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Monopoly and the Rate of Extraction of Exhaustible Resources: Note

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In a recent paper appearing in this *Review,* Joseph Stiglitz demonstrates under a set of familiar conditions that a monopoly-owned nonreplenishable resource will tend to be exhausted at a slower rate than is socially optimal.[1] This supports earlier views on the subject expressed by Harold Hotelling and Robert Solow. Stiglitz shows under the natural "first approximation" assumptions of stationary, iso-elastic demand and zero extraction costs, that monopolistic and socially optimal (competitive) extraction rates are identical. If demand elasticity increases with time or constant unit production costs are positive but possibly decrease with time, he shows that competitive extraction rates exceed monopolistic rates for at least an initial period of time.

In this note we present realistic, alternative extensions to the iso-elastic, zero cost analysis which tend to bias monopolistic extraction rates in the opposite direction, that is, towards excessive resource use. The first modification allows for costs that do not vary with the extraction rate. Occurring in the form of leasing fees, capital costs, and maintenance fees, these quasi-fixed costs[2] are incurred only during periods of production and often constitute a substantial portion of operating expenses.[3]

The second extension involves demand elasticities varying with consumption instead of time. In particular, we consider a stationary demand schedule with elasticity increasing in consumption. A justification for this assumption is that for small quantities demand may be inelastic if certain amounts of the resource are essential in the production of some goods. At lower prices, however, the resource may be used in other industries for which substitute inputs exist as well. Consequently, the elasticity of aggregate demand may increase. For example, the demand for natural gas by homeowners with gas appliances is inelastic since substitute fuels are difficult to use. Since these homeowners are the primary users at high prices, natural gas demand is inelastic at high prices (low consumption). However, at lower prices marginal usage occurs at the extensive margin as various manufacturers switch to natural gas, and aggregate demand may consequently become more elastic.[4] The result in this case is that if costs are quasi fixed, or if demand elasticity increases with consumption, then a monopolist depletes the resource too soon.

In general, competitive ownership of the resource will also result in socially nonoptimal production when fixed operation costs exist. For example, if all costs are quasi fixed, least cost production requires that only one mine operate at a time. Yet with discounting there always will be an in-

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[1]Similar analyses comparing monopolistic and socially optimal extraction rates appear in John Kay and James Mirrlees, Tracy Lewis, James Sweeney, and Milton Weinstein and Richard Zeckhauser. Milton Kamien and Nancy Schwartz compare extraction rates in a general equilibrium setting.

[2]Fixed costs of this variety which can be avoided by stopping production were categorized as "avoidable fixed costs" by Vernon Smith, pp. 257–59.

[3]For example, see James Hendry.

[4]Strong income effects may also tend to cause demand elasticity to increase with consumption.

centive for competitive mining firms to operate simultaneously. While several alternative forms of market intervention might limit the number of operating mines to the social optimum, in general the behavior of an unregulated competitive industry that has quasi-fixed costs is difficult to assess and beyond the scope of this note.[5] Consequently, we contrast monopolistic with socially optimal programs of resource extraction.

Letting $p(q)$ be the inverse demand function for the resource, Q_o be the initial resource supply, and $q_S(\mathrm{t})$ and $q_M(\mathrm{t})$ be the socially optimal and monopolistic rates of extraction, the respective maximization problems for the social maximizer[6] and the monopolist are:[7]

$$(1)\quad \underset{q_S(\mathrm{t}),T_S}{\text{maximize}} \int_0^{T_S} \left[\int_0^{q_S(\mathrm{t})} \cdot p(q)dq - F\right]e^{-r\mathrm{t}}d\mathrm{t}$$

subject to $\int_0^{T_S} q_S(\mathrm{t})d\mathrm{t} \leq Q_o; q_S(\mathrm{t}), T_S \geq 0$

$$(2)\quad \underset{q_M(\mathrm{t}),T_M}{\text{maximize}} \int_0^{T_M} \cdot [p(q_M(\mathrm{t}))q_M(\mathrm{t}) - F]e^{-r\mathrm{t}}d\mathrm{t}$$

subject to $\int_0^{T_M} q_M(\mathrm{t})d\mathrm{t} \leq Q_o; q_M(\mathrm{t}), T_M \geq 0$

where r is the discount rate and T_S and T_M are the terminal extraction dates. Note that these terminal dates are choice variables.

