ONLINE APPENDIX
Collusion through Communication in Auctions

Marina Agranov and Leeat Yariv

October 22, 2017

1 Structure

In this Online Appendix, we conduct additional analysis of the data and robustness checks as well as provide some theoretical analysis. We proceed as follows. At the end of Section 1, we describe the type of statistical analysis we perform to compare treatments and behaviors in different parts of the experiment. This section also contains definitions we use in the Online Appendix. In Section 2, we present additional analysis of the Pure Communication treatment. In Section 3, we present additional analysis of the Communication and Transfers treatment. In Section 4, we consider the choice of "small perturbation" that was fixed at two experimental units for the results reported in the paper. We show the robustness of our results to alternative levels of perturbation. In Section 5, we illustrate that the risk measure elicited in some sessions does not seem to explain subjects' behavior either during communication or while bidding. In Section 6, we discuss learning effects and compare outcomes and behavior in the first and second halves of the experiment. In Section 7, we present analysis of additional experimental sessions that we conducted with complete strangers matching protocol. Finally, in Section 8, we provide some theoretical analysis of one-shot independent private value sealed-bid auctions corresponding to our different treatments.

Our Statistical Analysis

To compare average outcomes between two groups (be that two treatments, two auctions, or two parts of the experiment), we use regression analysis. More precisely, we run a random-effects GLS regression in which we regress the variable of interest, e.g. an indicator of whether an outcome is efficient or collusive, or observed price, on a constant and an indicator for one of the two considered groups. We cluster standard errors by session. We report that there is a statistically significant difference between outcomes in these two groups if the estimated coefficient on the group indicator is significantly different from zero at the 5% level.

To compare median outcomes between two groups, we use the Wilcoxon rank-sum test and report results at the 5% significance level. To compare two distributions (for example, the distribution of prices or the distribution of reported values) we use the Kolmogorov-Smirnov test and report results at the 5% significance level.

Definitions

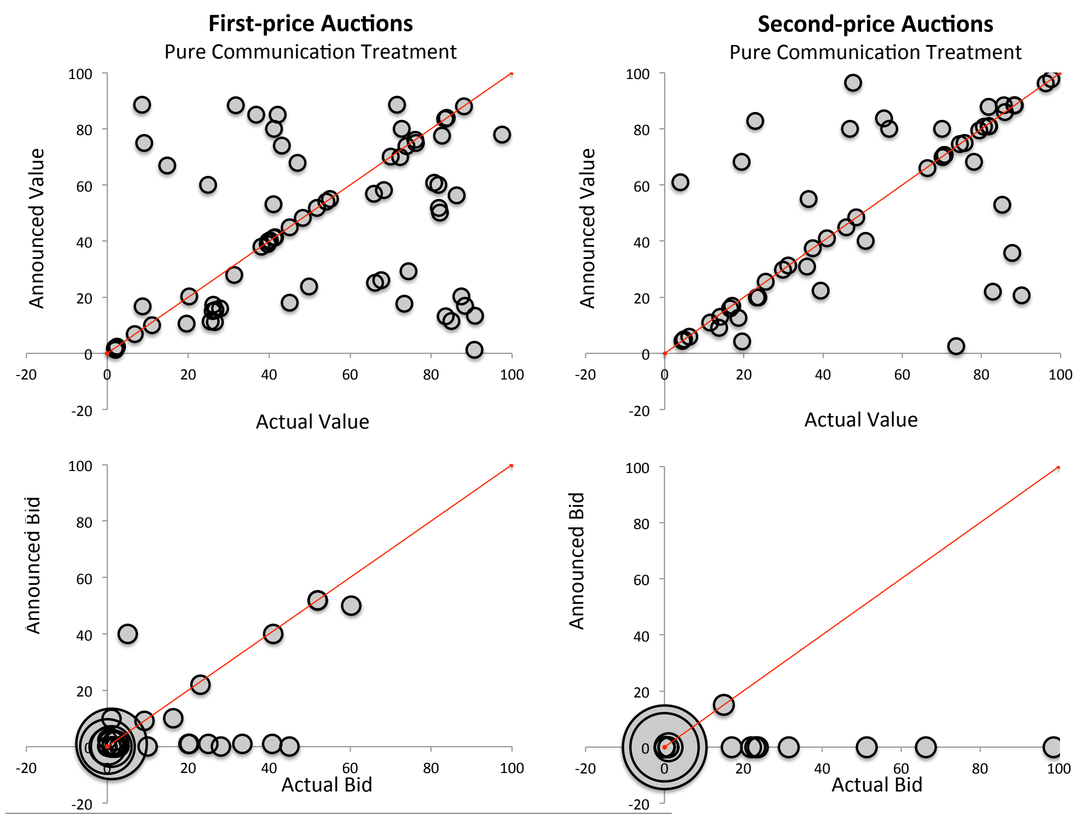
As discussed in the main text of the paper, we allow for perturbations of two experimental points in our classifications of outcomes. Below is the list of definitions we use in our analysis (both in the main text of the paper and in this Online Appendix):

- We call an outcome **efficient** if the winning bidder's value was at least as high as the losing bidder's value minus two experimental points.
- We use the term **minimal price** to refer to a price less or equal to two experimental points.
- When we discuss **collusion rates**, we say **an outcome is collusive** if the auction culminates in a minimal price (price below or equal to two experimental points).
- We use the expression **substantial transfer** to refer to a transfer of more than two experimental points.
- When analyzing communication between bidders, we distinguish between the following three categories of reports regarding bidders' values and bids. In cases in which a numerical value was reported, we say that a bidder **understates** her true value (bid) if her announced value (bid) is below her true value (bid) minus two experimental points. We say that a bidder **overstates** her true value (bid) if she reports a value higher than her true value plus two experimental points. Otherwise, we say that the bidder reports her value (bid) **truthfully**.

2 Pure Communication Treatment

In the main text of the paper, we have documented that our experimental subjects in the Pure Communication treatments often choose not to discuss their values and bids with each other. Figure 1 depicts the distribution of announced values and bids as a function of actual values and bids in the minority of cases in which subjects chose to report their values and/or bids in rounds 6 to 10. The size of each point in this figure is a proxy for the corresponding number of observations. As evident from Figure 1, the most common pattern of misrepresentation of values in both auction formats is an understatement of the true value. With respect to bids, much of the truthful revelation is linked with fairly low bids, as is much of the misrepresentation. The extensive amount of lying coupled with the low frequency of reporting values and bids explains why subjects in our Pure Communication treatments achieved a rather limited volume of collusive outcomes.

