Reply to "Comment on 'Solar System constraints to general $f(R)$ gravity'"

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(Received 18 February 2008; published 7 May 2008)

Here we respond to the Comment by Faraoni and Lanahan-Tremblay on our paper. We show that the criticisms of our paper presented in this Comment are incorrect.

DOI: 10.1103/PhysRevD.77.108502 PACS numbers: 04.25.Nx, 04.50.Kd

In Ref. [1], the authors showed that, under certain assumptions, the parametrized post-Newtonian (PPN) parameter γ of $f(R)$ gravity is $1/2$, in agreement with the result using the equivalence with scalar-tensor action [2]. In Ref. [3], Faraoni and Lanahan-Tremblay claim that our paper is incomplete. Here we show that their criticisms of our paper are incorrect and that the clarifications they propose are superfluous to the evaluation of Solar System constraints to $f(R)$ gravity.

Faraoni and Lanahan-Tremblay (FL) claim that our definition of the effective mass squared (m^2) in the equation for the Ricci scalar perturbation R_1 is incorrect. This is not the case. The definition of m^2 follows directly from the trace of the field equation for $f(R)$ gravity. When the Ricci scalar is expressed as the sum of two components,

$$R(r, t) \equiv R_0(t) + R_1(r), \tag{1}$$

where $R_0(t)$ is a spatially homogenous background curvature and $R_1(r)$ is a time-independent perturbation to this background curvature, and $\mathcal{O}(R_1^2)$ terms are neglected, the trace of the field equation becomes

$$\nabla^2 R_1 - m^2 R_1 = -\frac{\kappa\rho}{3f''(R_0)} \tag{2}$$

with the effective mass squared defined by

$$m^2 \equiv \frac{1}{3}\left(\frac{f'(R_0)}{f''(R_0)} - R_0 - 3\frac{\Box f''(R_0)}{f''(R_0)}\right), \tag{3}$$

as described in Sec. II of Ref. [1]. Contrary to the claims of FL, we do not assume that R_0 is time-independent; we allow it to vary on cosmological time scales. In this way, our analysis differs from the prior work cited by FL, and that is why the expression for m^2 has an extra term. This extra term in m^2 vanishes if the background is assumed to be static.

The second criticism in Ref. [3] addresses the instability associated with negative values of m^2. As FL admits, this point does not affect our final results since we only consider cases where $|m^2|r^2 \ll 1$. If $m^2 < 0$ and $|m^2|r^2 \ll 1$ inside the Solar System, then the evolution of the unstable background is negligible because the time scale of the instability is much longer than the dynamical time scale of the Solar System. Consequently, we do not consider the dynamical stability of the cosmological background; this issue is not relevant to the analysis of Solar System constraints to $f(R)$ gravity provided that $|m^2|r^2 \ll 1$.

Finally, Ref. [3] includes an extended discussion of why the C_1/r term in their expression for Ψ_1 [their Eq. (10)] should be neglected and claims that there was an implicit assumption in our analysis that led us to ignore this term in Ref. [1]. This is also incorrect. Aside from continuity of Ψ_1 across the boundary of the source, no assumption is required to determine the value of C_1. The value of C_1 is uniquely determined by integrating both sides of

$$\nabla^2\Psi_1 = -\frac{1}{2}R_1 \tag{4}$$

over a sphere centered at the origin and applying Gauss's law; this calculation reveals that the C_1/r term in the exterior solution for Ψ_1 is negligible, and that is why it was ignored in Ref. [1]. (The exact same technique is applied in electromagnetism to solve for the electric field sourced by a sphere of charge, so we did not feel that it required elaboration.) Contrary to the statements of FL, the potential $\Psi_1(r)$ cannot be singular at the origin because R_1 is not singular there.

[1] T. Chiba, T. L. Smith, and A. L. Erickcek, Phys. Rev. D **75**, 124014 (2007).
[2] T. Chiba, Phys. Lett. B **575**, 1 (2003).
[3] V. Faraoni and N. Lanahan-Tremblay, preceding Comment, Phys. Rev. D **77**, 108501 (2008).

