DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

A Protocol for Factor Identification

Kuntara Pukthuanthong
University of Missouri

Richard Roll
California Institute of Technology

Avanidhar Subrahmanyam
University of California, Los Angeles

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Kuntara Pukthuanthong, Richard Roll, and Avanidhar Subrahmanyam

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Contacts

	Pukthuanthong	Roll	Subrahmanyam
Voice:	1-619- 807-6124	1-626-395-3890	1-310-825-5355
E-mail:	pukthuanthongk@missouri.edu	rroll@caltech.edu	asubrahm@anderson.ucla.edu
Address:	Department of Finance University of Missouri Columbia, MO 65211	California Institute of Technology Pasadena, CA 91125	Anderson School UCLA Los Angeles, CA 90095-1481

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A Protocol for Factor Identification

Abstract

We propose a protocol for identifying genuine risk factors. The underlying premise is that a risk factor must be related to the covariance matrix of returns, must be priced in the cross-section of returns, and should yield a reward-to-risk ratio that is reasonable enough to be consistent with risk pricing. A market factor, a profitability factor, and traded versions of macroeconomic factors pass our protocol, but many characteristic-based factors do not. Several of the underlying characteristics, however, do command material premiums in the cross-section.

I. Introduction

Let us express a conditional linear factor model as

$$\mathbf{R}_t = E_{t-1}(\tilde{\mathbf{R}}_t) + \boldsymbol{\beta}_{t-1}\mathbf{f}_t + \boldsymbol{\gamma}_{t-1}\mathbf{g}_t + \boldsymbol{\varepsilon}_t \,, \tag{1}$$

where **R** is an N-asset column vector of returns in period t.[1] In this model, assume that stochastic factors **f** command risk premiums and stochastic factors **g** do not. Assuming there are K true risk factors, **f** is a KX1 mean zero column vector; the true risk factor loadings are in a matrix, **β**, with N rows and K columns. Similarly, if there are J diversifiable factors, **g** is a JX1 mean zero column vector and the associated loadings, **γ**, is a matrix with N rows and J columns. Finally, **ε** is an idiosyncratic NX1 mean zero column vector whose covariance matrix is diagonal.

Notice that the loadings of both the true risk factors and the diversifiable factors have time subscripts t-1 to allow for time variation. The loadings are assumed to be known one period in advance of the returns.

In an arbitrage-free economy (Ross, 1976) with many assets, the expected returns as of t-1 conform to their own linear cross-sectional relation,

$$\mathbf{E}_{t-1}(\mathbf{R}_t)=\mathbf{R}_{F,t-1}+\boldsymbol{\beta}_{t-1}\,\boldsymbol{\lambda}_{t-1} \,, \tag{2}$$

where the first term on the right is an NX1 column vector with the riskless rate at the beginning of the period in every position, **λ** is a possibly time-varying KX1 column vector of non-zero risk premiums corresponding to factor class **f**.[2] This implies that the factor set **g** is not priced in the cross-section of assets. In turn, this means that **g** drives cross-correlations in the N assets, but is diversifiable when all tradeable and non-tradeable assets, and portfolios, are considered.

Empirically, how should one determine whether a particular candidate factor is in the set **f**,

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[1] Hereafter, bold face indicates a vector or a matrix.

[2] The arbitrage pricing theory represented by (2) holds exactly in an economy with infinitely many assets, and approximately otherwise.

the set **g**, or none of these sets? The job of any such procedure for factor identification should be to ascertain whether a particular factor candidate is in the **f** class and hence is unpredictable, is related to systematic volatility and has an associated risk premium; or is in the **g** class and hence is unpredictable, is related to volatility but does not earn a risk premium, or is neither priced nor related to asset volatility. A principal goal of our paper is to present a protocol for identifying whether a particular proposed factor is indeed a priced risk factor, i.e., belongs to class **f**.

Note that Eq. (2) holds in a market where arbitrage is perfect and assets are not mispriced because of behavioral biases and arbitrage constraints. If asset mispricing is allowed, then deviations from Eq. (2) are permissible, and such deviation will be associated with "characteristics" that proxy for investor biases. Indeed, numerous factor candidates and firm-specific return predictors (characteristics) have been proposed in a voluminous literature. For example, Lewellen, Nagel, and Shanken (2010) list several of the most prominent predictor candidates in their opening paragraph and note that although they explain some empirical regularities, they have "…little in common economically with each other" (p. 175.) Subrahmanyam (2010) surveys more than fifty characteristics that various papers contend to be cross-sectionally related to mean returns. McLean and Pontiff (2016) examine 95 characteristics that were claimed in previous papers to explain returns cross-sectionally but find that predictability declines after publication. Lewellen (2015) finds strong predictive power of actual returns using 15 firm characteristics. Harvey, Liu, and Zhu (2016) enumerate 316 "factor" candidates suggested in 313 papers and suggest that any newly proposed factor should have to pass a much higher hurdle for statistical significance than the level habitually used in the literature, simply because of the extensive data mining. However, they do not attempt to relate the "factors" to the covariance matrix of returns, and do not draw a sharp distinction between firm-specific return predictors (characteristics) and priced factors. Green, Hand, and Zhang (2013) identify 330 firm characteristics and Green, Hand, and Zhang (2017) test whether 100 of them are priced (i.e., are associated with risk premiums.) They find that only 24 characteristics are priced with an absolute t-statistic ≥ 3.0.

Something needs to be done when more than 300 candidates have been suggested in the factor literature, and when there seems to be some confusion between priced "factors" and predictor "characteristics." New return predictors seem to be proposed in every issue of the major finance journals, adding to the existing ones, but there is no well-accepted process for determining their qualities. In addition, sometimes characteristic predictors are converted to their factor counterparts by computing the return differential across long-short decile portfolios formed based on the extreme values of the characteristics (Fama and French, 1993, 2008). At this point, there has been no protocol proposed in the literature to separately classify priced factors and non-priced factors. We need a process to evaluate them and to assess each additional predictor that will inevitably be nominated in the future.

We also note that there are few topics in finance, arguably none, that are more important than factor identification; factors are the main principal determinants of investment performance and risk. Indeed, the comparative values of well-diversified portfolios are determined almost completely by their factor exposures. Whether investors know it or not, every diversified portfolio is absolutely in thrall to factor drivers. Moreover, there seem to be more than one of them.

The multiplicity of factors is strongly suggested by two striking empirical regularities about portfolios. First, even really well-diversified portfolios are quite volatile. The volatility of a large positively-weighted portfolio is often around half as large as the average volatility of its constituents. For example, during the decade from 2001 through 2010, the monthly total return on the S&P 500 had an annualized volatility (standard deviation) of 16.3%. Over the same period, the average volatility for the S&P's constituents was 36.1%.

Second, although well-diversified portfolios are highly correlated within the same asset class, they are much less correlated across classes; e.g., across bond vs. equities vs. commodities or across countries or across industry sectors. From 2001 through 2010, the monthly total return correlation between the S&P 500 and Barclay's Bond Aggregate Index was -0.0426. The return correlations between these two indexes and the Goldman Sachs Commodity index were 0.266 and 0.0113, respectively. Similarly modest correlations are typical between real estate assets and assets in other classes.[3]

The first empirical fact indicates the existence of at least one common underlying systematic influence, (or "risk driver" or "factor") that limit diversification within an asset class; otherwise diversified portfolios would have much smaller volatilities. The second fact implies the presence of multiple systematic factors across assets; otherwise diversified portfolios would be more correlated across asset classes, countries, and sectors.

Almost all academics and probably the vast majority of finance professionals now recognize that pervasive factors are among the main drivers of observed returns, but there is considerable disagreement about the identities of factors and even about whether they represent risks, anomalies, or something else.

Theory suggests that a true risk factor (in the class **f** in Eq. (1)) has three fundamental attributes:

(1) It varies unpredictably in a time series sense

(2) Its variations induce changes in asset prices

[3] Cotter and Roll (2015) report that real estate investment trusts have rather low betas against the S&P 500.

(3) It is associated with a risk premium.

Quasi-factors (in the set **g**) influence the returns of few securities and are diversifiable in aggregate. A factor of this type possesses two attributes:

(1) It varies unpredictably in a time series sense

(2) Its variations do not induce changes in expected returns.

Characteristics are sometimes associated with factors, but a characteristic

(1) Is known in advance

(2) Might be cross-sectionally related to the expected returns of some assets, and/or

(3) Might be cross-sectionally related to the loadings on true risk factors or the loadings on quasi factors.

Our main goal is to popularize a process to identify factors that will be broadly acceptable to both scholars and practitioners. We believe this is the first attempt to suggest a complete normative process for dealing with one of the most fundamental questions in finance: how to identify systematic risk factors that are reliably associated with expected returns. Our protocol has the potential to identify factors associated with risk premiums or true factors, but also factors that move some returns but do not have associated risk premiums, and characteristics that are associated with systematic return differences but are not related to risk. Characteristics that are reliably associated with returns but not risks are perhaps the most interesting of all, since they offer potential profit opportunities.[4]

[4] Engelberg, McLean, and Pontiff (2016) show that the profitability from anomalies is higher around earnings announcement days after controlling for risk, which indicates that the anomalies in fact capture mispricing. Linnainmaa and Roberts (2017) argue that several recently discovered return predictors ("characteristics") might be spurious as they do not survive in the data from earlier time periods.

Our protocol is composed of a sequence of steps to check necessary conditions and one final step that examines a sufficient condition. From Eq. (1) (suppressing time subscripts and assuming orthogonal factors), we have the familiar relation

$$\mathbf{cov(R)=\beta\beta'var(f)+\gamma\gamma'var(g)+(\gamma\beta'+\beta\gamma')cov(f,g)}, \tag{3}$$

which implies that a fundamental necessary condition for any factor candidate is that it must be related to the covariance matrix of returns. Although this necessary condition for the factor existence (correlation with the assets in question) is well known and used to various extents in much of the empirical work, our protocol presents a more systematic treatment of the subject. Note that this necessary condition does not distinguish between pervasive priced factors, those with risk premiums, and non-pervasive, fully diversifiable factors, which are related to covariances of some individual assets but do not influence the aggregate portfolio. Our sufficient condition tests provide for this distinction.

Factor candidates that do not satisfy the fundamental necessary condition are not without interest, particularly to practical investors. If a factor candidate is reliably related to mean returns but is unrelated to the (conditional) covariance matrix, it represents a potential arbitrage opportunity.

