Appropriative Water Rights and the Efficient Allocation of Resources

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Historically, water rights to surface water in the United States have developed under two distinct legal doctrines—the English common law notion of riparian rights and the appropriative doctrine. Generally speaking, the riparian doctrine forms the basis for water law in the eastern states, while the western states have adopted the appropriative doctrine. Under the riparian doctrine, each property owner fronting on a lake or stream has a right to the unimpaired use of the waterway, regardless of the location of his property along the waterway and regardless of the time at which the property is acquired or use made of the waterway. Consequently, rights to water are only usufructuary: strictly speaking the right holder may not diminish the flow of water by physically consuming it as this would impair the rights of other riparians.

In practice the courts have held that "reasonable" diversions of water by riparian rights holders are permissible, but there are still severe restrictions on such diversions, coupled with uncertainty as to how a court will view any specific diversion. As a practical matter, the riparian doctrine is especially suited to an environment in which the use of water involves no diversions, for example, in the use of a stream for fishing, swimming, boating, transportation, or power generation.

In contrast, under the appropriative doctrine the right to a certain amount of water is established and maintained only through use; if there is a lapse in usage or a change in the nature of the usage, the right

to the water can be lost.[1] Moreover, the right enables the holder to physically consume the water to which he is entitled, provided it is put to a beneficial use. Seniority of rights is based on the chronological order in which the right was obtained, the earliest user of water along a waterway being the most senior rights holder with priorities superior to those of junior rights holders. Under the appropriative doctrine, "first in time means first in right."

The appropriative doctrine was adopted in the West (and is spreading to eastern states as well) because it is well suited to the exploitation of a waterway under conditions in which the major uses of the waterway involve physical diversions of water, say for irrigation or for municipal or industrial uses. There are obvious advantages under such circumstances to a system of rights based on the appropriative doctrine, as discussed in the authors (1976); Charles Meyers; Jerome Milliman. In particular, an allocation of rights based on the appropriative doctrine preserves incentives for investment that would be foregone under the riparian scheme because of the common property characteristics of water under riparian allocation of rights.

This is not to say that the appropriative doctrine is without drawbacks from the point of view of economic efficiency. For example, inefficiencies can arise under the appropriative doctrine when an individual diverts more water than he can presently use profitably in order to establish a right to the use of such water in the future when the use might be profitable. To protect against this, state water laws limit appro-

[1] For a discussion of the legal principles involved under the riparian and appropriative doctrines, see Richard Dewsnut and Dallice Jensen.

priative rights to diversions that qualify as "beneficial consumptive use," thus excluding wasteful types or methods of water use. But there are obvious difficulties in establishing that water is being wasted by a rights holder, so that the protection afforded by the restriction of appropriations to beneficial consumptive use might be more illusory than real.[2]

Most of the allocative problems associated with the appropriative doctrine would be eliminated if water rights could be freely transferred or sold. But in every state operating under the appropriative doctrine, there are limitations on the transfer and sale of water rights. The statutes apply with most force to transfers that involve a change in use or in diversion location, as, for example, in the transfer of a water right from irrigation to industrial or power use, or in the transfer of water outside the property limits of the original rights holder.[3] Moreover, even when restrictions on intrastate transfers are relatively weak, sale or transfer of a water right that involves removal of water to another state is a practical impossibility, at least in the western states.

Independent of these statutory restrictions the appropriative doctrine provides an interesting scenario for the analysis of the efficiency aspects of water allocation.

In this paper, we examine the efficiency implications of the appropriative doctrine at a long-run competitive equilibrium under simplified assumptions as to the legal status of water rights.[4] Briefly, our conclusions are the following. In the absence of a competitive market for the purchase and sale of water rights, the appropriative doctrine leads to an inefficient use of water. Inefficiency arises under the appropriative doctrine because of the unequal sharing of risks among the users of a waterway; senior appropriators bear less risk than junior appropriators. As an application of the Coase theorem, the introduction of a competitive market in water rights and use of diversion facilities eliminates allocative inefficiencies. However, increasing returns in the construction and maintenance of diversion facilities interferes with the establishment of competitive markets in the leasing of diversion capacity; furthermore, monopoly problems can arise in the market for water rights as well. Beyond this, limitations on entry can lead to problems associated with a suboptimal investment in diversion capacity.

For the special case of a waterway utilized by firms with identical production functions, allocative efficiency requires the equal sharing of risk (hence water) by all firms. But an assignment of water rights on the basis of equal sharing (a variant of the legal doctrine of correlative rights) leads to

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[2]Meyers and A. Dan Tarlock cite a case in which use of water during the off-season to flood gophers from their holes was not deemed beneficial consumptive use. Furthermore, some court decisions have specified maximum amounts of water usage per irrigated acre that qualify as beneficial consumptive use. On the other hand, a large unnamed western irrigation district loses from 150,000 to 500,000 acre-feet of water yearly due to seepage in an unlined diversion canal, a method of use which could be considered wasteful. As this amount is included in its rights total, should it decide to line the canal, it could use the salvaged water.

[3]For example, in 1974, the Metropolitan Water District (MWD) of Southern California was able to transfer a portion of its rights to Colorado River water to the Southern California Edison Company, but only after the passage of enabling legislation by the California State Legislature, as Southern California Edison intended to use this water outside of the geographic limits of the MWD.

[4]For example, many existing statutes allow for revisions in priority in times of drought. As a consequence, junior domestic and municipal or industrial users might be satisfied prior to senior agricultural users although not without compensation. We ignore this complication in our analysis. The relevance of this point arises in conjunction with the recent western drought and the prediction of lower long-term water availability (for example, as suggested from the tree ring studies performed by the Lake Powell Research Group relative to the Colorado River). It is difficult to assess the importance of these matters, at least in the case of the Colorado River, as large accumulations of stored water and the Bureau of Reclamation's implicit policy of releasing enough water to satisfy all downstream users (which in total are limited to mean stream flow) suggest that it will be quite a while until such constraints become effective. We explore these questions to some extent in our 1977 paper.

much the same common property problems as the riparian scheme; and similar difficulties arise for the case of firms using diverse technologies. Thus, in the absence of freely transferrable property rights, the appropriative doctrine leads to an allocation of water that is inefficient, but alternative schemes for assigning water rights are generally not incentive compatible with a competitive environment.