Performing the indicated maximizations, manipulation of the necessary conditions for (1) and (2) yields, respectively;

$$(3)\quad \dot{q}_S(q) = \frac{rp(q)}{p'(q)} = -re(q)q$$

$$(4)\quad \dot{q}_M(q) = \frac{rR'(q)}{R''(q)} = -re(q)q\left[1 - \frac{e'(q)q}{e(q)-1}\right]^{-1}$$

where $R(q) = p(q)q$ is the revenue function, which we presume is concave, and $e(q)$ is the demand elasticity. We assume $e > 1$ to ensure positive monopolistic output so that (3), (4), and $e'(q) \geq 0$ imply[8]

$$(5)\quad 0 > \dot{q}_S(q) \geq \dot{q}_M(q)$$

The necessary terminal time conditions can be expressed as[9]

$$(6)\quad q_M(T_M)f'(q_M(T_M)) = f(q_S(T_S)) = F$$

where the function $f(q)$ is defined by

$$(7)\quad f(q) \equiv \int_0^q p(x)dx - qp(q)$$

Note that $f''(q) \equiv p'(q)e(q)^{-1} - p(q)e'(q)e(q)^{-2} < 0$ since $e'(q) \geq 0$. The concavity of f together with $f(0) = 0$ and (6) imply[10]

$$(8)\quad q_M(T_M) \geq q_S(T_S)$$

Changing variables of integration from t to q in the resource constraint equations yields

$$(9)\quad -Q_o = \int_{q_M(T_M)}^{q_M(0)} [q/\dot{q}_M(q)]dq = \int_{q_S(T_S)}^{q_S(0)} [q/\dot{q}_S(q)]dq$$

Consistency between (5), (8), and (9) requires that $q_M(0) \geq q_S(0)$; i.e., the monopoly initially extracts at a rate no slower than is socially optimal. Since inequality (5) is strict if $e'(q) > 0$, and inequality (8) is strict if $F > 0$, the initial monopoly extraction rate will be excessive in either case. From equation (5) the time path $q_M(\mathrm{t})$ crosses $q_S(\mathrm{t})$ at most once, and only from above. Thus the monopolist either extracts

[5]This topic is currently being pursued by us in a subsequent manuscript.

[6]Subject to the usual caveats, the social maximizer is assumed to maximize consumer's surplus, the area beneath the demand curve.

[7]Since cost minimization with zero variable and positive quasi-fixed costs requires that one mine operate at a time, F represents the fixed operating costs for one mine.

[8]Lest the point of this section be made vacuously we hasten to assert that demand functions satisfying these requirements exist. In particular if the social welfare function is $U(q) = ln\ q + 2q^{1/2}$ then $dU/dq = q^{-1} + q^{-1/2} = p(q)$. From this one easily obtains $e > 1$ and $e'(q) > 0$. Moreover $R''(q) < 0$ everywhere.

[9]Terminal time conditions can be obtained by maximizing the Lagrange expression for this problem with respect to T_S and T_M.

[10]If $F = 0$, we have $T_M = T_S = \infty$ and $q_M(\infty) = q_S(\infty) = 0$. This follows because $e'(q) \geq 0$ implies $\lim_{q\to 0} p(q) = \infty$.

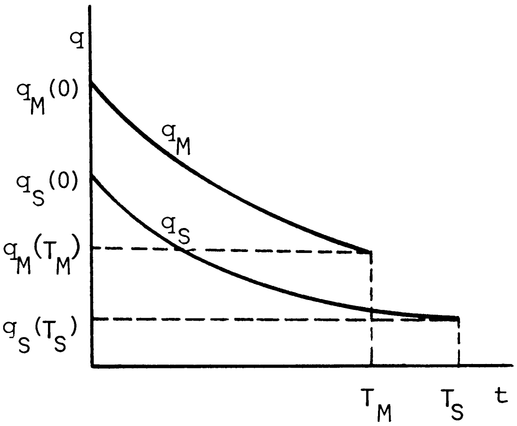

FIGURE 1. NONINTERESECTING EXTRACTION PATHS (ISO-ELASTIC DEMAND AND POSITIVE FIXED COSTS)