We also note that our paper is not aimed at testing a particular asset-pricing model, in contrast to studies by Lewellen and Nagel (2006) and Campbell et al. (2017), both of which examine the validity of the “conditional” CAPM. For reasons mentioned above, we think that any single factor theory, albeit conditional, cannot explain why diversified portfolios in different asset classes are so weakly correlated.

A single stochastic discount factor (SDF) or, equivalently, a conditional mean/variance efficient portfolio, is always available to <u>conditionally</u> explain the cross-section of asset returns with a single exposure coefficient.[5] There, however, is a mapping from the SDF to the factor model. To see this, let us suppress time subscripts and note that the Euler equation states the following:

$$E(\mathbf{R}M)=\mathbf{1},$$

where M is the SDF and **1** is the unit vector. Substituting for the factor model from (1), we have

$$E(\mathbf{R})=a_1+a_2\,\boldsymbol{\beta}+k,$$

where a_1 and a_2 are constants and $k=-E(\boldsymbol{\varepsilon}M)/E(M)$. Now, the linear form of the APT equation holds as long as $k=0$,[6] for which it suffices that $E(\boldsymbol{\varepsilon}M)=0$. This is true from Eq. (1) as long as the SDF M is a linear function of the factors. Thus, multiple factors are a practical way to explain unconditional returns over any finite sample.

From a practical perspective, either of an investor or a financial econometrician, incomplete information, via the finite sample problem, is inevitable. Our aim is to popularize an identification procedure for risk and non-risk factors that is useful, though perhaps not theoretically pristine in the sense of being congruent with a SDF. We illustrate our protocol using popular factors, which are based on fundamentals-driven, or characteristics-driven, arguments.

II. Related Research

One related study is by Charoenrook and Conrad (2008) (hereafter CC.) Their approach is motivated by Section 6.3 in Cochrane (2001), which derives a relation between the conditional variance of a true factor and that factor's associated risk premium. CC notice an important implication; viz., that time variation in a factor candidate's volatility should be correlated

[5] As emphasized by Cochrane (2001) and Singleton (2006, chapter 8)

[6] With the additional observation that the risk free rate is the reciprocal of the expected value of M (see, for example, Campbell and Cochrane, 2000, for details).

positively with time variation in its expected return. Consequently, if (a) a proposed factor has significant intertemporal variation, (b) its mean return also has significant variation, and (c) the two are positively correlated, then the factor candidate satisfies a necessary condition to be proxying for a true underlying priced factor. As CC emphasize though, such an empirical finding is not a sufficient condition.

CC find empirically that several proposed factor candidates, including size, book/market, and a liquidity construct, satisfy the above necessary condition. Momentum[7] does not. Momentum's estimated mean/volatility relation has the wrong sign. If this finding is upheld, it implies strongly that the momentum characteristic offers a free lunch, supposedly an arbitrage opportunity.

In the last section of their paper, CC, motivated by the recognition that their empirical condition is only necessary, examine whether the risk premiums associated with size, book/market and liquidity are in a plausible range. They find that the Sharpe ratios of size and book/market are plausible, but the Sharpe ratio for liquidity is not. We are left in doubt as to which of these are priced factors. We note that although size, book/market and liquidity satisfy a necessary condition to be risk factors, a rigorous test of sufficiency would build on their work. Also, since time variation in risk premiums is required for the CC necessity condition, a method that identifies factor candidates with stable risk premiums or with statistically small variation would also be complementary to their work.

Another related and recent paper is Harvey and Liu (2016). They propose a bootstrap method to select among a large group of candidate factors. They ascertain the factor from a pool of candidates that yields the lowest intercept in a cross-sectional model. They then find a second

[7] A factor candidate originally proposed by Carhart (1997).

factor that yields the lowest intercept from the base model that has the first successful factor. The process is repeated until no further factor passes a significant hurdle.

To illustrate, suppose they have seven candidate factors and the market factor is the first successful factor, so it is added to the model. Then, in the second round, HML is the best factor, so it is added to the model that already has the market factor. In the third round, suppose SMB is the best factor but it has an insignificant p-value. They stop at this point and declare that the market and HML are the only significant factors.

In another recent paper, Fama and French (2017) propose squared Sharpe ratios to select factors. But an anomaly can have a high Sharpe ratio and not be risk related. Both the Harvey and Liu (2016) and the Fama and French (2017) protocols are useful, but linking the candidate factors to the sample covariance matrix is an additional step that would advance their work.

Barillas and Shanken (2017) propose a Bayesian asset pricing test that allows comparison of all possible asset pricing models from subsets of given factors. Feng, Giglio and Xiu (2017) propose the combination of the double-selection LASSO method of Belloni et al. (2014) with two-pass regressions such as Fama-MacBeth to systematically select the best control model out of the large set of factors, while explicitly taking into account that in any finite sample we cannot be sure to have selected the correct model. Applying the key principle that true factors have to be related to the covariance matrix, which these papers do not do, would again be a useful exercise that would supplement their work.

Our protocol identifies not only factors associated with risk premiums or true factors, but also factors that move some returns but do not have associated risk premiums, and factors or characteristics that are associated with systematic return differences but not risks. Factors or characteristics that are reliably associated with returns but not risks are perhaps the most interesting of all, since they offer potential profit opportunities. Although the papers mentioned above have the same goal as ours, they do not distinguish among these categories of factors.

III. Factors and the Covariance Matrix

A necessary condition for any empirically measurable candidate (like Fama and French's (1993) HML) to be a factor is that it be related to the principal components of the covariance matrix. This condition represents the motivation for the analysis in Moskowitz (2003), who checks it for three candidates, size, book/market, and momentum. Moskowitz finds that size satisfies the condition; it is related to covariation and its associated risk premium is positively associated with its volatility. Book/market is close to satisfying but momentum is not. This agrees with the results of CC discussed above in the case of momentum, and it more or less agrees with CC in the case of book/market.

Unfortunately, in our imperfect world, factor extraction from the covariance matrix faces a number of serious difficulties, including

a. It produces only estimates for linear combinations of the true underlying factors, not the factors themselves;

b. It is compromised by non-stationarity since there is no plausible reason why the number of factors or their relative importance should be constant through time[8];

c. It includes true risk drivers, pervasive non-diversifiable factors (or linear combinations thereof) along with diversifiable factors, perhaps such as industry factors, that are not associated with risk premiums.

[8] Moreover, it seems that non-stationarity is an empirical fact. Moskowitz (2003) finds that "…significant time variation in the covariance structure of asset returns distorts the ability of these time-invariant factors (principal components extracted from the unconditional covariance matrix) to capture second moments, suggesting that unconditional factors miss important dynamics in return volatility," (p. 436).

Fortunately, there is a remedy, perhaps imperfect, for each of these conundrums. For (a), the linear combinations extracted by PCA could be related to other candidate factors, such as macro-economic variables, through canonical correlation or a similar method. This wouldn't prove anything but it would at least give some reassurance or raise some serious doubt. For (b), PCAs could be estimated for subperiods. For (c), a second stage method as in Fama and MacBeth (1973) could be employed to distinguish priced (presumably non-diversifiable) factors from others. Needless to say, none of these cures is without its own problems.[9]

IV. What are the Underlying Factors?

What exactly are the salient features of factors, the underlying risk drivers? Cochrane (2001) says unequivocally, "The central and unfinished task of absolute asset pricing[10] is to understand and measure the sources of aggregate or macroeconomic risk that drive asset prices." (p. xiv.) He particularly has in mind aggregate consumption as a driver and even goes so far as to say that "…the only consistent motivation for factor models is a belief that consumption *data* are unsatisfactory," (p. 170, emphasis in original.) In other words, if we only had adequate measures of aggregate consumption, we wouldn't need much else for risk modeling. The absence of adequate consumption data motivates the study of other indicators of macroeconomic activity, even hundreds of such indicators.

The underlying drivers cannot be the infrequently-published official numbers about macro-economic variables because market prices move around much too rapidly. Instead, the drivers

[9] It is known that PCA will do a rotation that makes it seem that the first factor is more dominant than in the true underlying structure. Brown (1989) offers a remedy. However, this problem is not all that troubling for our protocol because we do not need to determine the true number of underlying factors, but merely that a factor candidate is related to some PCA extracted from the covariance matrix. Just one, the first PCA, is sufficient for a factor candidate to pass the necessary conditions.

[10] As opposed to relative asset pricing such as comparing an option price to the underlying stock price.

must be high-frequency changes in privately-held market perceptions of pervasive macro-economic conditions. Perceptions could include (a) rational anticipations of change in macro conditions that are truly pervasive such as real output growth, real interest rates, inflation, energy, etc., and (b) behavior-driven pervasive shocks in confidence or risk perceptions such as panics, liquidity crises, etc.

To do a really good job, we must be able to identify and measure the pervasive factor perceptions and then to estimate factor sensitivities (betas) for every real asset. The first job is to identify and measure the factors. Existing literature has studied several alternative approaches. As discussed in the previous section, one approach relies on an entirely statistical method such as principal components or factor analysis, (e.g., Roll and Ross, 1980; Connor and Korajczyk, 1988.) A second approach pre-specifies macro-economic variables that seem likely to be pervasive and then pre-whitens the official numbers pertaining to such low frequency constructs as industrial production, inflation, and so on, (e.g., Chen, Roll and Ross, 1986.) Then there is the approach of relying on asset characteristics to develop proxies that are empirically related to average returns (e.g., Fama and French, 1993, Carhart, 1997.)

V. Putting it All Together: Linking Proposed Factors to the Covariance Matrix

Given the discussion above, we are ready to outline the first stage of our protocol for identifying factors. This stage identifies factors that move asset prices systematically but it does not distinguish between pervasive priced factors (with risk premiums) and diversifiable factors. That crucial information is postponed to a later stage. Here are the recommended steps for this first stage.

First, collect a set of N equities for the factor candidates to explain. The test assets should belong to different industries and have enough heterogeneity so that the underlying risk premium associated factors can be detected.