I. The Model

The problems that arise for an efficient allocation of water under the appropriative doctrine rest ultimately on the random nature of water flows. We consider the case of a waterway with a flow of x acre-feet per year, where x is a random variable with known probability density function $f(x)$.[5] For simplicity, we ignore the autocorrelation of streamflows over time and concentrate instead on the characteristics of a steady-state situation. We assume that there are a number of potential users of the stream and that no institutional barriers to entry exist, except those associated with the rights of senior appropriators.

Under the appropriative doctrine, rights to water are established only through use. In order to use a_i units of water each period, the ith appropriator must have access to a diversion facility with a capacity at least equal to a_i units. In particular, firm i is assumed to possess a profit function $\pi^i(a_i, \bar{a}_i)$ where a_i is the use of water per period by firm i and $\bar{a}_i$ is the capacity of the diversion facility owned by firm i, subject to the restriction $a_i \leqq \bar{a}_i$. For simplicity we ignore other factors of production although clearly substitution of other factors for water could play an important role in the production process, particularly for firms with relatively junior rights.

We assume that there is no charge to an appropriator for the water he uses,[6] that $\pi^i_1 \equiv \partial \pi^i / \partial a_i > 0$, $0 \leq a_i \leq \bar{a}_i$, $\pi^i_1 = 0$ otherwise, and $\pi^i_{11} \equiv \partial^2 \pi^i / \partial a_i^2 < 0$. Costs incurred in production are associated with the construction and maintenance of diversion facilities. It is clear that there are certain economies of scale associated with facilities such as pipelines and aqueducts. We assume that such nonconvexities apply for a certain range, after which problems of coordination, etc., overwhelm the natural economies of scale. In particular, we assume that $\pi^i_2 \equiv \partial \pi^i / \partial \bar{a}_i < 0$, for $\bar{a}_i \geq 0$; $\pi^i_{22} \equiv \partial^2 \pi^i / \partial \bar{a}_i^2 > 0$, for $\bar{a}_i < \bar{a}_i^*$; $\pi^i_{22} < 0$ for $\bar{a}_i > \bar{a}_i^*$. Moreover, in most of what follows we will assume that the profit-maximizing choices of diversion capacities occur in the range $\bar{a}_i > \bar{a}_i^*$, so that the marginal cost of adding diversion capacity is increasing. Finally, it is convenient to assume that diversion facilities deteriorate only through aging and not through use, so that $\pi^i_{12} \equiv \partial^2 \pi^i / \partial a_i \partial \bar{a}_i = 0$. Under this assumption the profit function is separable in a_i and $\bar{a}_i$, so that $\pi^i(a_i, \bar{a}_i) = R^i(a_i) - C^i(\bar{a}_i)$, where R^i and C^i are the revenue and cost functions for the ith firm.

Our primary purpose is to identify the sources of allocative inefficiency associated with the appropriative doctrine. These sources are most easily identified in the simplest possible setting. Hence our approach in the body of this paper is to examine in detail the special case where all appropriators have identical profit functions, with each appropriator acting to maximize expected profits. Extensions of these results to cases of dissimilar or risk-averse firms are noted when of interest.

With this as background we examine the long-run equilibrium of a waterway being exploited under the system of appropriative rights. We label rights holders in order of seniority, with firm 1 being the most senior rights holder, firm 2 second in seniority, etc. Clearly, in long-run equilibrium with

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[5] We assume $f(x) \geq 0$ for $x \geq 0$, $f(x) = 0$ for $x < 0$. Letting

$$F(x) = \int_0^x f(c)\, dc$$

we have $F(0) = 0$ and $\lim_{x \to \infty} F(x) = 1$.

[6] Generally, the presence of charges do not affect the results, hence the simpler formulation; note that delivery charges are incorporated into the costs of constructing and maintaining diversion facilities.

known probability density function $f(x)$, no expected profit-maximizing firm would acquire a diversion facility with capacity in excess of its rights to use water; moreover, rights in excess of diversion capacity would not be approved by the state rights administrator. Hence $\bar{a}_i$ can be identified as the appropriative rights of firm i. As a matter of notation, let

$$A_i = \sum_{j=1}^{i} \bar{a}_j$$

Then A_i denotes the aggregate amounts of claims to water senior to the claims of firm $i + 1$; alternatively, A_i is the total amount of diversion capacity owned (or leased) by firms 1 through i ($A_0 = 0$).

Identifying water rights with diversion capacities, the assignment of rights under the appropriative doctrine can be summarized in the vector $(\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_N)$, where there are N firms exploiting a stream. Expected profits for firm i, $E^i\pi$, are then given by

$$E^i\pi = F(A_{i-1})\pi(0, \bar{a}_i)$$
$$+ \int_{A_{i-1}}^{A_i} \pi(x - A_{i-1}, \bar{a}_i)f(x)\,dx$$
$$+ [1 - F(A_i)]\pi(\bar{a}_i, \bar{a}_i)$$

where $\pi^i = \pi$ for $i = i, \ldots, N$ (all firms are identical) and $F(c) = \int_0^c f(x)\,dx$. Thus, firm i receives zero units of water if the streamflow x is no more than enough to satisfy senior claimants; the probability that river flows do not exceed A_{i-1} is $F(A_{i-1})$ while profits for the ith firm in this case are $\pi(0, \bar{a}_i)$, so that the expected value of this outcome is represented by the first term in the expression for expected profits. If the flow exceeds senior claims and can be handled by firm i's diversion capacity, then expected profits are given by the second term in this expression; that is, expected profits are $\pi(x - A_{i-1}, \bar{a}_i)$ times $f(x)$ summed over the interval of river flows which yield increasing amounts of water to the ith firm. If river flow exceeds the capacity of claimants 1 through i, then the ith firm receives its entire appropriation. The probability of this occurrence is $1 - F(A_i)$ and ith firm profits are $\pi(\bar{a}_i, \bar{a}_i)$, hence the third term in the expression.