Figure 1: Misrepresentation Patterns in Pure Communication Treatments



Next, we inspect whether conversations that preceded the bidding stage in the Pure Communication treatment affect outcomes in terms of efficiency and the auctioneer's revenues (or prices). Instead of looking at individual communication strategies, here we consider the conversation as a whole and count each conversation as a unit of observation. In particular, a conversation is classified as "relevant" if at least one of the bidders sent a message about values, bids, or strategy. The remaining conversations are classified as "irrelevant." Most irrelevant conversations contained greeting messages such as "hi," "how are you doing there?," or some other short message having nothing to do with the experiment.

Table 1: Frequencies of Conversations in the Pure Communication Treatment

	First-price Auctions		**Second-price Auctions**	
	rounds 1 to 5	rounds 6 to 10	rounds 1 to 5	rounds 6 to 10
No Conversation	0.40	0.35	0.30	0.51
Irrelevant Conversations	0.21	0.19	0.27	0.13
Relevant Conversations	0.38	0.46	0.43	0.36

Table 2: Efficiency Depending on Conversations in the Pure Communication Treatment

	First-price Auctions		**Second-price Auctions**	
	rounds 1 to 5	rounds 6 to 10	rounds 1 to 5	rounds 6 to 10
No Conversation	0.92	0.93	0.78	0.81
Irrelevant Conversations	0.96	0.90	0.86	0.68
Relevant Conversations	0.66	0.75	0.75	0.78

Table 1 depicts the frequencies of relevant conversations in each auction format both in the first half and the second half of the experiment. Table 2 lists the corresponding efficiency levels in each of these categories. Regression analysis reveals that bidders who discuss relevant topics (values, bids, or strategies) in our first-price auction with pure communication achieve significantly lower efficiency levels than those that do not talk at all or discuss irrelevant topics ($p < 0.01$ in rounds 1 to 5 and $p = 0.003$ in rounds 6 to 10). This result combined with the excessive lying about values and bids documented in the paper suggests that bidders often do not trust their opponents' announcement. This has a statistically significant detrimental effect on overall efficiency. There is no such effect in our second-price auctions, in which in the last 5 rounds of the experiment, more than 50% of groups do not talk.

Finally, we study the effects of relevant conversations on the prices that emerge in both auction formats. Figures 2 and 3 present price distributions in our first- and second-price auctions with pure communication broken down into the three categories: pairs that discussed relevant topics, pairs that discussed irrelevant topics, and pairs that chose not to talk with one another. In both auction formats and throughout the experiment, there is no statistically significant difference between prices in auctions in which bidders chose not to communicate with one another and auctions in which bidders discussed irrelevant topics ($p > 0.05$ in all regressions). However, we observe lower prices in auctions in which bidders discussed relevant matters compared with auctions in which bidders either did not communicate or discussed irrelevant topics ($p < 0.01$ in all regressions).

Figure 2: The Effect of Relevant Conversations on Prices in First-price Auctions

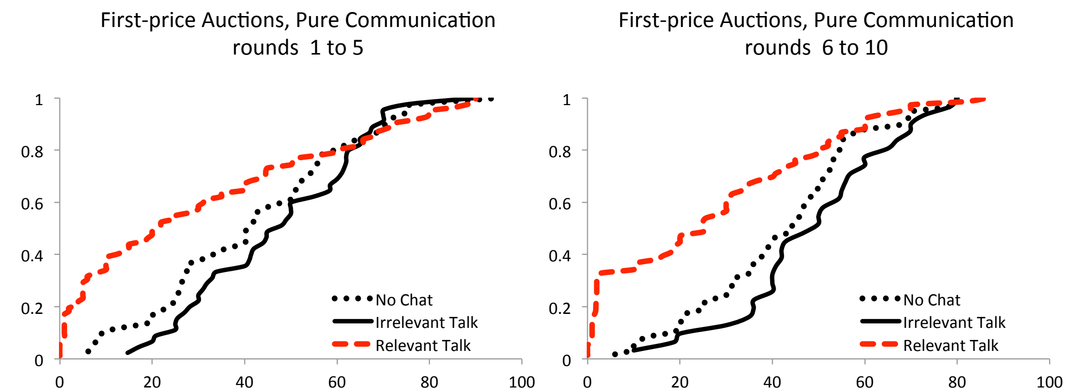


Figure 3: The Effect of Relevant Conversations on Prices in Second-price Auctions

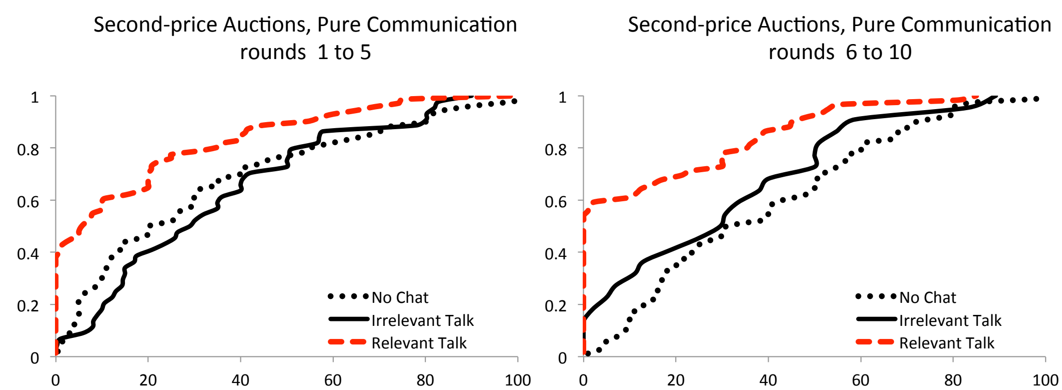


Table 3: Collusive Rates and Substantial Transfers in the Last Round, by Session

	Collusive Rates		Substantial Transfers Rates			
			all auctions		minimal price	
	first-price	second-price	first-price	second-price	first-price	second-price
all sessions	0.76	0.86	0.67	0.66	0.84	0.71
session 1	1.00	0.80	1.00	0.60	1.00	0.75
session 2	0.67	0.67	0.50	0.33	0.75	0.50
session 3	0.83	1.00	0.67	0.50	0.80	0.50
session 4	0.80	1.00	0.80	0.80	1.00	0.80
session 5	0.40	0.80	0.40	1.00	0.50	1.00
session 6	0.80	1.00	0.60	0.80	0.75	0.80