Second, extract L principal components from the return series, using the asymptotic approach of Connor and Korajczyk (CK) (1988). With T time-series units up to time t, the procedure involves computing the TxT matrix $\mathbf{\Omega_t = (1/T)\,RR'}$, where **R** is the return vector. CK show that for large N, analyzing the eigenvectors of $\mathbf{\Omega_t}$ is asymptotically equivalent to factor analysis. The first L eigenvectors of $\mathbf{\Omega_t}$ form the factor estimates. The cutoff point for L < N should be designated in advance; for instance, L could be chosen so that the cumulative variance explained by the principal components is at least ninety percent. Note that since, in most finance applications, N>>T, the approach has the virtue of allowing us to work with the smaller-dimension TxT matrix $\mathbf{\Omega_t}$, as opposed to the traditional NxN covariance matrix used for factor analysis.

Third, collect a set of K factor candidates. These could be well known characteristics-based candidates such as size, book/market, momentum, or any of the 50 or so documented in Subrahmanyam (2010), or the 316 from Harvey et al. (2016), or any new candidate as yet to be suggested.

Fourth, using the L eigenvectors from step #2 and the K factor candidates from step #3, calculate the covariance matrix over a period up to time t, $\mathbf{V_t}$ **(L+K x L+K)**.

Fifth, from the covariance matrix $\mathbf{V_t}$, in each period t, break out a sub-matrix, the cross-covariance matrix, which we denote $\mathbf{C_t}$. It has K rows and L columns (i.e., K x L); the entry in the i^{th} row and j^{th} column being the covariance between factor candidate i and eigenvector j. It will also be necessary to break out the covariance sub-matrix of the factor candidates, $\mathbf{V_{f,t}}$ **(K x K)** and the covariance sub-matrix of the real eigenvectors, $\mathbf{V_{e,t}}$ **(L x L)**.

Sixth, compute canonical correlations between the factor candidates and the corresponding eigenvectors from the second step. This involves first finding two weighting column vectors, a_t and b_t, on the factor candidates and eigenvectors, respectively (a_t has K rows and b_t has L rows) to maximize the correlation between the two weighted vectors. The covariance between the weighted averages of factor candidates and eigenvectors is $a_t'C_tb_t$, and their correlation is

$$\rho = \frac{a_t' C_t b_t}{\sqrt{a_t' V_{f,t} a_t b_t' V_{e,t} b_t}}.$$

The correlation is maximized over all choices of a_t and b_t. It turns out that the maximum occurs when the weights satisfy $a_t = V_{f,t}^{-1/2} h_t$ where h_t is the eigenvector corresponding to the maximum eigenvalue in the matrix $V_{f,t}^{-1/2} C_t V_{e,t}^{-1} C_t' V_{f,t}^{-1/2}$. The vector b_t is proportional to h_t. One then maximizes the correlation again, subject to the constraint that the new vectors are orthogonal to the old one, and so on. This way, there are min(L,K) pairs of orthogonal canonical variables sorted from the highest correlation to the smallest. Each correlation can be transformed into a variable that is asymptotically distributed as Chi-Square under the null hypothesis that the true correlation is zero.[11] This provides a method of testing whether the factor candidates as a group are conditionally related (on date t) to the covariance matrix of real returns (as represented by Eq. (3)). Also, by examining the relative sizes of the weightings in a_t, one can obtain an insight into which factor candidates, if any, are more related to real return covariances. We describe the latter procedure in detail within our empirical application in Section VII.

The intuition behind the canonical correlation approach is straightforward. The true underlying drivers of real returns are undoubtedly changes in perceptions about macroeconomic variables (see Section IV above). But the factor candidates and the eigenvectors need not be

[11] See Anderson (1984, ch. 12) or Johnson and Wichern (2007).

isomorphic to a particular macro variable. Instead, each candidate or eigenvector is some linear combination of all the pertinent macro variables. This is the well-known "rotation" problem in principal components or factor analysis.[12] Consequently, the best we can hope for is that some linear combination of the factor candidates is strongly related to some different linear combination of the eigenvectors that represent the true factors in Eq. (1). Canonical correlation is intended for exactly this application.

Any factor candidate that does not display a significant (canonical) correlation with its associated best linear combination of eigenvectors can be rejected as a viable factor. It is not significantly associated with the covariance matrix of real asset returns.

VI. Putting it All Together: Testing for Whether a Risk Factor is Priced

Factor candidates that are associated with the covariance matrix of returns but do not entail risk premiums must, according to theory, be fully diversifiable. In principle, this sufficiency stage of ascertaining whether factor candidates command risk premiums is easy. We simply run a pooled cross-section/time series panel with real returns as dependent variables and betas on surviving factors as the explanatory variables, taking account of correlations across assets and time (Cf. Petersen, 2009). This should be done with <u>individual</u> real asset returns on the left side, not with portfolio returns, because portfolios might diversify away and thus mask relevant risk- or return-related features of individual assets. Diversification into portfolios can mask cross-sectional phenomena in individual assets that are unrelated to the portfolio grouping procedure. Roll (1977) argues that the portfolio formation process makes it difficult to reject the null hypothesis of no

[12] The rotation problem is resolved by placing restrictions on the extracted factors. In principal components, the restriction is that successive factors explain the maximum amount of remaining variance. In factor analysis, restrictions are imposed on the factor covariance matrix; (e.g., it is diagonal or lower triangular.)

effect on security returns. Advocates of fundamental indexation (Arnott, Hsu and Moore, 2005) argue that high market value assets are overpriced and vice versa, but any portfolio grouping by an attribute other than market value itself could diversify away such mispricing, making it undetectable.

Second, test portfolios are typically organized by firm characteristics related to average returns, e.g., size and book-to-market. Sorting on characteristics that are known to predict returns helps generate a reasonable variation in average returns across test assets. However, Lewellen, Nagel, and Shanken (2010) point out sorting on characteristics also imparts a strong factor structure across test portfolios. Lewellen et al. (2010) show that even factors that are weakly correlated with the sorting characteristics would explain the differences in average returns across test portfolios regardless of the economic theories underlying the factors. They caution about the low dimensionality issue when portfolios are used, decreasing test power. That is, there are fewer explanatory variables with portfolios than with individual assets. Lo and MacKinlay (1990) support this strand of the argument and show that, in contrast to Roll (1977), forming portfolio on characteristics makes it likely to reject the null hypothesis too often because of a “data snooping” bias.

Third, forming portfolios might mask cross-sectional relation between average returns and factor exposures (“betas”). To illustrate, the cross-sectional relation between expected returns and betas under the single-factor CAPM holds exactly if and only if the market index used for computing betas is on the mean/variance frontier of the individual asset universe. Errors from the beta/return line, either positive or negative, imply that the index is not on the frontier. But if the individual assets are grouped into portfolios sorted by portfolio beta and the individual errors are not related to beta, the analogous line fitted to portfolio returns and betas will display much smaller errors. This could lead to a mistaken inference that the index is on the efficient frontier.

Finally, the statistical significance and economic magnitudes of risk premiums could depend on the choice of test portfolios. For example, the Fama and French size and book-to-market risk factors are significantly priced when test portfolios are sorted based on corresponding characteristics, but they do not command significant risk premiums when test portfolios are sorted only based on momentum. Brennan, Chordia, and Subrahmanyam (1998) also show different results for different sets of portfolios depending on characteristics used to form such portfolios.

The preceding discussion indicates that a properly-specified regression analysis based on individual securities is more desirable than a portfolio approach to identify risk premia. A variant of the panel approach of Petersen (2009) is standard in finance; it was originated by Fama and MacBeth (FM) (1973). The only real difficulty is that the regression betas, the factor loadings, are not known quantities and must be estimated. This implies a classic errors-in-variables (EIV) problem because the betas are the explanatory variables in each FM cross-section. Since the estimated betas inevitably contain measurement errors, the cross-sectional regressions have biased coefficients. To see this effect, let $\mathbf{R_t}$ represent a cross-sectional vector of excess returns, the dependent variable at time t, and suppose the true model is

$$\mathbf{R}_t = \boldsymbol{\beta}(\boldsymbol{\lambda} + \mathbf{f}_t) + \boldsymbol{\varepsilon}_t \equiv \boldsymbol{\beta}\boldsymbol{\gamma}_t + \boldsymbol{\varepsilon}_t$$

Here $\mathbf{R_t}$ is an N-vector (for N assets), $\mathbf{f}_t$ is a set of K unexpected (i.e., mean zero) factor realizations at t, $\boldsymbol{\lambda}$ is a K-vector of risk premiums, $\boldsymbol{\beta}$ is an NXK matrix of factor exposures and $\boldsymbol{\varepsilon}_t$ is an N-vector of idiosyncratic disturbances that are unrelated to everything. For convenience of exposition, we assume stationarity in all the parameters and ignore any possible intercept.[13]

The operational cross-sectional regression involves estimates of $\boldsymbol{\beta}$. If they have been estimated with an ordinary least squares (OLS) time series, the cross-sectional regression equation can be written as

$$\mathbf{R}_{\mathrm{t}} = \hat{\boldsymbol{\beta}}_{\mathrm{OLS}}\hat{\boldsymbol{\gamma}}_{\mathrm{OLS,t}} + \hat{\boldsymbol{\varepsilon}}_{\mathrm{OLS,t}}$$

where the "chapeaus" denote estimated values; i.e., $\hat{\boldsymbol{\gamma}}_{\mathrm{t}}$ contains the K cross-sectional estimated slope coefficients at t and $\hat{\boldsymbol{\varepsilon}}_{\mathrm{t}}$ is the vector of estimated cross-sectional residuals.