II. Water Rights and Water Usage: Appropriative System

Clearly, senior claimants obtain a preferred position due to their priority in access to the streamflow. Let x_i denote the flow available for use by firm i and let $G^i(x_i)$ denote the cumulative probability distribution over this flow. Then $G^i(x_i)$ is given by

$$G^i(0) = F(A_{i-1})$$
$$G^i(x - A_{i-1}) = F(x), \quad A_{i-1} \leq x \leq \infty$$

Since $A_j > A_i$ for $j > i$, it follows that $G^i(b) \leq G^j(b)$ for $b \geq 0$, $j > i$, with strict inequality for $b \geq A_{i-1}$, assuming $f(x) > 0$ for $A_{i-1} \leq x \leq A_i$.

Hence the probability distribution of streamflows facing a junior appropriator is *stochastically dominated* (in the sense of first-degree stochastic dominance) by the distribution facing any senior appropriator. Under the assumption of positive marginal profitability of water use ($\pi_1(a_k) > 0$ for $a_k \geq 0$, $k = i, j$) stochastic dominance implies that for any monotonically increasing measurable utility function U over profits,

$$E_{G^i}U(\pi) > E_{G^j}U(\pi)$$

for $i < j$ (see Quirk and Rubin Saposnik or Josef Hadar and W. R. Russell). This result can be summarized as follows:

PROPOSITION 1: *Under the appropriative doctrine, the probability distribution facing any senior appropriator is unambiguously preferred by any potential user of the waterway to that facing a junior appropriator.*

To analyze the consequences of the appropriative doctrine for the allocation of water among potential users, we consider the distribution of rights that would arise under stationary conditions, assuming free entry coupled with an absolute prohibition on the sale or transfer of water rights to other water users or alternative uses. Given that senior claims $(\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_{i-1})$ exist, firm i chooses the diversion capacity $\bar{a}_i$ which maximizes the expected utility of

profits. Assuming that firm i is risk neutral, expected utility maximization implies expected profit maximization where

$$
E^i\pi = F(A_{i-1})\pi(0,\bar{a}_i)
$$
$$
+ \int_{A_{i-1}}^{A_i} \pi(x - A_{i-1}, \bar{a}_i) f(x)dx
$$
$$
+ [1 - F(A_i)]\pi(\bar{a}_i, \bar{a}_i)
$$

For $\bar{a}_i > 0$ we have

$$
\frac{dE^i\pi}{d\bar{a}_i} = F(A_{i-1})\pi_2(0,\bar{a}_i)
$$
$$
+ \int_{A_{i-1}}^{A_i} \pi_2(x - A_{i-1}, \bar{a}_i) f(x)dx
$$
$$
+ [1 - F(A_i)\{\pi_1(\bar{a}_i,\bar{a}_i)
$$
$$
+ \pi_2(\bar{a}_i,\bar{a}_i)\} = 0
$$

Under separability of the profit function ($\pi_{12}^i = 0$) we can write $\pi_1(z, w) = \pi_1(z)$ and $\pi_2(z, w) = \pi_2(w)$ and the first-order condition reduces to

$$
\pi_2(\bar{a}_i) + [1 - F(A_i)]\pi_1(\bar{a}_i) = 0
$$

which we write as

$$
M_i(\bar{a}_i) = \pi_2(\bar{a}_i)
$$
$$
+ [1 - F(A_{i-1} + \bar{a}_i)]\pi_1(\bar{a}_i) = 0
$$

At a regular maximum of expected profits we have $\partial M_i(\bar{a}_i)/\partial \bar{a}_i < 0$. Observing that

$$
\left(\frac{\partial M_i(\bar{a}_i)}{\partial A_{i-1}}\right)_{\bar{a}_i \text{constant}} = -\pi_1(\bar{a}_i)f(A_i) < 0
$$

we have

PROPOSITION 2: *Given two expected profit-maximizing firms with identical separable profit functions, the firm with senior rights claims a larger quantity of water (constructs a larger diversion capacity) than does a firm with junior rights.*

The incentive rationale underlying Proposition 2 is obvious since senior firms face preferred probability distributions over streamflows relative to junior firms. Already there is some indication of the allocative inefficiency of the appropriative system in the absence of a competitive market for water rights. At an optimum, firms with identical production and profit functions should presumably divert and use identical

amounts of water. The appropriative system biases the distribution of water use in favor of firms with earlier filing dates for water rights over firms filing later in time.

Proposition 2 generalizes directly when firms are risk-averse expected utility maximizers. If firms face different technologies, then the ordering of diversion capacities does not generalize. However, the intent of the proposition does. Fundamentally, the first-order conditions imply that with all firms operating in competitive output markets, junior appropriators are more productive at the margin in the sense that the ratio of marginal revenue to marginal cost increases with decreasing seniority. If firms are identical, this implies that senior firms appropriate more water.

An issue of some importance to allocative efficiency of the appropriative doctrine is the extent to which the flow of a waterway is appropriated. To put it in other terms, how much diversion capacity is built under the system of appropriative rights, assuming each firm builds its own capacity?

Let N denote the number of appropriators exploiting a waterway at a long-run competitive equilibrium so that the last firm just finds it worthwhile to appropriate a portion of the stream by building a diversion capacity. Assuming risk neutrality and separable profit functions for all firms, if the waterway is appropriated by N firms we have $E^N\pi \geq 0$ for $\bar{a}_N > 0$, $E^{N+1}\pi < 0$ for $\bar{a}_{N+1} > 0$, where

$$
\pi_2(\bar{a}_N) + [1 - F(A_N)\pi_1(\bar{a}_N) = 0
$$
$$
\pi_{22}(\bar{a}_N) - f(A_N)\pi_1(\bar{a}_N)
$$
$$
+ [1 - F(A_N)]\pi_{11}(\bar{a}_N) \leq 0
$$

From the first-order condition we have

$$
F(A_N) = 1 + \frac{\pi_2(\bar{a}_N)}{\pi_1(\bar{a}_N)}
$$

where $\pi_2(\bar{a}_N) < 0$, $\pi_1(\bar{a}_N) > 0$ for all $\bar{a}_N \geq 0$. The entire stream is completely appropriated only if $\bar{a}_N \to 0$ with

$$
\lim_{\bar{a}_N \to 0} \pi_1(\bar{a}_N) = +\infty
$$

So long as π_1 is bounded from above, the total amount of diversionary capacity

built is less than the maximum flow of the stream.