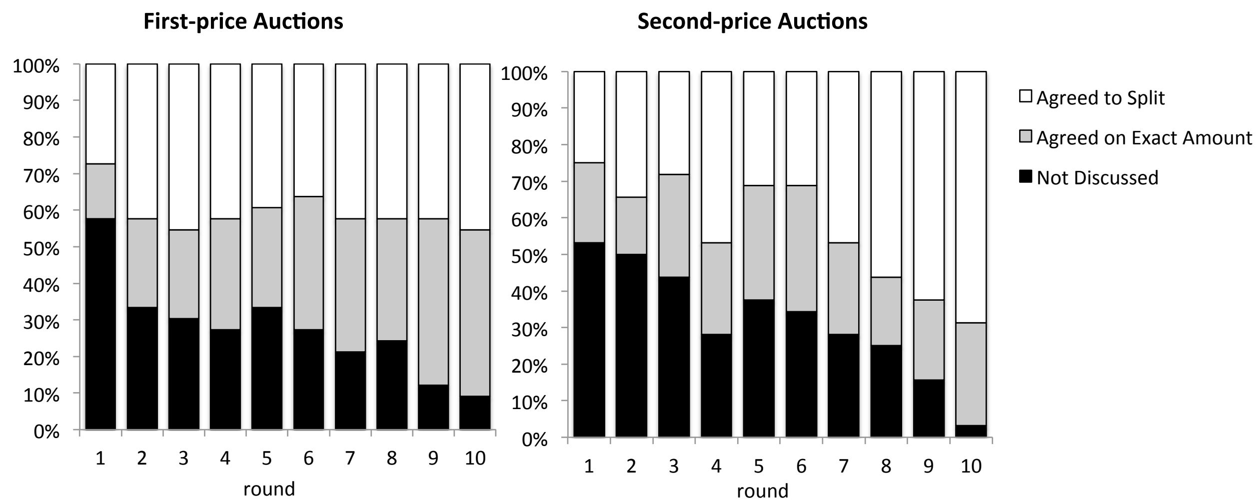
3 Communication and Transfers Treatment

In this section we report additional statistical analysis of the Communication and Transfers treatment. We start by showing that even in the very last round of sessions subjects successfully achieved collusive outcomes. In our second-price auctions, the last round of each session was round 10. In our first-price auctions, the last round of sessions 2, 4, 5, and 6 was round 10, while the last round of session 1 was round 11 and the last round of session 3 was round 12. Table 3 reports collusion rates and how often we observe substantial transfers in the very last round of each session.

Next, we observe that some pairs of bidders agreed on exact amounts to be transferred from the winning bidder to the losing bidder while other pairs decided to split the surplus. Figure 4 depicts how often these two types of agreements occurred in each of the auction formats by round. In both auction formats, as subjects experience the game, they learn to discuss transfers more often. In our first-price auctions, we detect higher frequencies of exact agreements in later rounds as compared with earlier rounds ($p < 0.05$). In our second-price auctions, the fraction of exact agreements is stable throughout the experiment ($p > 0.10$).

Table 4 summarizes the average amount of transfers when breaking the data by the type of agreement reached in the communication stage and the ultimate price. In both auction formats and throughout the experiment, transfers are significantly higher when winning bidders obtain the object at a minimal price than at a strictly higher than minimal price ($p < 0.05$). Related to this point, Figure 5 depicts the fraction of winners per round who transfer nothing to the losing bidder after receiving the object at a minimal price. This fraction is quite stable across rounds in both our first- and second-price auctions. This indicates that a majority of winning bidders continue to share the surplus with their coun-

Figure 4: Frequency of Bidders Agreeing on Transfers



terparts even after gaining a lot of experience.

Figures 6 and 7 present transfers as shares of surplus captured by winning bidders for all auctions and for auctions in which the object's price was zero. Figure 7 shows that the most common behavior of winning bidders is to share equally the surplus captured with the losing bidder when the price is very low.

Finally, Table 5 reports results from a Probit regression in which whether or not substantial transfers were passed is regressed on the winning bidder's surplus and other control variables, in analogy to Table 4 in the main text of the paper. The results reported in Table 5 have similar implications as those reported in Table 4 in the main text of the paper. Most importantly, the winning bidder's surplus has a statistically significant impact on whether transfers occur.

Table 4: Average Transfers and Agreements Reached During Communication

	First-price Auctions				Second-price Auctions			
	rounds 1 to 5		rounds 6 to 10		rounds 1 to 5		rounds 6 to 10	
	mean	# of obs	mean	# of obs	mean	# of obs	mean	# of obs
Transfers are Not Discussed	2.88	60	2.49	31	5.74	68	6.40	34
Agreed on Exact Transfers								
minimal price	26.97	35	25.30	57	20.07	35	23.85	36
price higher than minimal	6.40	5	0.00	8	0.00	4	0.00	5
Agreed to Split Surplus								
minimal price	28.13	49	27.47	57	26.40	47	21.82	79
price higher than minimal	6.34	16	4.46	12	5.36	6	7.50	6

Notes: Minimal price is less or equal to two experimental units.

