Topics in Shear Flow

Chapter 3 – Channel Flow

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Chapter 3

CHANNEL FLOW

3.1 Generalities

3.1.1 Preamble

Channel flow will be taken here to include plane Couette flow, and thus some aspects of lubrication theory. An important application is in ducting for ventilation. There is a strong parallel between pipe flow and channel flow, extending to the techniques used, the laboratories involved, and even the investigators. This parallel does not extend to applications.

3.1.2 Equations and integrals

Channel flow is described by the equations of motion in rectangular coordinates $(x,\ y,\ z)$, with velocity components $(u,\ v,\ w)$. The mean flow is two-dimensional and rectilinear. Thus $v = w = 0$ and $\partial/\partial x = \partial/\partial z = 0$, except for the driving term $\partial p/\partial x$. The continuity equation is automatically satisfied. The momentum equations in the appendix[1] become

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y} - \rho\overline{u'v'}\right) \ ; \tag{3.1}$$

$$0 = -\frac{\partial p}{\partial y} - \frac{\partial}{\partial y}(\rho\overline{v'v'}) \ ; \tag{3.2}$$

$$0 = -\frac{\partial}{\partial z}\,(\rho\overline{w'w'}) \ . \tag{3.3}$$

As in pipe flow, the quantities in parentheses do not depend on x, so that

$$\frac{\partial^2 p}{\partial x^2} = \frac{\partial^2 p}{\partial x \partial y} = 0 \ . \tag{3.4}$$

It follows that

$$\frac{\partial p}{\partial x} = \text{constant} = \frac{\mathrm{d}p}{\mathrm{d}x} \ . \tag{3.5}$$

The second equation has the integral

$$p + \rho\overline{v'v'} = p_w = x\frac{\mathrm{d}p}{\mathrm{d}x} + \text{constant} \ . \tag{3.6}$$

The shearing stress is conveniently defined as

$$\tau = \mu\frac{\partial u}{\partial y} - \rho\overline{u'v'} \tag{3.7}$$

and equation (3.1) has the integral

$$\tau - \tau_w = y\,\frac{\mathrm{d}p}{\mathrm{d}x} \tag{3.8}$$

where the origin for y is taken (say) as the lower channel wall. If the channel height is h, then with $\tau = 0$ at $y = h/2$ and $\tau = -\tau_w$ at $y = h$,

$$\tau_w = -\frac{h}{2}\,\frac{\mathrm{d}p}{\mathrm{d}x} \ . \tag{3.9}$$

This expression can also be obtained by an overall force balance over a length of the channel.

The third equation (3.3) needs a comment about sublayer vortices; see section x. Note that no information is obtained about two of the Reynolds normal stresses, $\overline{u'u'}$ and $\overline{w'w'}$.

[1] This appendix was not found.

3.1.3 Laminar flow

If the flow is laminar, the velocity profile is determined by a combination of equations (3.7) and (3.8),

$$\mu\frac{\partial u}{\partial y} - \tau_w = y\frac{\mathrm{d}p}{\mathrm{d}x} \tag{3.10}$$

with the integral

$$u = \frac{\tau_w}{\mu}\left(y - \frac{y^2}{h}\right) + \text{constant} \ . \tag{3.11}$$

With $u = 0$ at $y = 0$, the constant of integration is zero. With $u = u_c$ at $y = h/2$,

$$u_c = \frac{\tau_w h}{4}\mu \ . \tag{3.12}$$

The velocity profile is the parabola

$$\frac{u}{u_c} = 1 - \left(1 - \frac{y}{h/2}\right)^2 \ . \tag{3.13}$$

The mean velocity is defined by

$$h\widetilde{u} = \int\limits_o^h u\,\mathrm{d}y \ . \tag{3.14}$$

Equation (3.13) then implies

$$\widetilde{u} = \frac{2}{3}u_c \ . \tag{3.15}$$

3.1.4 Development length

The same argument and the same numbers apply as for pipe flow.