Given a neoclassical production function and a competitive output market for the firm's product, then

$$\lim_{\bar{a}_N \to 0} \pi_1(\bar{a}_N) = +\infty$$

This does not necessarily guarantee a completely appropriated stream, however. In fact, under extreme conditions of scale economies in diversion capacity

$$\lim_{\bar{a}_N \to 0} \pi_2(\bar{a}_N) = -\infty$$

We will assume that increasing returns "dominate" for small diversion capacities in the sense that

$$\lim_{\bar{a}_i \to 0} \left(\frac{-\pi_2(\bar{a}_i)}{\pi_1(\bar{a}_i)} \right) > 1$$

Under such circumstances, the number of firms exploiting the stream is finite, with each firm of noninfinitesimal size.

PROPOSITION 3: *If increasing returns dominate for small diversion capacities and if the potential users of a stream are risk neutral with identical separable profit functions, then the aggregate amount of water rights (diversion capacity) at a long-run competitive equilibrium is less than the maximum flow of the stream; further, each appropriator is of noninfinitesimal size, and the number of appropriators is finite.*[7]

COROLLARY: *Under the conditions of Proposition 3, the expected value of stream-flows exceeds the expected value of diversions.*

III. Allocative Inefficiency of the Appropriative System

Consider next a waterway operating in long-run competitive equilibrium, exploited

by N risk-neutral firms with identical separable profit functions, with water rights determined under the appropriative doctrine. Suppose the conditions of Proposition 3 hold, so that N is finite. Let $(\bar{a}_1, \ldots, \bar{a}_N)$ denote the vector of diversion capacities for the N firms. Associated with this pattern of appropriations is a value of aggregate expected profits E^A, given by

$$
\begin{aligned}
E^A = \sum_{i=1}^{N} \{ &F(A_{i-1}) \pi(0, \bar{a}_i) \\
&+ \int_{A_{i-1}}^{A_i} \pi(x - A_{i-1}, \bar{a}_i) f(x) dx \\
&+ [1 - F(A_i)] \pi(\bar{a}_i, \bar{a}_i) \}
\end{aligned}
$$

Given A_N the diversion capacity under the appropriative system, is the pattern of investment in diversion capacity and use of water associated with the appropriative system efficient? The answer is that the appropriative system is not efficient. We establish this by showing that there exists a feasible alternative to the appropriative system, namely equal sharing, that produces a higher value of output for every possible streamflow.

Given the aggregate diversion capacity A_N and given any streamflow x, then aggregate profits associated with an arbitrary feasible allocation of diversion capacities and water usage are given by

$$\sum_{i=1}^{N} \pi(a_i, \bar{a}_i) \qquad \text{where } a_i \leq \bar{a}_i,$$

$$i = 1, \ldots, N,$$

$$\sum_{i=1}^{N} \alpha_i \leq x, \qquad \sum_{i=1}^{N} \bar{a}_i = A_N$$

It is immediate that with $\pi_{11} < 0$, then so long as $\pi_{22} < 0$ (marginal cost of adding diversion capacity is increasing), aggregate profits are maximized with $a_i = x/N$ for $x \leq A_N$, $a_i = A_N/N$ for $x > A_N$, with $\bar{a}_i = A_N/N$ for $i = 1, \ldots, N$. In fact, writing aggregate profits as

$$\sum_{i=1}^{N} \pi(a_i, \bar{a}_i) = \sum_{i=1}^{N} \{ R(a_i) - C(\bar{a}_i) \}$$

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[7]Proposition 3 generalizes even for nonidentical risk-averse expected utility maximizers facing diverse technologies. To see this observe that the last appropriator must be of noninfinitesimal size else the dominance of increasing returns for small diversion capacity is violated. Given this, complete appropria-

tion of the river implies that the costs of constructing diversion capacity be negative at some level, a clear impossibility (see Appendix A).

it also follows that under equal sharing, aggregate revenue $\Sigma_{i=1}^{N} R_i(a_i)$ is maximized and aggregate cost $\Sigma_{i=1}^{N} C(\bar{a}_i)$ is minimized, subject to the feasibility constraints. Finally, for $N > 1$, aggregate profits (and aggregate revenue) are clearly less under the appropriate system than under equal sharing, for any value of x, because of the ordering of capacities from Proposition 2. Assuming competitive input and output markets, the results in turn imply that Pareto optimality implies equal sharing, while the appropriative system is inefficient.

PROPOSITION 4: *Assume that N firms exploit a waterway, with each firm having an identical separable profit function strictly concave in water usage. If the marginal cost of adding diversion capacity is increasing, then equal sharing is the efficient allocation of diversion capacity and water usage; allocation under the appropriative system is inefficient.*[8]

The assumption that the marginal cost of adding diversion capacity is increasing at an equal sharing allocation is restrictive. As it turns out, when this assumption is relaxed to permit declining marginal costs of adding diversion capacity, then it can still be shown that the appropriative system is inefficient in the sense that expected profits under other feasible allocations exceed those under the appropriative system. However, equal sharing is no longer necessarily the efficient allocation. Details are given in Appendix B.