Figure 5: Fraction of Winning Bidders that Transfer Nothing to Losing Bidders

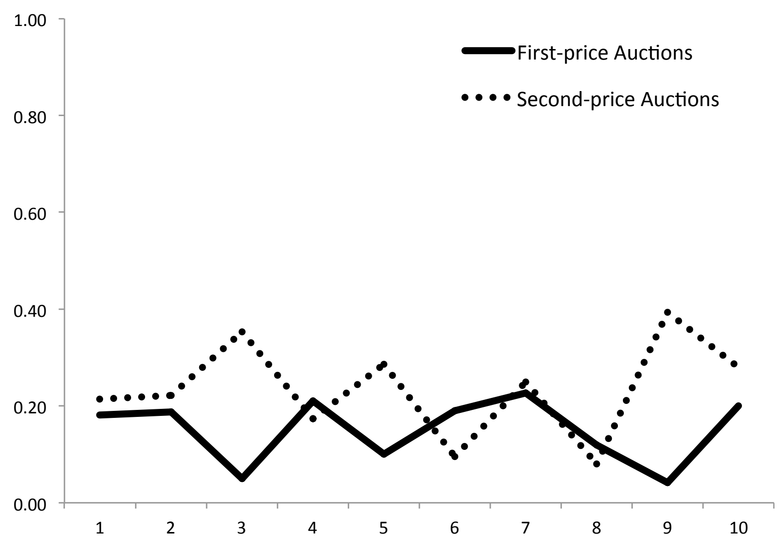


Figure 6: Transfers as Shares of Winning Bidders' Surplus

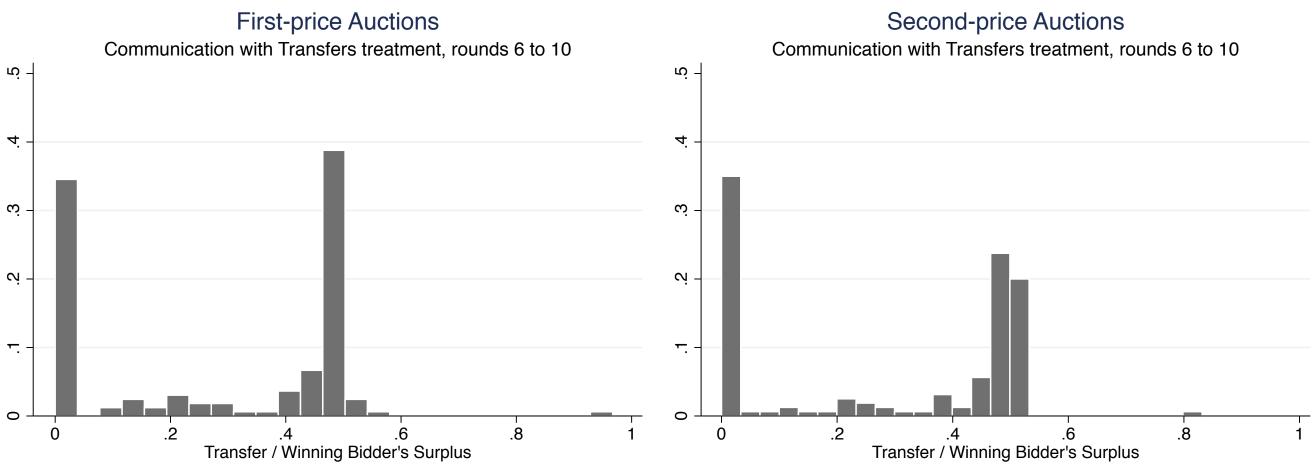


Figure 7: Transfers as Shares of Winning Bidders' Surplus when the Price is Minimal

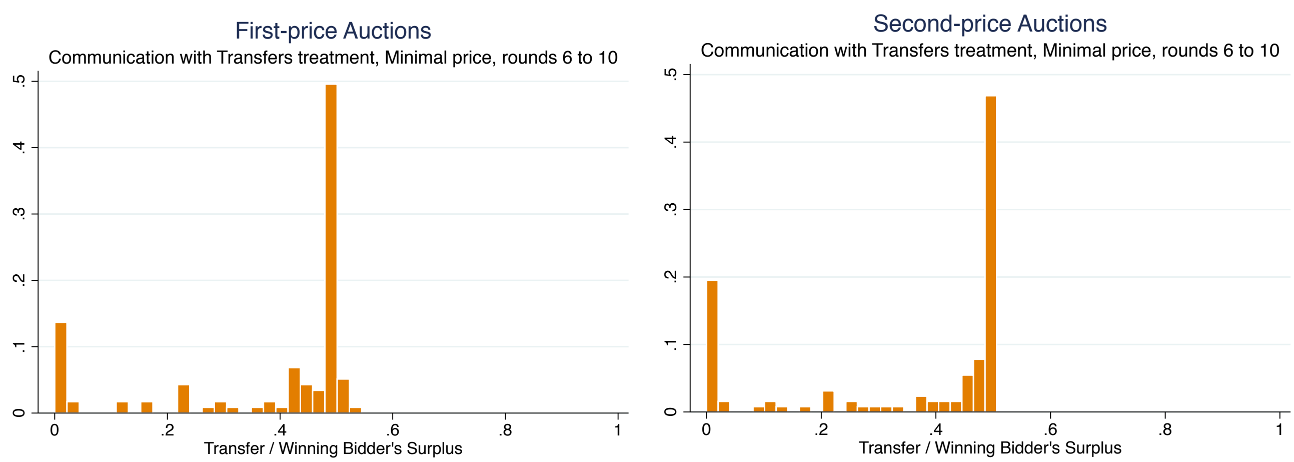


Table 5: Probit Estimates Explaining when Transfers Occur

	First-price	**Second-price**
Winning Bidder's Surplus	0.03** (0.003)	0.02** (0.007)
Losing Bidder's Value	-0.01** (0.003)	-0.01 (0.007)
Indicator if Efficient Outcome	0.80* (0.48)	0.33 (0.31)
Indicator if Winning Bidder Lied in the Past about Values or Bids	-0.50** (0.21)	-0.70** (0.27)
Constant	-1.19** (0.36)	-0.27 (0.40)
# of obs	165	160
# of sessions	6	6
Pseudo R-square	0.3415	0.3162

Remarks: Estimates are from the Probit regressions with a dependent variable that takes the value of 1 if the winning bidder transferred a substantial amount (more than 2 experimental points) to the losing bidder and zero otherwise. Robust standard errors are clustered at the session level. ** (*) indicates significance at the 5% (10%) level.

Table 6: Frequency of Efficient Outcomes when Small Perturbation is One Experimental Point

	First-price Auctions		**Second-price Auctions**	
	rounds 1 to 5	rounds 6 to 10	rounds 1 to 5	rounds 6 to 10
No Communication	0.89 (0.02)	0.88 (0.03)	0.81 (0.03)	0.79 (0.03)
Pure Communication	0.80 (0.04)	0.84 (0.04)	0.78 (0.04)	0.78 (0.05)
Comm with Transfers	0.83 (0.03)	0.86 (0.04)	0.84 (0.03)	0.84 (0.04)

Notes: In the parentheses we report robust standard errors clustered by session. An outcome is considered efficient if the winning bidder's value is greater or equal to the losing bider's value minus 1 experimental point.

Table 7: Collusion Rates when Small Perturbation is One Experimental Point

	First-price Auctions		**Second-price Auctions**	
	rounds 1 to 5	rounds 6 to 10	rounds 1 to 5	rounds 6 to 10
No Communication	0.00 (0.00)	0.00 (0.00)	0.02 (0.001)	0.02 (0.01)
Pure Communication	0.08 (0.03)	0.06 (0.02)	0.17 (0.06)	0.23 (0.09)
Comm with Transfers	0.44 (0.06)	0.65 (0.08)	0.58 (0.03)	0.80 (0.05)

Notes: In the parenthesis we report robust standard errors clustered by session.

4 Choice of "Small" Perturbation

In the analysis of our experimental data we allowed for a small perturbation in measurements that we set at two experimental points. The choice of this small perturbation is somewhat arbitrary and intended to account for small errors and rounding. We now demonstrate that if we change our threshold to one experimental point, results remain virtually identical. In particular, in Table 6 we report the frequency of efficient outcomes when the small perturbation is taken to be one, while in Table 7 we report the frequency of collusive outcomes when the small perturbation is, again, taken to be one experimental point.

5 Risk Attitudes

In this section we investigate whether the risk measure that we elicited in several sessions correlates with subjects' bidding behavior and the, tendency to discuss relevant topics during the communication phase. Recall that for half of sessions in each treatment, at the end of the session we asked subjects to allocate 100 points (translating into $2) between a safe investment, which had a unit return (i.e., returning point for point), and a risky investment, which with probability 50% returned 2.5 points for each point invested and with probability 50% produced no returns for the investment.