A cross-sectional OLS produces the following estimates

$$\hat{\boldsymbol{\gamma}}_{\mathrm{OLS,t}} = (\hat{\boldsymbol{\beta}}_{\mathrm{OLS}}'\hat{\boldsymbol{\beta}}_{\mathrm{OLS}})^{-1}\hat{\boldsymbol{\beta}}_{\mathrm{OLS}}'\mathbf{R}_{\mathrm{t}} \ .$$

Assume now that the estimated OLS betas above conform to $\hat{\boldsymbol{\beta}}_{\mathrm{OLS}} = \boldsymbol{\beta} + \boldsymbol{\zeta}$ where the estimation error, $\boldsymbol{\xi}$, is independent of $\boldsymbol{\beta}$ and all other variables. Then,

$$\hat{\boldsymbol{\gamma}}_{\mathbf{OLS,t}} = (\hat{\boldsymbol{\beta}}_{\mathbf{OLS}}'\hat{\boldsymbol{\beta}}_{\mathbf{OLS}})^{-1}\hat{\boldsymbol{\beta}}_{\mathbf{OLS}}'(\boldsymbol{\beta}\boldsymbol{\gamma}_{\mathbf{t}} + \boldsymbol{\varepsilon}_{\mathbf{t}}) = (\hat{\boldsymbol{\beta}}_{\mathbf{OLS}}'\hat{\boldsymbol{\beta}}_{\mathbf{OLS}})^{-1}\hat{\boldsymbol{\beta}}_{\mathbf{OLS}}'[(\hat{\boldsymbol{\beta}}_{\mathbf{OLS}} - \boldsymbol{\zeta})\boldsymbol{\gamma}_{\mathrm{t}} + \boldsymbol{\varepsilon}_{\mathrm{t}}] \ .$$

Asymptotically, since $\boldsymbol{\varepsilon}_{\mathrm{t}}$ is not related to any other variable,

$$\hat{\boldsymbol{\gamma}}_{\mathbf{OLS,t}} = \boldsymbol{\gamma}_{\mathrm{t}} - (\hat{\boldsymbol{\beta}}_{\mathbf{OLS}}'\hat{\boldsymbol{\beta}}_{\mathbf{OLS}})^{-1}(\hat{\boldsymbol{\beta}}_{\mathbf{OLS}}'\boldsymbol{\zeta})\boldsymbol{\gamma}_{\mathrm{t}}$$

And the second term does not disappear, even asymptotically, because $\hat{\boldsymbol{\beta}}_{\mathbf{OLS}} = \boldsymbol{\beta} + \boldsymbol{\zeta}$, which contains the common term $\boldsymbol{\xi}$. Asymptotically,

$$\hat{\boldsymbol{\gamma}}_{\mathbf{OLS,t}} = \boldsymbol{\gamma}_{\mathrm{t}} - (\hat{\boldsymbol{\beta}}_{\mathbf{OLS}}'\hat{\boldsymbol{\beta}}_{\mathbf{OLS}})^{-1}(\boldsymbol{\zeta}'\boldsymbol{\zeta})\boldsymbol{\gamma}_{\mathrm{t}}$$

The bias depends on the variance of the errors in the OLS time series betas. The underlying

[13] The dependent variables are excess returns relative to the riskless rate; hence the intercept should be zero if assets are priced rationally.

problem is that when running a cross-sectional regression, the estimates of betas can be somehow related to the estimation error in the average returns, leading to endogeneity. The estimation error in betas is vanishing (it is of the order $1/\sqrt{T}$ as $\hat{\beta}$ converges to the true beta). Therefore, generally even if there is a non-zero correlation, it cannot affect the consistency of the risk premia estimates, only finite sample properties and the asymptotic distribution, e.g. standard errors.

The error variances for individual assets are almost certainly greater than they are in betas estimate for portfolios, which explains why Fama and French (1992) used the latter. In our analysis, we adopt their procedure in using portfolios to obtain beta estimates, assigning portfolio betas to the constituent individual stocks, and then checking to see if the factor is priced via FM regressions. While this exercise is performed on the market factor in Fama and French (1992), it has not been performed consistently on other factors. Indeed, Fama and French (1993) do not perform this second stage exercise on individual securities for their SMB and HML factors. To address the EIV problem, we do a double sorting. That is, we sort stocks based on size into ten portfolios and then in each size decile, we sort stocks into ten portfolios by market beta. Then, we independently do the same double sorts but instead of sorting by market beta, we sort by HML beta. We do the same for the factors that pass necessary conditions. Then, we assign the relevant portfolio betas on stocks. Following this assignment procedure, we consider the significance of the betas in FM regressions.

As a final check following the FM regression, we propose that for a genuine risk factor, its reward-to-risk ratio must be within reasonable limits. To take an extreme example, if a candidate traded version of a risk factor delivers a Sharpe ratio of three, it would be difficult to argue that this magnitude is consistent with the factor capturing a source of priced risk, given that Sharpe ratios for most well-diversified market indices are usually less than unity over periods of a decade or more (MacKinlay, 1995). Thus, we propose an investigation of factor-based Sharpe ratios exceed a "reasonable bound." Our bound is the one proposed by MacKinlay (1995). He argues that based on the historical mean excess return and volatility of the CRSP value-weighted index, a reasonable annualized Sharpe ratio for a risk factor is 0.6 (corresponding for example, to an annualized excess return of 10% and a standard deviation of 16%). We propose to test whether each individual proposed factor delivers a Sharpe ratio is statistically higher than the proposed MacKinlay bound.

VII. An Empirical Analysis

This section presents an example of the suggested protocol using simultaneous monthly return observations over a half century, 1965-2014 inclusive. The sample assets are individual U.S. equities listed on CRSP. We select stocks based on Fama and French (1992).

As candidate factors, we include the five Fama-French (2015) market (Rm-Rf), SMB, HML, profitability (RMW), and investment (CMA) factors, the Carhart (1997) momentum factor (MOM), the riskfree rate (Rf),[14] a traded liquidity factor (LIQ), and ST_REV as well as LT_REV, the factors based on short-term (monthly) and long-term reversals, respectively. We also include the Chen, Roll, and Ross (1986) factors. Specifically, ΔDP, ΔIP, ΔTS, UNEXPI, and ΔEI are the traded versions of the Chen, Roll, and Ross (1986) factors.

We obtain Rm-Rf, SMB, HML, RMW, CMA, MOM, ST_REV, and LT_LEV from Ken French's data library. We construct the traded liquidity factor, and Chen, Roll, and Ross (1986)'s five factors using Cooper and Priestley (2011, henceforth CP)'s methodology. We first obtain the

[14] The risk free rate proxy (obtained from Federal Reserve Bank of St. Louis) is a three-month Treasury Bill rate. Fluctuations in this proxy can be priced when investors have longer horizons, and these fluctuations are not readily diversifiable.

raw CRR factors as follows. The default premium, ΔDP, is the yield spread between Moody's Baa and Aaa corporate bonds. The growth rate of industrial production, ΔIP is log (IP_t) subtracted by log (IP_{t-1}) where IP_t is the index of industrial production in month t. ΔTS is the term premium defined as the yield spread between the long-term (10-year) and the one-year Treasury bonds. UNEXPI and ΔEI are unexpected inflation and change in expected inflation, respectively. Similar to Chen, Roll and Ross (1986), UNEXPI is derived from the total seasonally adjusted consumer price index (CPI). We collect the inputs for these five factors from the website of Federal Reserve Bank of St. Louis. We obtain Pastor and Stambaugh (2003)'s innovation (INNOV) series from Lubos Pastor's website to construct liquidity-traded portfolio. We do not apply their traded factor (VWF) because it is in fact a zero net investment portfolio formed by longing stocks with high loadings on INNOV and shorting stocks with low loadings. A properly specified factor loading requires a loading of one on the actual factor (the INNOV series) and zero on other factors. This concept is consistent with CP, so we use the CP method to construct a traded version of the Pastor and Stambaugh's factor using the INNOV series as the input.

In Table 1, we present summary statistics associated with our candidate factors. We observe that the liquidity, market, and MOM factors tend to be the most volatile, whereas RF and the Chen, Roll, and Ross (1986) factors tend to exhibit the least variations (except for industrial production.) The liquidity, momentum, and short-term reversal factors tend to exhibit the highest mean returns. The momentum factor has negative skewness, as does the market factor.

To construct mimicking portfolios of all six (the five CRR factors and liquidity factor), we collect the return of fifty portfolios (the returns of the ten equal-weighted size portfolios, ten equal-weighted book-to-market portfolios, ten value-weighted momentum portfolios, ten equal-weighted investment portfolios and ten equal-weighted operating profitability portfolios) from Ken French's website. We apply CP, who adopt the Lehmann and Modest (1988, Section 2) approach as follows. Returns of each of the 50 test assets are regressed on the five CRR factors and INNOV i.e., we perform 50 time-series regressions producing (50x6) matrix B of slope coefficients against the five CRR factors and INNOV. We generate the variance-covariance matrix (50x50) of the error terms for these regressions. The weight on the mimicking portfolios (W), a 6 x 50 matrix, is computed as $(B^{'}V^{-1}B)^{-1}B^{'}V^{-1}$. R, the returns of 50 portfolios, is a Tx50 matrix where T is a number of months. The return of the CRR mimicking portfolios is $WR^{'}$, a 6 x T matrix where each row represents mimicking portfolio return over the sample period. The CP procedure generates mimicking portfolio where beta is one with respect to a particular factor.

The next step in our analysis is to compute asymptotic principal components that represent the covariance matrix. Being sensitive to non-stationarity in the data, we split the overall sample into five subsamples with ten years each, while the first spans seven years because one of the potential factors was unavailable for the first three years, 1965-67 inclusive. For each subsample, we extract ten principal components from the return series, using the method of Connor and Korajczyk (CK) (1988). In Table 2 we present the summary statistics for the concatenated principal components. The components tend to be positively skewed and tend to exhibit positive kurtosis as well. We retain only the first 10 PCs because they account for close to 90% of the cumulative eigenvalues or the total volatility in the covariance matrix, suggesting these 10 PCs capture most of the stock variations. We admit that the number of retained PCs is somewhat arbitrary. If something is omitted, it is omitted for all stocks and should not have impact on the pattern of detected factors. Considering the average across 50 sample years of the cumulative percentage of variance explained within the estimation year, the first principal component explains about 38% of the variance and five PCs explain over 75%. Thus, this is evidence of multiple factors, not just one. We also find there is some variation in cumulative variance explained within each estimation year by the first 10 PCs from year to year and the total of variation throughout the sample period is about 90%.

Our protocol then proceeds to calculate canonical correlations. Since we have several factor candidates, there are several pairs of canonical variates, each pair being orthogonal to the others and having a particular intercorrelation. Canonical correlation sorts these pairs from largest to smallest based on their squared correlation, but the correlation itself can be either positive or negative. Panel A of Table 3 reports, in the second and third columns, the canonical correlations for the covariance matrices, and associated t-statistics, covering 1968-1974 monthly. The next few columns provide the correlations and t-statistics for subsequent periods.

As indicated by these results, the first and largest canonical correlation is dominant. Its mean conditional value is close to unity and strongly significant. Across all subperiods, only one correlations falls below .2 in absolute terms. The top five canonical correlations are significant in every subperiod we consider.