The source of the allocative inefficiencies of the appropriative system is unequal sharing of risk and diversion capacity among firms. The Coase theorem makes it clear that a solution to the problem involves the establishment of competitive markets in water rights. Let α_{ij} be the fraction of firm i's rights that is purchased by firm j, and let β_{ij} be the fraction of firm i's

diversion capacity leased to firm j; let p_i be the price for a 1 percent share of firm i's water and let q_i be the price for a 1 percent share of firm i's diversion capacity. Then given the investment vector $(\bar{a}_1, \ldots, \bar{a}_N)$ established under the appropriative allocation[9] and assuming risk neutrality, with competitive markets in rights and capacity, each firm picks α_{ij} and β_{ij} so as to maximize expected profits. At an equilibrium (see Appendix C for a complete derivation) with $\bar{a} = A_N/N$, we have

$$(1) \quad \int_{A_{i-1}}^{A_i} \pi_1\left(\frac{x}{N}\right)(x - A_{i-1})\, f(x)\,dx$$

$$+ \sum_{r=i+1}^{N} \int_{A_{r-1}}^{A_r} \pi_1\left(\frac{x}{N}\right)\bar{a}_i\, f(x)\,dx$$

$$+ [1 - F(A_N)]\pi_1(\bar{a})\bar{a}_i = p_i + q_i,$$
$$i = 1, \ldots, N$$

$$(2) \quad q_j/\bar{a}_j = q_i/\bar{a}_i \qquad i, j = 1, \ldots, N$$

Condition (1) can be written as

$$(1') \quad \int_{A_{i-1}}^{A_i} \pi_1\left(\frac{x}{N}\right)\left(\frac{x - A_{i-1}}{\bar{a}_i}\right) f(x)\,dx$$

$$+ \int_{A_i}^{A_N} \pi_1\left(\frac{x}{N}\right) f(x)\,dx$$

$$+ [1 - F(A_N)]\pi_1(\bar{a}) =$$
$$p_i/\bar{a}_i + q_i/\bar{a}_i$$

In (2), $q_i > q_j$ for $i < j$; the price for 1 percent of a senior firm's capacity exceeds that of a junior firm, as the senior firm has larger capacity; however, $q_i/\bar{a}_i = q_j/\bar{a}_j$ for $i, j = 1, \ldots, N$, so that price per unit of capacity is equal among all firms. Since p_i is the price for 1 percent of firm i's water rights, 100 x $p_i/\bar{a}_i$ is the price of obtaining one unit of firm i's water when available by streamflow. Then the left-hand side of (1') (multiplied by 100) is the expected marginal

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[8]Proposition 4 generalizes as well. Note, however, that with diverse technologies, at the optimum water is prorated among firms according to their productivity so the expected marginal profitability is zero across firms, a condition that implies equal sharing if firms are identical.

[9]Should the eventuality of resale of water rights be foreseen, one might question the determinancy at the investment vector $(\bar{a}_1, \ldots, \bar{a}_N)$: what prevents a senior appropriator from "over-appropriating" for possible future resale? Fortunately this poses no problem as the appropriative doctrine is clear on this matter: to obtain a right to water it must be diverted, and diversions are limited to beneficial consumptive use. However in practice this may be problematic (see fn. 2).

profitability of a unit of water obtained from firm i, set equal to its marginal cost, including the increase in cost due to the added diversion capacity necessary to deliver the water. It is clear from (1) and (1') that $i < j$ implies $p_i > p_j$ and $p_i/\bar{a}_i > p_j/\bar{a}_j$, but at the equilibrium prices p_i, $i = 1, \ldots, N$, purchasers of water are indifferent among suppliers at the margin.

Clearly conditions (1) and (2) are consistent with market clearing. Hence an efficient mix of capacities and usages (equal sharing) can be attained under the appropriative system, given competitive markets in water rights and diversion capacity and given a fixed diversion capacity. Thus we have

PROPOSITION 5: *Given that all firms have identical separable profit functions and are risk neutral, and given that increasing returns dominate for small diversion capacities, the appropriative system of rights allocation coupled with competitive markets in rights and diversion facilities leads to an efficient outcome (namely, equal sharing), given the fixed aggregate installed diversionary capacity. The price per unit of water varies monotonically with the seniority of the water supplier; the price per unit of capacity is constant across firms.*[10]

However, there are some problems with the conclusion of Proposition 5. Economies of scale in the delivery system for water are pervasive enough that it is difficult to maintain a competitive environment in the market for diversionary facilities and hence in the market for water rights as well. In fact, it is this phenomeon that no doubt accounts for the creation of publicly controlled irrigation districts in the Southwest, designed to achieve the savings from scale while minimizing monopolistic distortions; and this also accounts for the (rarely enforced) acreage limitations on recipients of water from the Bureau of Reclamation projects. Admittedly, in principle monopoly distor-

[10] Proposition 5 generalizes in the same manner as Proposition 4.

tions could also be eliminated through appropriate bribes, but excessive transactions costs impose impediments to such a policy.

Proposition 5 takes as given the aggregate diversion capacity that is built under the appropriative system and asserts that competitive markets in rights and in leases of diversion facilities lead to an efficient outcome given that diversion capacity. This still leaves unanswered the issue of an optimal level of aggregate diversion capacity for a waterway.

We begin by examining a variant of this problem. Suppose that each firm owns its own diversion facility, and that increasing returns dominate for small diversion capacities, with N firms exploiting a waterway in long-run equilibrium under the appropriative doctrine. Aggregate capacity is A_N units. Consider in contrast the same N firms (all with identical separable profit functions) operating under equal sharing of water, with identical diversion capacities. What can be said about the amount of capacity that will be installed under equal sharing if aggregate expected profits are to be maximized?

First-order conditions require that

$$\pi_2(\bar{a}_N) + [1 - F(A_N)]\pi_1(\bar{a}_N) = 0$$

for the appropriative scheme, while

$$\pi_2(\bar{a}) + [1 - F(N\bar{a})]\pi_1(\bar{a}) = 0$$

for the equal sharing scheme. Thus,

$$F(A_N) = 1 + \frac{\pi_2(\bar{a}_N)}{\pi_1(\bar{a}_N)}$$

and

$$F(N\bar{a}) = 1 + \frac{\pi_2(\bar{a})}{\pi_1(\bar{a})}$$

$$F(A_N) - F(N\bar{a}) = \frac{\pi_2(\bar{a}_N)}{\pi_1(\bar{a}_N)} - \frac{\pi_2(\bar{a})}{\pi_1(\bar{a})}$$

We have $\pi_1 > 0$ for $a_i \geq 0$, $\pi_2 < 0$ for $\bar{a}_i \geq 0$. Further, assume that all firms operate under nondecreasing marginal costs of diversion capacity; in particular, $\bar{a} \geq \bar{a}_N$ implies $\pi_2(\bar{a}) \leq \pi_2(\bar{a}_N)$.

