4) \) Hz, for a source at 10 kpc is \( \lesssim 1 \) s). These short duration, burst-like scalar GW signals may be detected using strategies similar to those used to search for standard core collapse supernovae in GR. However, for these light scalar fields the observational constraints on the coupling constants \( \alpha_0 \) and \( \beta_0 \) rule out the hyper-scalarized signals shown in Fig. 2 and the amplitudes are similar to those reported in [23].

**Discussion** – The main results of our work are the following points. (i) Weaker constraints on the coupling parameters \( \alpha_0, \beta_0 \) in ST theory with scalar masses \( \mu \gtrsim 10^{-15} \) eV allow for scalarization in stellar core collapse orders of magnitude above what has been found in massless ST theory. The scalar signature is rather insensitive to the EOS parameters and varies only weakly with the ST parameters \( \alpha_0 \) and \( \beta_0 \) for sufficiently negative \( \beta_0 \). (ii) The strong scalar GW signal disperses as it propagates over astrophysical distances, turning it into an inverse

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1 For \( \mu = 10^{-14} \) eV, for example, we obtain for SN1987A a frequency \( \Omega/(2\pi) \approx 128 \) Hz and rate of change \( \dot{\Omega}/(2\pi) \approx 2 \) Hz/yr, using distance \( D := r - r_{\text{ex}} = 51.2 \) kpc and time \( t - D = 30 \) yr.
chirp signal spread out over years with a near monochromatic signature on timescales of \( \sim 1 \) month. (iii) We identify three existing GW search-strategies (continuous wave, stochastic and burst searches) that have the capacity to observe these signals for galactic sources or infer unprecedented bounds on the massive ST theory’s parameter space through non-detection.

A significant consequence of the dispersion is that the signal to be detected is largely insensitive to details of the original source. Instead, it is mainly characterized by the overall magnitude of the scalarization and the ST parameters, most notably the mass \( \mu \). We tentatively conjecture that other prominent astrophysical sources, such as NS binary inspiral and merger, may result in a similar inverse-chirp imprint on the GW signal in massive ST theory. A natural extension of our work is the exploration of other theories of gravity with massive degrees of freedom (e.g. \[32\]), but the results reported here already demonstrate the qualitatively new range of opportunities offered in this regard by the dawn of GW astronomy.

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