In Table 8 we report the results of the regression analysis in which we regress bids submitted in our sessions without communication on bidders' values and different measures of risk attitudes. In both the first- and the second-price auctions without communication, we find that risk attitude of subjects does not play a statistically significant role in determining bidding strategies.

Table 9 reports p-values corresponding to the pairwise correlations between announcing one's value or bid and various risk measures in the treatment with pure communication. We find that risk attitude as measured by CRRA and CARA coefficients has some effect on the tendency of subjects to announce their values or intended bids in the first-price but not in the second-price auctions.

Finally, Table 10 summarizes results from our regression analysis, in which we investigate the relationship between risk attitudes and bidding strategies in pure communication treatment. We present regressions for all auctions that occurred in rounds 6 to 10 irrespective of whether or not any form of conversation occurred prior to the auction. Regressions that focus only on auctions in which there was no conversation at all produce very similar results and are available from the authors upon request. The resulting estimates clearly indicate that elicited risk attitudes have no statistically significant impact on subjects' bidding behavior.

Table 8: Relationship between Risk Attitudes and Bidding Strategies in the No Communication treatment, rounds 6 to 10

	First-price Auctions			
	regression 1	regression 2	regression 3	regression 4
Bidder's Value	0.77** (0.03)	0.77** (0.03)	0.77** (0.03)	0.77** (0.03)
Risky Investment	-0.06 (0.05)			
Risk-averse		2.42 (3.42)		
β from CARA			1519.36 (1224.04)	
ρ from CRRA				13.99 (11.38)
Constant	5.19 (3.76)	-0.51 (3.32)	-3.91 (4.95)	-3.48 (4.67)
# of observations	150	150	120	120
# of clusters	30	30	24	24
Log Likelihood	-555.00	-555.39	-453.12	-453.13
	Second-price Auctions			
	regression 1	regression 2	regression 3	regression 4
Bidder's Value	0.83** (0.06)	0.83** (0.06)	0.79** (0.07)	0.79** (0.07)
Risky Investment	0.02 (0.10)			
Risk-averse		-1.02 (6.25)		
β from CARA			462.75 (301.59)	
ρ from CRRA				4.51 (2.99)
Constant	21.27** (7.60)	23.27** (5.92)	21.58** (5.17)	21.68** (5.17)
# of observations	150	150	105	105
# of clusters	30	30	21	21
Log Likelihood	-578.92	-578.92	-411.72	-411.76

Notes: Random-effects TOBIT regressions. The dependent variable is Observed Bid. Standard errors are clustered by subject. Risky investment stands for the number of tokens invested in the risky project in the Investment Task (a number between 0 and 100). Risk-averse is an indicator that takes the value 1 if the subject invested a strictly less than 100 tokens, which indicates that she is risk-averse. β is the estimated parameter from the CARA utility function $u(x) = 1 - e^{-\beta x}$. ρ is the estimated parameter from the CRRA utility function $u(x) = \frac{x^{1-\rho}}{1-\rho}$. ** indicates significance at the 5% level. The number of observations in regressions 3 and 4 is smaller than in regressions 1 and 2 since we can estimate parameters β and ρ only for subjects who chose a risky investment strictly greater than 0 and lower than 100.

Table 9: Correlations between Tendency to Announce Values and Bids and Risk Measures

	First-price Auctions Pure Communication, rounds 6 to 10			
	risky investment	risk-averse	β from CARA	ρ from CRRA
Indicator for Announcing Value	0.1008	-0.0692	0.3932**	0.3902**
Indicator for Announcing Bid	-0.1298	0.0537	0.4633**	0.4646**
	Second-price Auctions Pure Communication, rounds 6 to 10			
	risky investment	risk-averse	β from CARA	ρ from CRRA
Indicator for Announcing Value	0.0702	0.0713	0.1613	0.1988
Indicator for Announcing Bid	0.1132	0.0473	0.2071	0.1984

Notes: We report correlation scores corresponding to the pairwise correlations between dummy variables that indicate whether a subject announced his or her value or bid and different measures of risk attitudes. ** indicates that correlation is significant at the standard 5% level. Risky investment stands for the number of tokens invested in the risky project in the Investment Task (a number between 0 and 100). We exclude 5 subjects who reported that they want to invest 0 tokens in the risky project. Risk-averse is an indicator that takes the value 1 if the subject invested a strictly less than 100 tokens, which indicates that she is risk-averse. β is the estimated parameter from the CARA utility function $u(x) = 1 - e^{-\beta x}$. ρ is the estimated parameter from the CRRA utility function $u(x) = \frac{x^{1-\rho}}{1-\rho}$.

Table 10: Relationship between Risk Attitudes and Bidding Strategies in the Pure Communication Treatment, round 6 to 10

	First-price Auctions			
	regression 1	regression 2	regression 3	regression 4
Bidder's Value	0.54** (0.04)	0.54** (0.04)	0.50** (0.05)	0.50** (0.05)
Risky Investment	0.02 (0.08)			
Risk-averse		1.54 (4.32)		
β from CARA			-2650.95 (1591.80)	
ρ from CRRA				-25.40 (15.14)
Constant	-4.44 (6.38)	-4.15 (3.98)	8.97 (6.82)	8.55 (6.58)
# of obs	170	170	105	105
# of subjects	34	34	21	21
Log Likelihood	-667.09	-667.05	-415.54	-415.52
	Second-price Auctions			
	regression 1	regression 2	regression 3	regression 4
Bidder's Value	0.67** (0.08)	0.68** (0.08)	0.83** (0.13)	0.83** (0.13) (3.02) (0.12)
Risky Investment	-0.04 (0.12)			
Risk-averse		20.10** (8.75)		
β from CARA			-7737 (5235.81)	
ρ from CRRA				-78.11 (43.06)
Constant	7.49 (9.70)	-8.78 (8.35)	30.02 (17.69)	29.10** (14.67)
# of obs	150	150	75	75
# of subjects	30	30	15	15
Log Likelihood	-645.96	-643.55	-308.27	-307.80

Notes: Random-effects TOBIT regressions. The dependent variable is Observed Bid. Standard errors are clustered by subject. Risky investment stands for the number of tokens invested in the risky project in the Investment Task (a number between 0 and 100). Risk-averse is an indicator that takes the value 1 if the subject invested a strictly less than 100 tokens, which indicates that she is risk-averse. β is the estimated parameter from the CARA utility function $u(x) = 1 - e^{-\beta x}$. ρ is the estimated parameter from the CRRA utility function $u(x) = \frac{x^{1-\rho}}{1-\rho}$. ** indicates significance at the 5% level. The number of observations in regressions 3 and 4 is smaller than in regressions 1 and 2 since we can estimate parameters β and ρ only for subjects who chose a risky investment strictly greater than 0 and lower than 100.