Information on significant relations between factor candidates and the principal components is reported in Panel B of Table 3. We use the following procedure to derive the significance levels of each factor candidate. First, for each of the ten canonical pairs,[15] the eigenvector weights for the 10 CK PCs are taken and the weighted average CK PC (which is the canonical variate for the 10 CK PCs that produced the canonical correlation for this particular pair) is constructed. Then, a regression using each CK PC canonical variate as the dependent variable and the actual candidate factors values as independent variables is run over the sample months in each subperiod. The square root of the R-square from this regression (not the adjusted R-square)

[15] Recall that there are min(L,K) possible pairs, and in our application, L=10 and K=15.

is the canonical correlation. The coefficients of the regression are equal (after proper normalization) to the eigenvector's weighting elements for the candidate factors. The t-statistics from the regression then give the significance level of each candidate factor. Since there are 10 pairs of canonical variates in each of the five subperiods and a canonical correlation for each one, there is a total of 50 such regressions. The first row presents the mean t-statistic of all canonical correlations. The second row shows the mean t-stat across cases where the canonical correlation itself is statistically significant. The fifth through ninth rows present the number of significant canonical correlations in each decade and the bottom row presents its average.

Since the t-statistics are those of coefficients that represent the square roots of eigenvalues, they are always positive, so a one-tailed cutoff is appropriate. We find that the mean t-statistics for the Fama-French three factors all exceed the one-tailed 2.5% level cutoff of 1.96. Further, the mean t-statistics for momentum are significant at the 5% levels for a one-tailed test. The average number of significant t-statistics exceeds two for all factors except CMA, ΔIP, ΔEI, and LIQ.

We adopt the following screening criteria based on Table 3. We deem a factor a candidate risk factor if in Table 3, Panel B, the average t-statistic of significant canonical correlation in the second row exceeds the one-tailed, 2.5% cutoff based on Chi-square value, <u>and</u> the average number of significant t-stats (last row of Table 3, Panel B) exceeds 2.5 (out of a maximum of five). We focus on the significant canonical correlation, rather than all canonical correlations, because insignificant CCs imply that none of the factors matter, so it is not desirable to use them. This results in nine factors including the three Fama-French (1992) market (Rm-Rf), SMB, HML factors, and one of the two new Fama-French (2015) factors, RMW, followed by MOM, LT_REV, ΔDP, ΔTS, and UNEXPI. These factors pass the screen of being materially related to the covariance matrix of returns across the subperiods we consider.

VIII. Are the Factors Priced?

A. Regression Analysis

The next step in the protocol is to check the sufficient conditions for a factor candidate that satisfies the necessary conditions for being priced in the cross-section of returns. To address this issue, our procedure is as follows. We estimate two versions of factor betas, one uncorrected, and one corrected for EIV. For the non-EIV estimation, OLS multiple regressions are run for each stock on the nine acceptable factors using all available observations for that stock. Then, for each calendar month in the sample, January 1965 through December 2014 inclusive, available individual stock returns are multiple-regressed cross-sectionally on the nine OLS beta estimates. The time series averages of the cross-sectional coefficients, termed the "risk premiums," along with associated sampling statistics, are then computed.

For the EIV calculations, stocks are sorted into ten groups (deciles) by market capitalization (Size), annually at the end of each June, based on NYSE size decile breakpoints. Then within each Size decile, stocks are sorted further by the OLS betas of the first factor (Rm-Rf) into ten deciles, thus resulting in 100 Size/first factor beta groups. Within each of the 100 groups, the equal weighted average first factor beta of the group is assigned to each stock within that group. This is repeated for each of the eight additional betas whose factors pass the necessary conditions. Hence, for each of the nine factors, the individual stock beta is replaced by the equally-weighted mean beta of the size/beta sorted group to which the stock belongs. Subsequently, over each of the following 12 calendar months, July through June of the next year, all available individual stock returns are multiply-regressed cross-sectionally on the nine EW mean betas assigned to that stock. This is repeated for each June in the sample period, 1965-2014; then the time series average of the cross-sectional coefficients, i.e., the risk premiums, along with associated sampling statistics are computed. There are 594 months in the time series; the first six months are not used because the first sort is done in June 1965 and the last sort in June 2014 has only six available subsequent months.

We also control for stock-specific characteristics corresponding to some of the risk factors (specifically, those that are in fact associated directly with characteristics). The idea is to conduct a horse race between factor betas and the characteristics in the spirit of Daniel and Titman (1997).[16] The characteristics are RetLag1 (the one month lagged return), Lag2_12 (the two to twelve month lagged return, Lag13_36 (the thirteen to thirty-six month lagged return), Size, Book/Mkt, ProfRatio (Profitability), AssetGrth (asset growth), and Amihud's (2002) illiquidity measure. These characteristics are defined in detail within the appendix.

In Table 4, we present summary statistics for the (non-EIV-corrected) betas as well as the characteristics. Variables are first averaged cross-sectionally, then in the time series. We find that the innovation to default premium carries the lowest (most negative) mean betas whereas SMB and the market factor present the highest (most positive) mean betas. Amongst the characteristics, the one-month lagged returns are the most volatile, and prior (2-12) returns also exhibit relatively high variation. SMB also exhibits considerable negative skewness.

Table 5 presents estimated risk premiums for both the non-EIV-corrected and EIV-corrected betas. The first two models presents the FM regression for the nine factors that pass our necessary conditions. Rm-Rf, RMW, Momentum and unexpected inflation are the only factors that command a risk premium. [Note that a negative premium of unexpected inflation is expected according to the way it is defined; high unexpected inflation is an adverse event which has a

[16] This horse race is relevant because, as Karolyi (2016) points out, characteristics may be related to factor loadings, and without a proper setting that includes both betas and characteristics, one may misleadingly conclude that a characteristic represents market inefficiency.

downward impact on stock prices.] Following the EIV correction, the excess market return, RMW, and unexpected inflation are significant.

In Models 3 and 4, we present results from FM regressions with the nine betas on the factors that pass necessary conditions, as well as the characteristics. The results show that Rm-Rf, HML, RMW, LT_Rev, ΔDP, UNEXPI and *all* characteristics command significant risk premia. After correcting for EIV, the same factors except LT_Rev, UNEXPI, and lag13_36 remain significant. Further, ΔTS becomes significant in this specification. Surprisingly, the HML risk premium is negative and significant, presumably because the beta estimates of HML (and CMA) are contaminated by multicollinearity. Indeed, we find that the correlation between HML and CMA is .71. It is noteworthy that in the EIV-corrected regression the insignificance result for lag13_36 is consistent with that of long-term reversals.

The Amihud measure causes a loss in sample size of more than 50%. Hence we do not report results that include this measure. However, in unreported analyses, we do run regressions with this measure (along with all of the other betas and characteristics), and find that the results remain qualitatively unchanged. These results are available upon request.

We note that the characteristics in general are far more significant than the betas. This appears to represent evidence against market efficiency. We also note that there is no *major* difference between correcting for EIV and not doing so. The results for the simplest cross-sectional regression risk premium estimates match the estimates using double sorts on all candidate factors (double sorts on size and beta and then portfolio betas replacing individual stock betas.) We note that the Fama-French (1992) method introduces its own EIV problem; using portfolio betas in place of individual stock betas. This is an EIV since the true but unknown individual stock beta is not used.[17]

B. Hedge Portfolio Returns

While in Section VI, we describe the desirability of using individual securities as test assets, in this subsection we perform a robustness check with the standard methodology of forming hedge portfolios. These are formed by going long (short) in the portfolios with the highest (lowest) beta in deciles.[18] Table 6 Panel A presents the hedge portfolios that are long the top decile and short the bottom decile after sorting by individual stock (EIV-corrected) betas on each factor with replacement. For example, the individual stock betas on Rm-Rf are sorted and a portfolio is formed from the stocks with the largest 10%, equal weighted, and then another portfolio is formed from the stocks with the smallest 10%. The second portfolio's return is subtracted from the first one. Then, for SMB, the same procedure is repeated. Individual SMB betas are sorted and then the hedge portfolio's return comes from the largest less the smallest decile. This is repeated for each candidate factor. The results, presented in Table 6, Panel A indicate that hedge portfolios for the market and RMW factors remain significant, whereas all of the Chen, Roll, and Ross (1986) factors, namely the innovations to the default premium, term spread, and inflation, also are significant. The sign of the hedged portfolio returns for unexpected inflation is negative. This is consistent with the sign of its risk premium in Table 5.

[17] A method that, at least in principle, is effective in eliminating EIV is an instrumental variable (IV) approach where odd month betas are regressed on even month betas as an instrument and vice versa (for regressions involving odd and even months, respectively), which we do not report because this IV approach does not work well in our multiple factor setting. Specifically, the approach is somewhat unstable in the multifactor case. Additionally, some outlier stocks cause bad errors and have very poor instruments. Details are available upon request.

[18] Kozak, Nagel, and Santosh (2017) argue that a return spread from sorted betas is not necessarily evidence that the associated factor is actually "priced" due to risk because behavioral biases can affect factor loadings; in turn, this implies that systematic investor mistakes can be reflected in factor pricing.

The final step in our protocol checks for whether the Sharpe ratios generated by the factors are within reasonable magnitudes. Note that the traded versions of our factors are zero net investment portfolios. We thus check if the Sharpe ratio of a portfolio that combines each of them with a representative long-only portfolio is below a reasonable bound, relative to a representative long-only equity portfolio. Accordingly, in Panel B, we present the mean, standard deviation, and Sharpe ratios for a portfolio that combines the market (i.e., the value-weighted CRSP index) with the zero net investment long-short portfolio from Panel A and tests whether the resulting Sharpe ratio is statistically greater than the bound of 0.6 recommended by MacKinlay (1995, p.13). We find that none of the Sharpe ratios exceed this threshold; indeed, eight of nine are below the threshold. Thus, our priced factors command Sharpe ratios of a magnitude consistent with risk-based pricing.

In Panels C and D, we repeat the analyses of Panels A and B, except that we form the long-short portfolio based on the top (bottom) 30% of stocks with the highest and lowest betas, instead of using deciles. The results are similar. In Panel D, almost all of the Sharpe ratios are lower than the MacKinlay (1995) threshold.