Suppose that $N\bar{a} \geq A_N$. Then $\pi_2(\bar{a}_N)/\pi_1(\bar{a}_N) - \pi_2(\bar{a})/\pi_1(\bar{a}) \leq 0$. But $N\bar{a} \geq A_N$ implies $\bar{a} > \bar{a}_N$ for $N > 1$. Hence $0 < \pi_1(\bar{a}) <$

$\pi_1(\bar{a}_N)$ and $0 > \pi_2(\bar{a}_N) > \pi_2(\bar{a})$ so that $\pi_2(\bar{a}_N)/\pi_1(\bar{a}_N) - \pi_2(\bar{a})/\pi_1(\bar{a}) > 0$, a contradiction. Hence $N\bar{a} < A_N$.

PROPOSITION 6: *Let N be the number of firms with identical separable profit functions exploiting a waterway in long-run equilibrium under the appropriative system. Then the aggregate diversion capacity constructed by these N firms exceeds that which would be constructed by the same firms under equal sharing, assuming that each firm builds its own diversion facility and that the marginal cost of constructing diversion facilities is increasing.*[11]

Thus there is a systematic overinvestment in diversion capacity under the appropriative scheme, assuming that the same N firms exploit a waterway under either equal sharing or the appropriative scheme, with each firm building its own diversion capacity.

IV. A Diversion Capacity Industry

It is clear, however, that with increasing returns operating with respect to diversion facilities, one would expect the development of an independent subindustry engaged in the construction and leasing of such facilities. As noted earlier, there are monopolistic problems present in such an industry; this has led to the formation of publicly operated and controlled irrigation districts which act in effect as lessors of diversion capacity. Suppose that the monopoly problems are overcome so that there is competitive pricing of leases. Let $C(\bar{a})$ denote the annualized cost associated with a diversion facility of capacity $\bar{a}$. Then under long-run competitive conditions, the aggregate diversion capacity for a waterway would be equal to $M\bar{a}^*$, where M is the number of leasing firms and $\bar{a}^*$ is the capacity owned by any one leasing firm, with $C(\bar{a}^*)/\bar{a}^* = C'(\bar{a}^*)$. That is, each leasing

firm builds a capacity such that the average annualized cost per unit of capacity is minimized. Under competitive conditions, the charge for leasing a unit of capacity would then equal $C'(\bar{a}^*)$. Thus lessees would face constant marginal costs of diversion facilities; i.e., $\pi_2(\bar{a}) = -C'(\bar{a}^*)$ is a constant independent of $\bar{a}$.

Recall that under the appropriative scheme,

$$\pi_2(\bar{a}_N) + [1 - F(A_N)]\pi_1(\bar{a}_N) = 0$$

With a leasing industry operating under competitive conditions, we have $\pi_2(\bar{a}_N)$ independent of $\bar{a}_N$. Given that $C'(\bar{a}^*)(\equiv -\pi_2(\bar{a}))$ is less than $\lim_{a\to 0} \pi_1(a,a)$, it is clear that the "marginal" firm chooses a capacity that approaches zero as $N \to +\infty$. (If $C'(\bar{a}^*) > \lim_{a\to 0} \pi_1(a)$, then no firm finds it profitable to exploit the waterway). It follows that

$$F(A_N) = 1 - \frac{C'(\bar{a}^*)}{\lim_{a\to 0} \pi_1(a)}$$

The same condition holds when aggregate expected profits are maximized, with all firms sharing equally in streamflows, since at a maximum of aggregate expected profits we have

$$\pi_2(\bar{a}) + [1 - F(N\bar{a})]\pi_1(\bar{a}) = 0$$

so that

$$F(N\bar{a}) = 1 - \frac{C'(\bar{a}^*)}{\lim_{a\to 0} \pi_1(a)}$$

Hence $A_N = Na$. We summarize this as

PROPOSITION 7: *Suppose there is an arbitrary number of firms, all with identical separable profit functions, exploiting a waterway. In addition suppose there is a competitive industry in diversion capacity which leases capacity to rights holders, with each leasing firm building the capacity which minimizes the average annualized cost per unit of capacity. If entry is free and unobstructed in both the diversion capacity and water-using industries, then in long-run equilibrium aggregate investment is the same*

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[11] Proposition 6 relies heavily on separability of the profit function. Although the proof is less direct the result generalizes for diverse technologies (maintaining the separability assumption) but not for risk averters.

under either the appropriative or equal sharing systems.[12]

Hence, the establishment of a competitive leasing industry that takes full advantage of the economics of scale in diversion facilities leads to the same aggregate diversion capacities under the appropriative scheme as under equal sharing.[13]

From Proposition 4 we know that for any given diversion capacity, equal sharing is the efficient solution given competitive markets with aggregate profits and revenue larger for any value of stream flows x than under the appropriative system. Thus with a competitive leasing industry operating to capture the limited economies of scale in building diversion capacity, equal sharing is a necessary condition for Pareto optimality, and involves the same aggregate diversion capacity as the appropriative system. Finally, the equal sharing allocation can be achieved under an appropriative system by competitive markets in water rights, from Proposition 5. By employing l'Hospital's rule we have

$$\lim_{N \to \infty} NR\left(\frac{a}{N}\right) = aR_1(0)$$

so that the aggregate revenue function for equal sharing is continuous at the origin. By a limiting argument we have

PROPOSITION 8: *Under the conditions of Proposition 7, at a long-run equilibrium equal sharing is a necessary condition for Pareto optimality and equal sharing can be achieved under an appropriative system through competitive markets in water rights.*

The appropriative system possesses one fundamental advantage over the riparian system or the equal sharing of rights in that it provides *tenure certainty* for each rights holder: rights holders are in principle pro-

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[12]This result generalizes directly for diverse technologies and risk-averse expected utility maximizers.