Table 11: Efficiency Levels, by Treatment

	First-price Auctions		**Second-price Auctions**	
	rounds 1 to 5	rounds 6 to 10	rounds 1 to 5	rounds 6 to 10
No Communication	0.91 (0.02)	0.89 (0.03)	0.81 (0.03)	0.80 (0.03)
Pure Communication	0.80 (0.04)	0.84 (0.04)	0.79 (0.04)	0.78 (0.05)
Comm with Transfers	0.83 (0.03)	0.87 (0.03)	0.84 (0.03)	0.84 (0.04)

Notes: In the parentheses we report robust standard errors clustered by session.

6 Learning

In this section we compare outcomes observed in the first five and second five rounds of our experimental sessions in terms of efficiency, collusion, and ultimate prices.

Table 11 reports efficiency levels by treatment. Efficiency levels are stable across the two parts of the experiment: rounds 1 to 5 and rounds 6 to 10. Regression analysis confirms this observation: we detect no statistically significant differences between efficiency levels in the first five and the ensuing five rounds of the experiment in any of our treatments ($p > 0.05$ in all six regressions).

Table 12 reports frequencies of collusive outcomes in each treatment and each auction format. Table 13 lists p-values from pairwise comparisons between frequencies of collusion in the two parts of the experiment for each treatment and auction format. As evident from these tables, there are very few collusive outcomes in treatments without communication or treatments with pure communication in both auction formats. At the same time, both in the first and in the second half of the experiment, collusive outcomes are frequent when bidders can communicate and use transfers. Moreover, collusion becomes significantly more frequent in later rounds in the treatment allowing for communication and transfers, under both auction formats.

Table 14 presents summary statistics of observed prices in our first- and second-price auctions in both parts of the experiment. This table also compares average observed prices between the first and the second part of the experiment, for each treatment separately. In our first-price auctions without communication or with pure communication, we detect no statistically significant difference in average prices in the two parts of the experiment. On the contrary, in the first-price auctions with communication and transfers average prices in the second part of the experiment are significantly lower than in the first part. In contrast, we observe a statistically significant increase in prices as subjects gain experience in our second-price auctions without communication, no statistically significant difference be-

Table 12: Frequency of Collusive Outcomes, by Treatment

	First-price Auctions		**Second-price Auctions**	
	rounds 1 to 5	rounds 6 to 10	rounds 1 to 5	rounds 6 to 10
No Communication	0.00 (0.00)	0.00 (0.00)	0.03 (0.01)	0.02 (0.01)
Pure Communication	0.10 (0.03)	0.14 (0.06)	0.18 (0.06)	0.23 (0.09)
Comm with Transfers	0.52 (0.08)	0.71 (0.06)	0.59 (0.04)	0.79 (0.05)

Notes: In the parentheses we report robust standard errors clustered by session.

Table 13: Regressions Comparing the Frequency of Collusive Outcomes Throughout Sessions

	First-price Auctions	**Second-price Auctions**
No Communication		0.672
Pure Communication	0.204	0.333
Comm with Transfers	0.000	0.000

Notes: We report p-values that correspond to the estimated coefficient of a dummy that takes the value of 1 if the auction took place in the second part of the experiment (rounds 6 to 10). All regressions are random-effect GLS regressions, with standard errors clustered at the session level.

Table 14: Prices, by Treatment

	First-price Auctions				
	rounds 1 to 5		**rounds 6 to 10**		**test of average prices b/w earlier and later rounds**
	mean (st err)	median	mean (st err)	median	p-value
No Communication	53.69 (3.3)	50.00	50.15 (3.1)	50.00	0.180
Pure Communication	38.41 (3.0)	40.00	36.37 (2.2)	38.31	0.445
Comm with Transfers	20.26 (4.6)	2.00	8.75 (1.7)	0.02	0.000
	Second-price Auctions				
	rounds 1 to 5		**rounds 6 to 10**		**test of average prices b/w earlier and later rounds**
	mean (st err)	median	mean (st err)	median	p-value
No Communication	41.86 (2.4)	40.00	48.30 (2.9)	50.00	0.035
Pure Communication	25.84 (3.0)	17.50	28.83 (4.0)	23.04	0.288
Comm with Transfers	14.47 (1.4)	1.00	8.11 (2.1)	0.00	0.007

Notes: Robust standard errors are computed by clustering observations by session. Comparison between average prices in earlier and later parts of the experiment is performed using random-effects GLS regressions, in which standard errors are clustered at the session level.

tween earlier and later rounds in the treatment with pure communication, and statistically significant decrease in prices in the treatment with communication and transfers.

7 Additional Sessions with Complete Strangers Protocol

We conducted several additional sessions for our Pure Communication and Communication with Transfers treatments in each auction format, in which we employed a complete strangers protocol. Under this protocol, each subject was never paired more than once with any other subject. While such sessions require more subjects, they allow us to eliminate repeated game effects altogether. The only difference between these additional sessions and the ones analyzed in the main text of the paper is the matching protocol. Table 15 below presents the number of sessions and the number of subjects participating in these additional sessions.

Table 16 presents revenues and efficiency observed in our experimental sessions separating out sessions that were conducted with random re-matching of participants between rounds and those conducted with complete strangers protocol.

Table 15: Sessions with Complete Strangers Protocol

Auction Format	Available Interaction	Nb of Sessions	Nb of Subjects
First-price	Pure Communication	3 sessions	40 subjects
	Communication with Transfers	1 session	12 subjects
Second-price	Pure Communication	2 sessions	28 subjects
	Communication with Transfers	1 session	12 subjects

Table 16: Revenues and Efficiency, rounds 6 to 10

	First-price Auctions			Second-price Auctions		
	efficiency	revenues	collusion	efficiency	revenues	collusion
Pure Communication	79%	31.08 (2.3)	16%	64%	41.04 (6.4)	3%
Comm + Transfers	80%	0.92 (0.28)	90%	77%	28.0 (6.3)	50%

Remarks: For revenues, we report average quantity as well as robust standard errors in the parenthesis where observations are clustered at the session level in case we have more than one session.

Table 16 shows that outcomes obtained in sessions with the complete strangers matching protocol exhibit similar patterns to those documented in the main text of the paper (using random re-matching across rounds). In particular, the availability of transfers does not affect efficiency levels conditional on auction format ($p = 0.904$ and $p = 0.225$ for the first- and second-price auctions with communication with and without transfers, respectively). Moreover, in both auction formats, the availability of transfers increases significantly the frequency of successful collusion ($p < 0.01$ in both regressions). Finally, the availability of transfers decreases the revenues of the auctioneer: this effect is highly significant in the first-price auction with $p < 0.01$, and goes in the correct direction but is not significant in the second price auction, with $p = 0.271$.