Table 7 presents hedge portfolio results similar to those in Table 6, but using characteristics rather than factor betas as the sorting criteria. Panel A demonstrates that RetLag1 (monthly reversals), Lag2_12 (momentum), Book/Mkt, ProfRato, and AsstGrth provide statistically significant average returns. The negative sign for RetLag1 and AssetGrth is consistent with their negative risk premium in Table 7. Panel B tests whether the Sharpe ratio from combining the market and the zero net investment portfolio from Panel A is greater than the MacKinlay bound of 0.6. The results show that RetLag1, Lag2_12, Book/Mkt and ProfRato provide SRs that are statistically higher than the bound. That is, the strategies associated with these characteristics, although linked with premiums, provide abnormally high Sharpe ratios. When we construct hedge portfolios using the top and bottom 30% of stocks in Panel C, the results are similar to those in Panel A. The Sharpe ratios in Panel D also have largely similar patterns as those in Panel B except that only Lag2_12 and Book/Mkt yield SR greater than 0.6.[19]

Table 8 presents the cross-correlations obtained using the hedge portfolios formed in Panel A of Tables 6 and 7, across the factors and the characteristics. 86% of the correlations are below 0.5 in absolute magnitude. The hedge portfolio for the profitability characteristic is negatively correlated with SMB, whereas HML, RMW, and MOM, not surprisingly, are positively correlated with their characteristic-based counterparts. The hedge portfolios corresponding to some of the CRR factors are also positively cross-correlated with some characteristic-based portfolios, but there is no ready explanation for these results, so we leave a full explanation for future research.

Overall, when subjected to a rigorous protocol, across both the regression and hedge portfolio method, a market factor, a profitability factor, and factors based on credit spreads, term spread, and unexpected inflation are related to the covariance matrix, command statistically significant risk premiums in all specifications, and yield reasonable Sharpe ratios. Almost all characteristics are associated with statistically significant premiums, but only momentum and the book/market anomaly yield Sharpe ratios that exceed a reasonable bound to be considered an abnormal profit opportunity.

[19] Note that the sample observations used are different for different runs. The non-EIV sample sizes are different from the double-sorted EIV sample sizes and the double-sort approach cannot be the same because it uses portfolio betas to replace the individual stock betas. This means that some observations are available after the double size sorts even when the individual betas are not available; e.g., because we require 24 observations to compute them. The hedge portfolios are formed by sorting on betas or characteristics one variable at a time. Obviously, the beta sorted hedge portfolios and characteristics sorted hedge portfolios have different stocks and, equally obviously, the particular stocks within the 10% and 30% portfolios are different for each beta and characteristic.

IX. Summary and Conclusions

Our goal in this paper is to suggest a protocol for sorting factors that potentially are the drivers of asset returns and for determining whether they are associated with risk premiums. We are striving for a procedure that will be acceptable to scholars and practitioners; a standard for future factor identification. The protocol we present here is just an outline and it will undoubtedly be modified by others to render it more acceptable. Ours is just a first attempt.

We begin with an empirical observation: asset returns reveal an underlying factor structure because diversification is not all that powerful. Moreover, weak correlations across diversified portfolios in different asset classes and/or countries suggest that there must be multiple factors.

An underlying factor cannot have movements that are easily predictable because asset prices adjust in advance. One implication is that a characteristic cannot be a factor. This rules out firm-specific attributes such as size, dividend yield, book/market and so on. Such characteristics can be related to factor loadings or exposures, but they cannot be factors per se because they are known. Over 300 factors and characteristics have been claimed by the extant literature to explain expected returns, and there seems to be confusion about factors versus characteristics. The characteristics might be related to risk exposures (i.e., to "betas" on unknown risk drivers) but they might also be symptomatic of arbitrage opportunities. Our protocol tries to ascertain their true nature.

Our suggested protocol has two stages. The first stage provides a sequence of steps that represent necessary conditions for factor candidates to be valid. A candidate that does not satisfy these conditions is not a risk factor, but this does not imply that rejected candidate is uninteresting, particularly to investors. Indeed, if such a rejected candidate is related to average returns on any set of assets, there is a potential profit opportunity. In principle, a diversified portfolio could be constructed to produce significant return with minimal risk. The second suggested stage entails testing whether factor candidates that satisfy the necessary conditions are pervasive and consequently have associated risk premiums or instead are diversifiable even though they affect some real assets but not all of them.

One very important application of our protocol would be to study the relative importance of industry, country, and global factors. Intuitively, some factors might be pervasive globally but there is some doubt because many or perhaps most countries do not share fully integrated macroeconomic systems. This leaves room for country factors and, indeed, most previous studies of factors have been exclusively domestic. Finally, at an even lower level of aggregation, industry factors clearly have the ability to explain some individual firm covariances; but are they diversifiable and carry no risk premiums or, instead, are at least some of them sufficiently pervasive to be genuine risk factors at either the country or global level?

Industry factors have been studied for a long time, from King (1966) through Moskowitz (2003). It seems to us that a very useful exercise would be to study industry factors globally. Following our suggested protocol, we would only need to assemble some international real asset returns, extract a time series of eigenvectors from their time-varying covariances, and check whether industry factors satisfy the necessary and sufficient conditions of Sections V and VI above.

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Appendix

Variable Definition

Variables	**Description**
Rm-Rf	Market excess return factor from Ken French's library
SMB	Small minus big factor from Ken French's library
HML	High minus low B/M factor from Ken French's library
RMW	Robust minus weak operating profitability from Ken French's library
CMA	Conservative minus aggressive investment from Ken French's library
MOM	Momentum factor from Ken French's library
Rf	3-month Treasury Bill rate from Federal Reserve Bank of St. Louis
LIQ	Traded factor of liquidity constructed from Pastor and Stambaugh (2003)'s innovation series, collected from Lubos Pastor's website
ST_Rev	Short-term reversal factor from Ken French's library
LT_Rev	Long-term reversal factor from Ken French's library
ΔDP	Traded factor of default risk premium where default risk premium is the yield spread between Moody's Baa and Aaa corporate bonds from Federal Reserve Bank of St. Louis
ΔIP	Traded factor of growth rate of industrial production, where industrial production is from the Federal Reserve Bank of St. Louis
ΔTS	Traded factor of term premium where term premium is the yield spread between the ten-year and the one-year Treasury bonds from Federal Reserve Bank of St. Louis
UNEXPI	Traded factor of unexpected inflation where unexpected inflation at time t is the difference between inflation at time t and expected inflation at time t-1. Both are available from the Federal Reserve Bank of St. Louis
ΔEI	Traded factor of change in expected inflation, where expected inflation is from the Federal Reserve Bank of St. Louis
RetLag1	Return in prior month from CRSP (%)
Lag2_12	Return in prior 2nd to 12th month from CRSP (in % per month)
Lag13_36	Return in prior 13th to 36th month from CRSP (in % per month)
SizeLag1	Natural log of size (market cap) lagged one-month relative to the return. For instance, if the return is for February 1965, SizeLag1 is the log market cap at the end of January 1965. Price and number of shares outstanding are from CRSP.
Book/Mkt	Book-to-market equity, the natural log of the ratio of the book value of equity to the market value of equity. Book equity is total assets (Compustat data item 6) for year t-1, minus liabilities (181), plus balance sheet deferred taxes and investment tax credit (35) if available, minus preferred stock liquidating value (10) if available, or redemption value (56) if available, or carrying value (130). Market equity is price times shares outstanding at the end of December of t-1, from CRSP.

ProfRato	Profit ratio for June of year t is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity for the last fiscal year end in t-1.
AsstGrth	Annual firm asset growth rate is calculated using the year-on-year percentage change in total assets (Compustat data item 6). The firm asset growth rate for year t is estimated as the percentage change in data item 6 from fiscal year ending in calendar year t-2 to fiscal year ending in calendar year t-1,
Amihud's illiquidity ratio	The annual illiquidity ratio of stock i in year t measured as the average ratio of the daily absolute return to the (dollar) trading volume on that day divided by number of days for which data are available for stock i in year t.

Table 1
Summary Statistics for the Candidate Factors

Here are summary statistics for candidate factor realizations in % per month. The sample period spans 600 months from January 1965 through December 2014. See the Appendix for variable definitions.

	Mean	Median	Sigma	Skewness	Kurtosis	Maximum	Minimum
Rm-Rf	0.492	0.840	4.513	-0.524	1.823	16.100	-23.240
SMB	0.289	0.115	3.116	0.380	3.494	19.180	-15.360
HML	0.354	0.335	2.897	0.000	2.583	13.910	-13.110
RMW	0.253	0.165	2.163	-0.403	11.22	12.190	-17.570
CMA	0.324	0.195	2.038	0.269	1.585	9.510	-6.810
Rf	0.411	0.410	0.261	0.554	0.815	1.350	0.000
MOM	0.690	0.775	4.283	-1.401	10.77	18.380	-34.580
ST_Rev	0.497	0.330	3.180	0.367	5.459	16.200	-14.580
LT_Rev	0.301	0.185	2.534	0.629	2.628	14.490	-7.790
ΔDP	-0.061	-0.066	0.267	-0.017	1.913	1.050	-1.361
ΔIP	0.375	0.419	2.893	-0.316	1.496	12.670	-12.810
ΔTS	-0.099	-0.098	0.876	0.143	0.590	3.093	-3.190
UNEXPI	-0.203	-0.203	0.783	-0.025	0.487	3.011	-3.121
ΔEI	0.012	0.011	0.118	0.062	1.401	0.575	-0.464
LIQ	1.610	2.344	16.90	-0.096	2.807	91.310	-71.283

Table 2

Summary Statistics for Principal Components

Here are summary statistics over 600 months for principal components (PCs) extracted using the Connor and Koraczyk (1986) cross-sectional method. The entire data period spans January 1965 through December 2014, 50 years of monthly observations. For each decade within the fifty years, a 120X120 cross-sectional covariance matrix is computed for all available stocks with full records, from which ten principal components are extracted. The number of stocks included is 1,259 in 1965-1974, 2,331 in 1975-1984, 2,660 in 1985-1994, 3,145 in 1995-2004, and 3,349 in 2005-2014. Each PC has a mean of exactly zero and is normalized to have the same standard deviation.