[13]Although Proposition 7 is instructive, one would not expect it to be operational in the real world because of spatial monopolies in the diversion leasing industry.

tected against loss of their rights through the legal actions of others. While the appearance of a new claimant to water can dilute the privileges of existing water users under either the riparian or equal sharing systems, the principle of "first in time means first in right" protects the privileges of existing users under the appropriative system.

Unfortunately in practice tenure certainty is difficult to guarantee even under the appropriative system. Due to spatial dispersion of appropriators, informational inadequacies and random elements (such as variability in return flows), it is often difficult to determine whether a diminished downstream flow to senior appropriators is the result of the stochastic nature of river flows or the improper actions of upstream junior appropriators. And, as we have seen, the principle of tenure certainty is bought at the cost of economic efficiency, so long as water rights are not freely transferable.

Limitations on the transferability of water exist in the form of federal and state statutes, and interregional and interstate compacts. Moreover, there are other impediments to transfers: fixed diversion capacities, transactions costs, and externalities. Externalities arise because a change in the nature or location of water diversions can affect return flows to a river and hence can impinge on the established water rights of third parties. Thus there are sound economic grounds for certain of the existing limitations on rights transfers.

However, we would argue that considerable potential latitude for the transfer of water rights still exists, and that economic efficiency could be improved by weakening existing legal constraints on such transfers. The usual argument in favor of transferable water rights identifies the higher productivity of water in industrial or municipal use as compared to present usage which is highly concentrated in irrigated farming. While we certainly agree with this argument, our conclusion goes even further: even among identical firms producing identical products, freely transferable water rights leads to increased economic efficiency.

Our approach in this paper has centered on the simple case of a static long-run competitive equilibrium with an uncontrolled river. We have not attempted to model the dynamics of the process by which rights are acquired and implemented under the appropriative system, nor have we examined the special problems that arise when a reservoir system is constructed with releases to downstream users being determined in an optimal fashion, subject to the priorities that hold under the appropriative system of rights.[14] It is clear to us, however, that whatever are the complexities introduced into the analysis by these factors, there are still advantages that can be gained by widening the possibilities for transferability of water rights.

APPENDIX A

Proposition 3 generalizes immediately for nonidentical firms as no comparisons are made across firms. The result generalizes as well for risk-averse producers—in fact even for nonidentical firms with nonidentical utility functions, given monotone preferences and risk aversion.

To see this first observe that first-order conditions for expected utility maximization require that

$$\frac{E^i u(\pi^i)}{d\bar{a}_i} = 0 = F(A_{i-1})u'[\pi^i(0,\bar{a}_i)]\pi_2^i(\bar{a}_i)$$
$$+ \int_{A_{i-1}}^{A_i} u'[\pi^i(x - A_{i-1},\bar{a}_i)]\pi_2^i(\bar{a}_i)f(x)dx$$
$$+ [1 - F(A_i)]u'[\pi^i(\bar{a}_i,\bar{a}_i)]$$
$$\cdot [\pi_1^i(\bar{a}_i) + \pi_2^i(\bar{a}_i)]$$

subject to $E^i u(\pi) \geq u(0)$. Suppose the last appropriator chooses capacity $\bar{a}_N = 0$, satisfying the first-order condition as an equality. Then for any u, monotonic and concave, the first-order condition becomes

$$\lim_{\bar{a}_N \to 0} \{\pi_2^N(\bar{a}_N) + [1 - F(A_N)]\pi_1^N(\bar{a}_N)\} = 0$$

This conflicts with the assumption that increasing returns dominates for small diver-

sion capacities. Hence $\bar{a}_N$ is noninfinitesimal and N is finite. If the entire river is appropriated these first-order conditions become

$$F(A_{N-1})u'[\pi^N(0,\bar{a}_N)]\pi_2^N(\bar{a}_N)$$
$$+ \int_{A_{N-1}}^{A_N} u'[\pi^N(x - A_{N-1},\bar{a}_N)]$$
$$\pi_2^N(\bar{a}_N)f(x)dx = 0$$

which is impossible in view of the negativity of π_2. Hence the river is not fully appropriated.

APPENDIX B

Consider an arbitrary reassignment of diversion capacities and water rights among the N firms such that firm j receives β_j percent of A_N as its diversion capacity along with α_{ij} percent of any streamflow within the range A_{i-1} to A_i where

$$\beta_j \geq 0, \sum_{j=1}^N \beta_j = 1, \alpha_{ij} \geq 0,$$
$$\sum_{j=1}^N \alpha_{ij} = 1, i, j = 1, \ldots, N$$

Let $\alpha = [\alpha_{ij}], \beta = [\beta_j]$ and let $E(\alpha,\beta)$ be the expected value of aggregate profits, where

$$E(\alpha,\beta) = \sum_{j=1}^N \{\sum_{i=1}^N \int_{A_{i-1}}^{A_i} \pi[\alpha_{ij}(x - A_{i-1})$$
$$+ \sum_{k=1}^{i-1} \alpha_{kj}\bar{a}_k, \beta_j A_N]f(x)dx$$
$$+ [1 - F(A_N)]\pi(\beta_j A_N, \beta_j A_N)\}$$

Thus $E(\alpha\beta) = E(\bar{\alpha},\bar{\beta}) = E^A$ for $\bar{\beta}_j = \bar{a}_j/A_N$, $\bar{\alpha}_{jj} = 1, \bar{\alpha}_{ij} = 0, i \neq j$, $i, j = 1, \ldots, N$

Let $L = E(\alpha,\beta) + \lambda\left(1 - \sum_{j=1}^N \beta_j\right)$
$$+ \sum_{i=1}^N \mu_i\left(1 - \sum_{j=1}^N \alpha_{ij}\right)$$

At a constrained maximum of $E(\alpha,\beta)$ we have

$$\frac{\partial E(\alpha,\beta)}{\partial \beta_r} - \lambda \leq 0 \quad r = 1, \ldots, N$$
$$(< 0 \text{ implies } \beta_r = 0)$$