Finally, in the treatments with Communication and Transfers, we observe that winners often transfer statistically significant amounts to their losing partners even when they know that they are never going to interact with the same partner again through the experimental protocol. Positive transfers occur in 70% of our first-price auctions and in 43% of our second-price auctions in sessions with the complete strangers protocol. The frequency of transfers depends on how the auction culminates: transfers are very frequent if the ultimate price is minimal (this happens in over 70% of the first-price and over 85% of the second-price auctions). Finally, conditional on transferring a positive amount, winners transfer on average 44% of their surplus to their losing opponent in our first-price auctions and 48% in our second-price auctions. This analysis confirms that high frequencies of transfers documented in our sessions are robust and are not an artifact of the fully random protocol of matching subjects.

8 Theoretical Analysis

In our setting, absent communication, the first-price auction admits a unique equilibrium in which each bidder submits half her valuation (see Lebrun, 2004 and Maskin and Riley, 2003). In the second-price auction, there is a unique symmetric equilibrium, which entails strategies that are not weakly dominated, where each bidder bids precisely her value (see Fudenberg and Tirole, 1991). Nonetheless, in the second-price auction, there exist multiple asymmetric equilibria (for instance, one bidder bidding 100 and the other 0, regardless of their private values, is an equilibrium).[1] It is important to note that when symmetric equilibria are played in the first- and second-price auctions, the resulting mappings between bidders' valuations to allocation of the object (i.e., the probability that either bid wins the object) coincide. Furthermore, in both auctions the bidder with a valuation of 0 expects 0 payoffs. In this case, the Revenue Equivalence Theorem applies and the first- and second-price auctions are expected to generate identical revenues, given by 100/3.

With communication, the extant literature does not provide clear guidance on what outcomes may emerge. One issue is that communication naturally introduces multiplicity of equilibria with communication. The other, however, is that the literature has not provided a full characterization of equilibria of independent private value first- and second-price auctions with communication, as the ones we discuss. In terms of first-price auctions, Lopomo, Marx, and Sun (2011) may be the closest. Their results illustrate that with two bidders, binary valuations, and finite possible bids with vanishing increments, communication does not allow bidders to achieve greater returns than in the non-cooperative case with no communication. Most of the literature on one-shot collusion in auctions relies on some level of commitment across cartel members. Nonetheless, as we describe in the paper, the underlying message—with the caveat of differing assumptions—is that private-value sealed-bid second-price auctions are more fragile to collusion than first-price auctions.

In order to illustrate the theoretical results we can develop, let us first describe formally the setting. Assume there are two risk-neutral bidders. Agents' private valuations are independently and uniformly drawn from $V = [0, 100]$. At the outset, each individual knows her own value realization, but not the other bidder's. Bids are restricted to $B = V = [0, 100]$.

[1] For three or more bidders, Blume and Heidhues (2004) characterize the full set of equilibria in second-price auctions without communication.

We concentrate on first- and second-price auctions in which the highest bidder receives the good for a price corresponding to the highest and second highest bid, respectively. In either auction format, upon a tie, the winner is randomly chosen.

In what follows, we consider the case in which a cheap-talk stage is available after agents learn their private valuations and prior to submitting their bids. Formally, we consider the cheap-talk extensions of the first- and second-price auctions and study the induced set of equilibria.

From the Revelation Principle, an equilibrium with communication is tantamount to a mapping $\mu : V^2 \to \Delta(B^2)$ satisfying two types of incentive constraints: truthful revelation and obedience. An equilibrium outcome is a mapping $\gamma : V^2 \to \Delta(\{1, 2\} \times [0, 100])$, which maps any reported value profile into a distribution over the winning bidder and the price paid for the object. Denote by Γ_k the set of equilibrium outcomes corresponding to the k-price auction with communication, $k = 1, 2$.

Consider first the scope of collusion. Notice that in a second-price auction with communication, bidders can guarantee a price of 0, even in a symmetric equilibrium. Namely, bidders can randomly select the winner of the auction, who then submits a bid of 100, while the other bidder submits a bid of 0. This cannot be replicated in the first-price auction, where a price of 0 requires both bidders to submit a bid of 0 and is subject to profitable deviations. In fact, in the first-price auction, there cannot be any equilibrium of the game with communication in which the price of the object is always below some p, for sufficiently small p. Intuitively, suppose in some equilibrium the price always falls below some $p > 0$. A bidder who submits a bid of $p + \varepsilon$ for an arbitrarily small $\varepsilon > 0$ would then win the object for sure. For a bidder with valuation $v > p$ to follow the equilibrium prescriptions instead requires that the bidder win the object with a sufficiently high probability and, in that case, pay a price sufficiently lower than $p + \varepsilon$. For instance, if $v - p > v/2$, or $v > 2p$, a bidder with a valuation of v would need to win the object with more than a 50% probability. But, since v is distributed uniformly on $[0, 100]$, if $p < 25$, more than half of the bidders have a probability greater than 50% of winning the object, which leads to a contradiction.[2] To summarize, we have the following observation:

Observation (Full Collusion under First- and Second-price Auctions): *In the first-price auction, there is no equilibrium with communication that generates a price lower than* $100/4 = 25$ *regardless of the realized valuations. In the second-price auction, there is a (symmetric) equilibrium with communication that yields a price of* 0 *always.*

This observation suggests that full collusion, generating 0 revenue, is possible under the second-price auction, but not under the first-price auction. The next proposition illustrates

[2]Lopomo, Marx, and Sun (2011) showed that when there are two bidders and two possible values for the object, if the bid increment is sufficiently small, profitable collusion is not possible. While the generalization of this result to our setup is beyond the scope of this paper, these observations are in line with their conclusions.

that, in fact, any outcome produced in the first-price auction with communication can be emulated in the second-price auction with communication. In that respect, with communication, the scope for collusion under the second-price auction is strictly greater than that under the first-price auction. Formally,

Proposition 1. *The set of equilibrium outcomes generated by second-price auctions with communication strictly contains all equilibrium outcomes generated by the first-price auction,* $\Gamma_1 \subsetneq \Gamma_2$.

Proof. Notice that the outcomes corresponding to equilibria of the second-price auction without communication (and any mixtures of those) remain equilibrium outcomes of the second-price auction with communication. We now show that the outcome corresponding to the unique equilibrium in the first-price auction can be emulated in the second-price auction with communication. Indeed, consider the following mapping:

$$\mu(v_1, v_2) = \begin{cases} \left(100, \frac{v_1}{2}\right) & v_1 > v_2 \\ \left(\frac{v_2}{2}, 100\right) & v_1 < v_2 \\ \frac{1}{2} \otimes \left(100, \frac{v_1}{2}\right) + \frac{1}{2} \otimes \left(\frac{v_1}{2}, 100\right) & v_1 = v_2 \end{cases},$$

where $\frac{1}{2} \otimes \left(100, \frac{v_1}{2}\right) + \frac{1}{2} \otimes \left(\frac{v_1}{2}, 100\right)$ denotes a $50-50$ mixture between the bid profile $\left(100, \frac{v_1}{2}\right)$ and the bid profile $\left(\frac{v_1}{2}, 100\right)$.