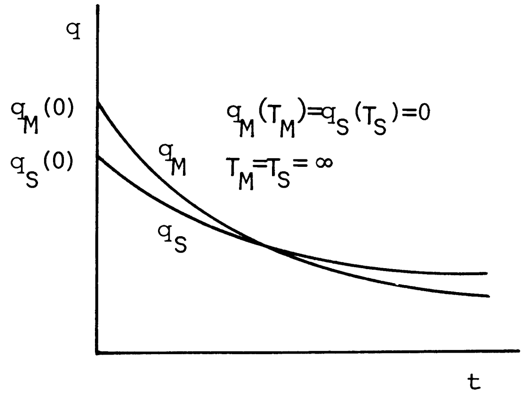

FIGURE 2. INTERSECTING EXTRACTION PATHS (FOR EXAMPLE, ZERO FIXED COSTS AND DEMAND ELASTICITY INCREASING WITH QUANTITY)

too fast for the entire extraction period before exhaustion (see Figure 1), or too fast initially and too slowly thereafter (see Figure 2). Hence, the resource remaining is always less than socially optimal and depletion occurs too soon.[11] This is clear for the case in Figure 1, and the case in Figure 2 follows directly from $q_M(0) > q_S(0)$, $q_M(\infty) = q_S(\infty) = 0$ and the fact that the paths $q_M(t)$ and $q_S(t)$ intersect only once.

Thus we have established the following:

PROPOSITION 1: *Suppose that demand for a nonrenewable resource is stationary and everywhere elastic, and that all variable costs are zero. If either* (a) *quasi-fixed costs are positive and demand elasticity is nondecreasing in consumption, or* (b) *quasi-fixed costs are nonnegative and demand elasticity is strictly increasing in consumption, then a monopolist extracts the resource faster than is socially optimal in the following ways:* (i) $T_M \leq T_S$ *(with* $T_M < T_S$ *if* $F > 0$), (ii) $q_M(t) > q_S(t)$ *initially (and for all* $t \leq T_M$ *if* $F > 0$ *and* $e' = 0$), *and* (iii) $Q_M(t) < Q_S(t)$ *for* $t < T_S$.

At first glance the excessive extraction rate of the monopolist when $e'(q) > 0$ appears mysterious, if only because derivatives of elasticities are second-order demand characteristics and do not affect usual (static) marginal analyses. But consider a simple two-period world, and suppose the monopolist is considering the socially optimal schedule (q_S^1, q_S^2) that is determined by setting the first period price equal to the discounted second period price. Marginal

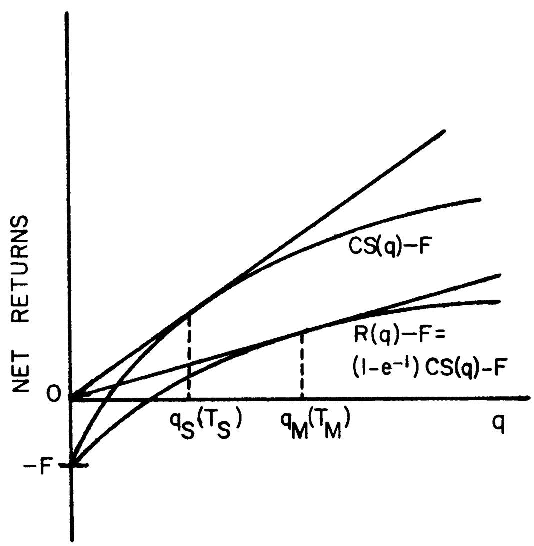

FIGURE 3. MONOPOLISTIC AND SOCIALLY OPTIMAL TERMINAL NET RETURNS AND OUTPUT (POSITIVE QUASI-FIXED COSTS AND ISO-ELASTIC DEMAND)

[11] For $F = 0$, we obtain the general result $Q_M(t) \lesseqgtr Q_S(t)$ as $e'(q) \gtreqless 0$. The analysis for $e'(q) < 0$ is in Lewis.

revenue is a fraction, $1 - e^{-1}$, of price, and that fraction increases with demand elasticity. Since $q_S^1 > q_S^2$ implies $e(q_S^1) > e(q_S^2)$, equality of the two discounted prices implies that discounted marginal revenue is greater in the first period than in the second. The monopolist therefore adjusts (q_S^1, q_S^2) by extracting more in period one and less in period two, until the two discounted marginal revenues are equal. When the number of periods is variable, this same reasoning indicates that the monopolist depletes the resource too soon.

The other polar case of positive quasi-fixed costs and iso-elastic demand can also be understood more heuristically. The instantaneous net returns of the monopolist and social maximizer are $(1 - e^{-1})\, CS(q) - F$ and $CS(q) - F$, respectively, where $CS(q)$ is consumer surplus. Both the monopolist and social maximizer bear the same costs F to operate in each period, but the monopolist captures but a fraction of the returns accruing to the social maximizer. There is, thus, a greater incentive for the monopolist to accelerate extraction to reduce total operating costs FT_m (see Figure 3).

We have shown that in two special cases a monopolist depletes a natural resource faster than is optimal. Since Stiglitz proves the opposite result for other special cases, the net effect of all these presumably realistic considerations is analytically indeterminate and must be ascertained empirically.

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