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[14]See the authors (1977) for a simplified treatment of the problem of reservoir management under the appropriative system.

$$\frac{\partial E(\alpha,\beta)}{\partial \alpha_{sr}} - \mu_s \leq 0 \ \ s, r = 1, \ldots, N$$

$$(<0 \text{ implies } \alpha_{sr} = 0)$$

where, given $\pi_{12} = 0$,

$$\frac{\partial E(\alpha,\beta)}{\partial \beta_r} = A_N \{ \pi_2(\beta_r A_N)$$

$$+ [1 - F(A_N)]\pi_1(\beta_r A_N)\} \quad r = 1, \ldots, N$$

$$\frac{\partial E(\alpha,\beta)}{\partial \alpha_{sr}} = \int_{A_{s-1}}^{A_s} \pi_1[\alpha_{sr}(x - A_{s-1})$$

$$+ \sum_{k=1}^{s-1} \alpha_{kr}\bar{a}_k](x - A_{s-1}) f(x)dx$$

$$+ \sum_{t=s+1}^{N} \int_{A_{t-1}}^{A_t} \pi_1[\alpha_{tr}(x - A_{t-1})$$

$$+ \sum_{k=1}^{t-1} \alpha_{kr}\bar{a}_k]\bar{a}_s f(x)dx$$

$$r, s = 1, \ldots, N$$

The question we pose is whether the appropriative system $(\bar{\alpha},\bar{\beta})$ qualifies as a candidate for a maximizer of $E(\alpha,\beta)$. Evaluate the expression immediately above at $\alpha_{ss} = 1$, $\alpha_{sr} = 0$, $s \neq r$, $\beta_r = \bar{a}_r/A_N$, $r, s = 1, \ldots, N$ so that, for $s = 1, \ldots, N$,

$$\frac{\partial E(\bar{\alpha},\bar{\beta})}{\partial \alpha_{ss}} = \int_{A_{s-1}}^{A_s} \pi_1(x - A_{s-1})$$

$$\cdot (x - A_{s-1}) f(x)dx + \int_{A_s}^{A_N} \pi_1(\bar{a}_s)$$

$$\cdot \bar{a}_s f(x)dx = \mu_s, \text{ since } \alpha_{ss} > 0;$$

$$\frac{\partial E(\bar{\alpha},\bar{\beta})}{\partial \alpha_{sr}} = \int_{A_{s-1}}^{A_s} \pi_1(0)(x - A_{s-1}) f(x)dx$$

$$+ \sum_{t=s+1}^{r-1} \int_{A_{t-1}}^{A_t} \pi_1(0)\bar{a}_s f(x)dx$$

$$+ \int_{A_{r-1}}^{A_r} \pi_1(x - A_{r-1})\bar{a}_s f(x)dx$$

$$+ \int_{A_r}^{A_N} \pi_1(\bar{a}_r)\bar{a}_s f(x)dx \leq \mu_s$$

$$\text{for } r > s$$

But $r > s$, $E(\bar{\alpha},\bar{\beta}) = E^A$ implies $\bar{a}_r < \bar{a}_s$ by Proposition 2, and $\pi_1(a_i)$ decreasing in a_i implies on a term-by-term basis that

$$\frac{\partial E(\alpha,\beta)}{\partial \alpha_{sr}} > \frac{\partial E(\alpha,\beta)}{\partial \alpha_{ss}}$$

This contradicts the first-order conditions, hence the appropriative allocation is non-optimal.

APPENDIX C

Given competitive markets in water rights and diversion facilities, each firm solves the problem

$$\max E^j\pi = \sum_{i=1}^{N} \int_{A_{i-1}}^{A_i} \pi[\alpha_{ij}(x - A_{i-1})$$

$$+ \sum_{k=1}^{i-1} \alpha_{kj}\bar{a}_k, \bar{a}_j] f(x)dx$$

$$+ [1 - F(A_N)]\pi(\sum_{k=1}^{N} \beta_{kj}\bar{a}_k, \bar{a}_j)$$

$$+ p_j - \sum_{i=1}^{N} p_i\alpha_{ij}$$

$$+ q_j - \sum_{i=1}^{N} q_i\beta_{ij}$$

subject to $\sum_{i=1}^{N} \alpha_{ij}a_i \leq \sum_{i=1}^{N} \beta_{ij}\bar{a}_i$

with first-order conditions

$$\frac{\partial E^j\pi}{\partial \alpha_{ij}} = \int_{A_{i-1}}^{A_i} \pi_1[\alpha_{ij}(x - A_{i-1})$$

$$+ \sum_{k=1}^{i-1} \alpha_{kj}\bar{a}_k](x - A_{i-1}) f(x)dx$$

$$+ \sum_{r=i+1}^{N} \int_{A_{r-1}}^{A_r} \pi_1[\alpha_{rj}(x - A_{r-1})$$

$$+ \sum_{k=1}^{r-1} \alpha_{kj}\bar{a}_k]\bar{a}_i f(x)dx$$

$$- p_i - \lambda_j\bar{a}_i = 0$$

$$\text{for } \alpha_{ij} > 0, i = 1, \ldots, N$$

$$\frac{\partial E^j}{\partial \beta_{ij}} = [1 - F(A_N)]$$

$$\pi_1(\sum_{k=1}^{N} \beta_{kj}\bar{a}_k)\bar{a}_i - q_i + \lambda_j\bar{a}_i = 0$$

for $\beta_{ij} > 0$, $i = 1, \ldots, N$, where λ_j is the multiplier associated with the constraint $\Sigma_{i=1}^{N} \alpha_{ij}\bar{a}_i \leq \Sigma_{i=1}^{N} \beta_{ij}\bar{a}_i$. Note that the allocation $\alpha_{ij} = \beta_{ij} = 1/N$ $i, j = 1, \ldots, N$ which maximizes $\Sigma_{j=1}^{N} E^j\pi$ also satisfies the first-order conditions for any j, given that p_i and q_i satisfy equations (1) and (2) of the main text.

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