	Median	Std. Dev.	Skewness	Kurtosis	Maximum	Minimum
PC1	0.011	0.091	-0.897	10.944	0.387	-0.733
PC2	-0.001	0.091	-0.144	16.288	0.657	-0.696
PC3	-0.002	0.091	2.408	40.851	0.968	-0.655
PC4	-0.006	0.091	1.419	18.751	0.689	-0.593
PC5	0.000	0.091	0.355	16.716	0.619	-0.709
PC6	-0.001	0.091	-0.337	11.743	0.450	-0.589
PC7	-0.001	0.091	0.160	11.438	0.523	-0.503
PC8	-0.004	0.091	1.715	17.438	0.780	-0.344
PC9	0.001	0.091	-0.244	13.472	0.630	-0.620
PC10	0.003	0.091	-0.408	8.2370	0.430	-0.509

Table 3

Canonical Correlations with Asymptotic PCs and Significance Levels of Factor Candidates

This table reports canonical correlations between Factor Candidates and Principal Components. The factor candidates include the five Fama-French (2015) factors, Rm-Rf, SMB, HML, RMW, and CMA along with RF, MOM, ST_REV, LT_REV, LIQ, ΔDP, ΔIP, ΔTS, UNEXPI, and ΔEI. See the Appendix for the variable definitions. The principal components are extracted as explained in Table 2 and the text using the Connor and Koraczyk (1986) cross-sectional method. Panel A reports ten canonical correlations for each decade, sorted in descending order by their estimated squares. Corresponding Newey-West T-statistics (ten lags) for the correlations are also reported. Panel B summarizes significance levels for factor candidates. The following procedure is implemented to derive the significance levels of each factor candidate: First, for each canonical pair, the eigenvector weights for the 10 CK PCs are taken and the weighted average CK PC, (which is the canonical variate for the 10 CK PCs that produced the canonical correlation for this particular pair) is constructed. Then, a regression using each CK PC canonical variate as the dependent variable and the candidate factor realizations as 15 independent variables is run over the sample months, 120 months for the last four decades and slightly fewer for the first decade.[20] The square root of the R-square from this regression (not the adjusted R-square) is the canonical correlation. The coefficients of the regression are equal (after proper normalization) to the eigenvector's elements for the candidate factors. The t-statistics from the regression then give the significance level of each candidate factor. There are 10 pairs of canonical variates in each decade and a canonical correlation for each one; thus, there is a total of 50 such regressions. In Panel B, the 1st row presents the mean t-statistic over all canonical correlations. The 2nd row reports the mean t-statistic when the canonical correlation itself is statistically significant. Rows#3 to #7 give the number of significant canonical correlation in each decade, respectively, and the bottom row (8) reports its average over the five decades. Critical rejection levels for the T-Statistic are 1.65 (10%), 1.96 (5%), and 2.59 (1%). T-Statistics breaching the 5% (1%) critical level are in boldface (boldface italic.)

(Table continued on next page)

[20] One of the factors is missing for the first 36 months.

Panel A: Canonical correlations

	1968-1974 84 Observations		1975-1984 120 Observations		1985-1994 120 Observations		1995-2004 120 Observations		2005-2014 120 Observations	
Canonical Variate	Canonical Correlation	NW T-Stat	Canonical Correlation	NW T-Stat	Canonical Correlation	NW T-Stat	Canonical Correlation	NW T-Stat	Canonical Correlation	NW T-Stat
1	0.999	***23.904***	0.998	***27.236***	0.995	***24.155***	0.989	***25.589***	0.997	***29.212***
2	0.967	***14.189***	0.906	***13.624***	0.869	***11.969***	0.943	***16.668***	0.913	***17.800***
3	0.885	***8.849***	0.833	***8.472***	0.727	***7.797***	0.791	***10.390***	0.858	***13.372***
4	0.779	***5.538***	0.649	***4.027***	0.705	***5.606***	0.760	***7.526***	0.756	***9.513***
5	0.641	***3.415***	0.602	**2.003**	0.566	***2.997***	0.633	***4.300***	0.746	***6.874***
6	0.560	**2.454**	0.428	-0.218	0.522	1.640	0.549	**2.253**	0.546	***3.325***
7	0.530	1.930	0.409	-0.851	0.422	0.195	0.387	0.562	0.514	**1.992**
8	0.508	1.334	0.261	-1.901	0.357	-0.648	0.363	0.299	0.385	0.299
9	0.390	0.413	0.228	-1.610	0.283	-1.351	0.307	-0.091	0.298	-0.438
10	0.344	0.290	0.185	-1.163	0.125	**-2.050**	0.246	-0.249	0.221	-0.632

Panel B: Significance levels for factor candidates

		Factor candidates													
	Rm-Rf	SMB	HML	RMW	CMA	RF	MOM	ST_Rev	LT_Rev	ΔDP	ΔIP	ΔTS	UNEXPI	ΔEI	LIQ
Mean_t	**5.787**	**3.921**	**2.034**	1.519	1.357	1.320	1.950	1.332	1.634	1.640	1.138	1.512	1.602	1.304	1.368
Mean_t of significant Canonical Corr	***8.649***	***5.616***	***2.924***	**2.002**	1.561	1.809	**2.455**	1.596	**2.066**	**2.141**	1.384	1.940	**2.064**	1.535	1.749
Decade #	Number of t-stat ≥ 1.96 out of 10 for each decade														
1	4	4	5	4	2	3	2	3	3	2	3	4	1	1	4
2	3	2	4	1	3	1	2	3	4	1	2	2	5	1	2
3	3	4	4	2	3	1	3	1	1	2	1	3	3	2	3
4	6	4	5	3	3	2	5	3	2	5	1	4	4	3	4
5	3	2	4	3	2	4	4	4	3	2	4	2	3	3	3
Mean	3.8	3.2	4.2	3.0	2.0	2.2	3.6	2.4	2.6	3.6	1.4	2.8	3.4	1.6	2.0

Table 4
Summary Statistics for the Candidate Factor Betas and Characteristics

Here are summary statistics for betas of the nine factors that pass our necessary condition, namely, Rm-Rf, SMB, HML, RMW, MOM, LT_REV, ΔDP, ΔTS and UNEXPI. Betas (OLS slope coefficients) are computed for each individual stock in a multiple regression of the stock's monthly return on monthly factor realizations, using all observations available for each stock. To be included, a stock must have at least 24 monthly observations. The summary statistics are for the cross-stock distributions of each beta. Also included here are corresponding summary statistics for individual stock characteristics (defined in the appendix). The sample period spans 564 months from January 1968 through December 2014. The first 36 months are lost due to the lagged -13 to -36 return, "Lag13-36."

	Mean	Median	Sigma	Skewness	Kurtosis	Maximum	Minimum
Rm-Rf	0.927	0.929	0.031	0.026	-0.658	0.994	0.861
SMB	0.800	0.808	0.045	-0.911	1.069	0.873	0.647
HML	0.151	0.153	0.067	-0.132	-0.874	0.276	0.023
RMW	0.083	0.089	0.061	-0.118	-0.474	0.218	-0.044
MOM	-0.111	-0.108	0.031	-0.180	-1.124	-0.055	-0.170
LT_Rev	-0.020	-0.017	0.018	-0.145	-0.309	0.031	-0.060
ΔDP	-1.009	-1.028	0.379	0.162	-1.190	-0.269	-1.709
ΔTS	-0.447	-0.433	0.179	0.003	-1.145	-0.141	-0.805
UNEXPI	0.435	0.474	0.192	-0.341	-1.259	0.703	0.112
RetLag1	1.069	1.503	5.426	-0.465	3.232	27.338	-27.660
Lag2_12	0.351	0.559	1.823	-0.518	0.932	5.866	-6.483
Lag13_36	0.437	0.567	1.134	-0.596	0.689	3.252	-3.342
SizeLag1	11.88	11.62	0.904	0.580	-0.892	13.810	10.385
Book/Mkt	0.874	0.797	0.319	1.722	4.112	2.221	0.453
ProfRato	0.274	0.208	0.457	6.199	38.34	3.332	-0.124
AsstGrth	0.170	0.174	0.054	-0.115	0.390	0.405	0.035

Table 5

Estimated Risk Premiums for Factors Candidates that Satisfy the Necessary Conditions

Risk Premiums (in %/month) are estimated from cross-sectional regressions computed using individual stock returns from 1968-2014 as dependent variables and, as explanatory variables, decile-sorted portfolio betas of the nine factors that pass necessary conditions including Rm-Rf, SMB, HML, RMW, MOM, LT_REV, ΔDP, ΔTS and UNEXPI in Models 1 and 2, and the nine factors that pass necessary conditions and associated characteristics in Models 3 and 4. Characteristics include RetLag1, Lag2_12, Lag13_36, Size, Book/Mkt, ProfRato, and AsstGrth. See variable definitions in the Appendix. These selected factors are those that are significantly related to any canonical variate in all decades or that have mean t-statistics in the second row of Table 3 Panel B that exceed the one-tailed, 2.5% cutoff based on the Chi-square value and an average number of significant t-stats exceeding 2.5 (see the bottom row of Table 3 Panel B). For the non-EIV estimation, OLS multiple regressions are run for each stock on all (nine) factors using all available observations for that stock. For the EIV calculations, stocks are sorted into ten groups (deciles) by market capitalization (Size), annually at the end of each June, based on NYSE size decile breakpoints. Then within each Size decile, stocks are sorted further by the OLS betas of the first factor (Rm-Rf) into ten deciles, thus resulting in 100 Size/first factor beta groups. Within each of the 100 groups, the equal-weighted average first factor beta of the group is assigned to each stock within that group. For each of the other eight factors, this procedure is repeated independently; ultimately, each stock's beta (for all nine betas) is replaced by the equal-weighted portfolio beta of the double sorted size/beta group to which the stock belongs. This same procedure is redone every June, 1965-2014; then cross-sectional regressions are calculated in the 12 subsequent months of individual stock returns on the double-sorted portfolios betas (six months only after the 2014 sort.) The time series average over all months of the cross-sectional coefficients, termed the "risk premiums," along with associated sampling statistics, are reported in the table. Critical rejections levels for the T-Statistic are 1.65 (10%), 1.96 (5%), and 2.59 (1%). T-Statistics breaching the 5% (1%) critical level are in boldface (boldface italic.)

(Table continued on next page)

	No EIV correction (Model 1)		EIV correction (Model 2)		No EIV correction (Model 3)		EIV correction (Model 4)	
	Mean	T-Stat	Mean	T-Stat	Mean	T-Stat	Mean	T-Stat
Constant	0.600	***9.467***	0.603	***6.895***	2.211	***9.014***	1.829	***5.726***
Rm-Rf	0.479	**2.503**	0.435	***2.872***	0.445	**2.214**	0.347	***2.713***
SMB	0.036	0.276	0.049	0.486	-0.240	-1.823	-0.077	-0.962
HML	-0.184	-1.484	-0.127	-1.456	-0.371	***-2.843***	-0.247	***-3.613***
RMW	0.244	***2.686***	0.190	***2.919***	0.250	**2.513**	0.098	**2.057**
MOM	0.480	***2.606***	0.264	1.875	0.303	1.620	0.190	1.653
LT_Rev	-0.197	-1.827	-0.113	-1.408	-0.259	**-2.334**	-0.112	-1.645
ΔDP	0.016	1.457	0.014	1.659	0.025	**2.104**	0.017	**2.211**
ΔTS	0.049	1.316	0.058	1.952	0.063	1.576	0.065	**2.462**
UNEXPI	-0.087	***-2.702***	-0.074	***-2.801***	-0.083	**-2.365**	-0.027	-1.189
RetLag1					-0.062	***-21.89***	-0.060	***-19.07***
Lag2_12					0.106	***9.274***	11.759	***8.632***
Lag13_36					0.038	***2.878***	2.141	1.362
SizeLag1					-0.130	***-7.444***	-0.103	***-4.100***
Book/Mkt					0.210	***6.267***	0.176	***4.256***
ProfRato					0.044	**2.038**	0.262	***5.619***
AsstGrth					-0.272	***-6.832***	-0.337	***-6.403***
RSquare	0.125		0.082		0.124		0.084	
SamplSize	4687		2839		2688		1706	

Table 6
Returns of Hedge Portfolios Associated with Factors that Satisfy Necessary Conditions

Here are summary statistics for returns (in %/month) of hedge portfolios associated with the nine factors that satisfy the necessary conditions in Table 3. The factors include Rm-Rf, SMB, HML, RMW, MOM, LT_REV, ΔDP, ΔTS, and UNEXPI. For each candidate factor, hedge portfolios are formed by a long position in a group of stocks with the highest betas on the factor and a short position a group with the lowest betas; this is done with replacement. Panel A (C) shows the returns of the hedge portfolios with the top and bottom deciles (the top 30% and bottom 30%). Panel B (D) show the excess returns of the market (Rm-Rf) and augmented returns, which is the hedge portfolio return added to the market excess return (Rm-Rf). The augmented return of Rm-Rf is itself appended with its own hedge portfolio. The other eight to the right of it are for the other eight hedge portfolios. The Sharpe ratio (Sharpe) and a t-statistic of the Sharpe ratio against 0.6 (Sharpe t), the MacKinlay (1995) threshold, are also reported. Critical rejections levels for the T-Statistic are 1.65 (10%), 1.96 (5%), and 2.59 (1%). T-Statistics breaching the 5% (1%) critical level are in boldface (boldface italic.)

Panel A: Hedge portfolio from 10% top and bottom

	Rm-Rf	SMB	HML	RMW	MOM	LT_Rev	ΔDP	ΔTS	UNEXPI
Mean	0.691	-0.107	-0.415	0.667	0.360	-0.035	0.504	0.875	-0.994
Std. Dev	6.559	6.411	6.264	5.873	5.604	5.029	5.335	5.663	5.943
t(Mean)	**2.502**	-0.395	-1.572	***2.698***	1.527	-0.164	**2.245**	***3.670***	***-3.971***

Panel B: Market return plus 10% top and bottom hedge portfolio returns

	Rm-Rf	Augmented returns								
		Rm-Rf	SMB	HML	RMW	MOM	LT_Rev	ΔDP	ΔTS	UNEXPI
Mean	0.490	1.181	0.383	0.075	1.157	0.850	0.455	0.995	1.365	-0.504
Std. Dev	4.584	10.739	8.711	7.191	7.140	6.595	7.032	7.207	7.200	7.821
t(Mean)	2.539	2.612	1.045	0.249	3.849	3.062	1.538	3.277	4.503	-1.529
Sharpe	0.370	0.381	0.152	0.036	0.561	0.447	0.224	0.478	0.657	0.223
Sharpe t	***-5.275***	***-5.02***	***-10.57***	***-13.38***	-0.850	***-3.472***	***-8.811***	***-2.744***	1.225	***-8.842***

Panel C: Hedge portfolio 30% top and bottom

	Rm-Rf	SMB	HML	RMW	MOM	LT_Rev	ΔDP	ΔTS	UNEXPI
Mean	0.368	-0.154	-0.122	0.522	0.246	0.015	0.281	0.609	-0.664
Std. Dev	4.327	4.433	4.046	3.734	3.488	2.989	3.178	3.666	3.867
t(Mean)	**2.017**	-0.824	-0.715	***3.321***	1.675	0.123	**2.102**	***3.945***	***-4.080***

Panel D: Market return plus 30% top-and-bottom hedge portfolio return

	Rm-Rf	Augmented returns								
		Rm-Rf	SMB	HML	RMW	MOM	LT_Rev	ΔDP	ΔTS	UNEXPI
Mean	0.490	0.858	0.336	0.368	1.012	0.736	0.506	0.771	1.099	-0.174
Std. Dev	4.584	8.669	7.335	5.319	5.589	4.920	5.756	5.540	5.588	6.469
t(Mean)	2.539	2.349	1.089	1.644	4.302	3.554	2.086	3.307	4.672	-0.640
Sharpe	0.370	0.343	0.159	0.240	0.627	0.518	0.304	0.482	0.681	0.093
Sharpe t	***-5.275***	***-5.939***	***-10.41***	***-8.433***	0.596	-1.821	***-6.865***	***-2.644***	1.743	***-12.007***

Table 7

Returns of Hedge Portfolios Associated with Characteristics

Here are summary statistics for returns (in %/month) on hedge portfolios associated with the seven characteristics RetLag1, Lag2_12, Lag13_36, Size, Book/Mkt, ProfRato, and AsstGrth. See variable definitions in the Appendix. For each characteristic, hedge portfolios are formed by a long position in a group of the stocks with high values of the characteristic and a short position a group with low characteristic values; this is done with replacement. Panel A (C) shows the returns of the hedge portfolios with the top and bottom deciles (the top 30% and bottom 30%). Panel B (D) show the returns of market (Rm-Rf) and augmented returns, which is the hedged portfolio returns added to the market return (Rm-Rf). The seven to the right of it are for the characteristics. The Sharpe ratio (Sharpe) and t-statistic of the Sharpe ratio against 0.6, (Sharpe t), the MacKinlay (1995) threshold, are reported. Critical rejections levels for the T-Statistic are 1.65 (10%), 1.96 (5%), and 2.59 (1%). T-Statistics breaching the 5% (1%) critical level are in boldface (boldface italic.)

Panel A: Hedge portfolio from 10% top and bottom

	RetLag1	Lag2_12	Lag13_36	SizeLag1	Book/Mkt	ProfRato	AsstGrth
Mean	-1.791	1.814	0.009	0.024	0.797	0.904	-0.510
Std. Dev	4.157	5.247	4.445	5.358	3.824	3.932	2.603
t(Mean)	***-10.23***	***8.212***	0.046	0.105	***4.951***	***5.458***	***-4.651***

Panel B: Market return plus 10% top and bottom hedge portfolio returns

	Rm-Rf	Augmented returns						
		RetLag1	Lag2_12	Lag13_36	SizeLag1	Book/Mkt	ProfRato	AsstGrth
Mean	0.490	-1.300	2.304	0.499	0.514	1.287	1.394	-0.020
Std. Dev	4.584	5.255	6.775	6.819	7.333	4.859	5.788	5.951
t(Mean)	2.539	-5.877	8.077	1.737	1.664	6.292	5.719	-0.079
Sharpe	0.370	0.857	1.178	0.253	0.243	0.918	0.834	0.011
Sharpe t	***-5.275***	***5.223***	***10.55***	***-8.104***	***-8.363***	***6.331***	***4.791***	***-13.98***

Panel C: Hedge portfolio 30% top and bottom

	RetLag1	Lag2_12	Lag13_36	SizeLag1	Book/Mkt	ProfRato	AsstGrth
Mean	-1.011	1.040	-0.035	0.097	0.562	0.420	-0.347
Std. Dev	2.870	3.643	3.032	3.958	2.643	2.533	1.738
t(Mean)	***-8.368***	***6.778***	-0.278	0.583	***5.045***	***3.940***	***-4.747***

Panel D: Market return plus 30% top-and-bottom hedge portfolio return

	Rm-Rf	Augmented returns						
		RetLag1	Lag2_12	Lag13_36	SizeLag1	Book/Mkt	ProfRato	AsstGrth
Mean	0.490	-0.521	1.530	0.455	0.587	1.052	0.910	0.143
Std. Dev	4.584	4.640	5.562	5.591	6.302	4.330	5.094	5.505
t(Mean)	2.539	-2.666	6.533	1.931	2.213	5.768	4.244	0.616
Sharpe	0.370	0.389	0.953	0.282	0.323	0.841	0.619	0.090
Sharpe t	***-5.275***	***-4.833***	***6.950***	***-7.414***	***-6.416***	***4.926***	0.414	***-12.09***

Table 8
Correlations between Factor- and Characteristic-Based Hedge Portfolios

Here are correlations between the hedge portfolios corresponding to the nine factors that pass necessary conditions and the hedge portfolios corresponding to the seven characteristics. These are hedge portfolios that are long the largest 30% of the values and short the smallest 30% for each factor and characteristic. The construction of the hedge portfolios is explained in Tables 6 and 7. See the Appendix for variable definitions. Correlations that are greater than or equal to 0.5 in absolute value are in boldface.

	Rm-Rf	SMB	HML	RMW	MOM	LT_Rev	ΔDP	ΔTS	UNEXPI
RetLag1	-0.302	-0.305	0.117	0.049	0.437	-0.140	0.154	0.195	-0.288
Lag2_12	-0.115	-0.304	-0.011	0.114	**0.745**	-0.047	0.148	0.309	-0.280
Lag13_36	0.026	-0.343	-0.143	0.143	0.263	-0.307	0.175	0.388	-0.309
SizeLag1	0.145	**-0.642**	-0.092	0.175	0.366	-0.045	0.231	**0.639**	**-0.522**
Book/Mkt	-0.363	0.108	**0.643**	0.199	-0.006	0.078	-0.088	-0.142	-0.036
ProfRato	-0.032	**-0.508**	0.230	**0.611**	0.319	-0.034	0.248	**0.585**	**-0.593**
AsstGrth	0.390	0.023	-0.353	0.088	-0.059	-0.140	0.118	0.149	-0.067