We now show that μ constitutes an equilibrium of the second-price auction with communication. Notice first that it is never profitable for a bidder to deviate at the bidding stage when told to bid an amount lower than 100. In this case, the bidder knows the other bidder is bidding 100, and she can only win the object if she bids 100 too, in which case her profit would be at most 0. Now, suppose bidder i deviates by reporting $\hat{v}_i$ and bidding $\hat{b}_i \leq 100$ when told to bid 100. If $\hat{b}_i < \frac{\hat{v}_i}{2}$, she never wins the object and her expected payoff is 0. If $\hat{b}_i > \frac{\hat{v}_i}{2}$, her expected payoff is:

$$\left[\Pr(v_j < \hat{v}_i) + \frac{1}{2}\Pr(v_j = \hat{v}_i)\right]\left(v_i - \frac{\hat{v}_i}{2}\right) = \frac{2v_i\hat{v}_i - \hat{v}_i^2}{200},$$

which is maximized at $\hat{v}_i = v_i$, in which case bidding $\hat{b}_i > \frac{\hat{v}_i}{2}$ or 100 generates the same expected payoff. If $\hat{b}_i = \frac{\hat{v}_i}{2}$, then the bidder receives half of the expected payoff she would receive by bidding 100, which is not profitable.

The equilibrium μ implements the same outcome that would have been achieved in the first-price auction without communication. The highest valuation bidder receives the good and pays a price that is precisely half of her valuation (when valuations coincide, each bidder gets the good with a $50 - 50$ chance). This completes the proof. □

Intuitively, an outcome of the first-price auction can be emulated by the second-price auction as follows. Whenever the bidders are to submit different bids, say $b_1 > b_2$ in the first-price auction, bidder 1 submits 100, thereby assuring she will receive the object, and bidder 2 submits b_1, thereby assuring the price is b_1. When bids coincide in the first-price auction, $b_1 = b_2 = b$, bidders can toss a fair coin in the second-price auction to determine who will bid 100 and who will bid b, which guarantees an equal chance of winning at the price b. In the proof we also show that this procedure assures truthful revelation.

The set of equilibrium outcomes in the second-price auction with communication is large. Indeed, bidders can always publicly randomize during the communication phase over which equilibrium they intend to play, assuring that the set of equilibrium outcomes is a convex set. In particular, it contains the convex hull of the outcomes just discussed, those generated by the equilibrium of the first-price auction, as well as the symmetric and asymmetric equilibrium outcomes of the second-price auction (in fact, it strictly contains the convex hull of the set of equilibrium outcomes of second-price auctions without communication). The main message of the proposition is that communication has more of an impact on second-price auctions than it does on first-price auctions.

There are two notes on this theoretical result. First, the cheap-talk extension of the auctions we consider implicitly assumes the availability of an impartial mediator (for the use of the Revelation Principle). The general characterization of games in which unmediated communication generates the same outcomes as mediated communication is a difficult problem (see Gerardi, 2004 and references therein and note that Gerardi, 2004 suggests that with five or more bidders, a mediator is unnecessary). Nonetheless, even absent a mediator, the set of equilibrium outcomes strictly expands when communication is introduced to second-price auctions (for instance, bidders can randomize between equilibria of the auction without communication). Second, we do not preclude weakly dominated strategies. This certainly simplifies the analysis, but the ultimate validity of this allowance is in the data. As our experimental results suggest (as well as extant ones for auctions without communication), subjects do not seem to focus on weakly dominant actions.

Suppose now that agents can communicate freely prior to bidding *and* exchange (simultaneously) non-negative transfers after bidding and learning the identity of the object's winner. Formally, the game played is a first- or second-price auction followed by a transfer stage in which agents can simultaneously pick a non-negative number to transfer to the other bidder. Their ultimate payoff is then their payoff in the auction plus the net transfers they have received (the transfers the other bidder passed minus the transfers they had passed to the other bidder). The availability of such ex-post transfers has no impact on the behavior in the preceding communication and auction phases corresponding to equilibria. Indeed, optimality required by best responses would imply zero transfers in any equilibrium and a profile of behavior consistent with some equilibrium in the cheap-talk extension of our baseline auction before that. Let $\tilde{\Gamma}_k$ denote the set of equilibrium outcomes corresponding to the k-price auction with communication *and transfers*, $k = 1, 2$. That is, these are mappings from value profiles to distributions over winning probabilities and prices when both communication and transfers are available. We then have the following:

Proposition 2. *In any Bayesian Nash equilibrium of the first- or second-price auctions with communication and transfers, no positive transfers are passed. Therefore, the sets of equilibrium outcomes coincide with those of the first- or second-price auctions with communication. That is,* $\tilde{\Gamma}_1 = \Gamma_1 \subsetneq \tilde{\Gamma}_2 = \Gamma_2$.

To summarize, there are three insights that are relevant to our design. *First*, without communication, both auction formats entail unique equilibrium predictions when bidders use weakly dominant strategies, and these equilibria are symmetric; The second-price auction entails multiple asymmetric equilibria if the domination restriction is dropped. *Second*, with communication, second-price auctions generate substantially more equilibrium outcomes than first-price auctions (Proposition 1). In particular, full collusion, associated with 0 revenue, is possible under the second-price auction but not under the first-price auction. *Last*, transfers have no impact on outcomes in either auction format and outcomes are predicted to be identical to those in auctions with communication, but without transfers. Furthermore, no positive transfers are passed in any subgame-perfect equilibrium (Proposition 2).[3]

References

[1] Blume, Andreas and Paul Heidhues (2004), "All Equilibria of the Vickrey Auction," *Journal of Economic Theory*, **114(1)**, 170-177.

[2] Fudenberg, Drew and Jean Tirole (1991), *Game Theory*, The MIT Press.

[3] Gerardi, Dino (2004), "Unmediated Communication in Games with Complete and Incomplete Information," *Journal of Economic Theory*, **114**, 104-131.

[4] Lebrun, Bernard (2004), "Uniqueness of Equilibrium in First-Price Auctions," mimeo.

[5] Lopomo, Giuseppe, Leslie M. Marx, and Peng Sun (2011), "Linear Programming for Mechanism Design: An Application to Bidder Collusion in First Price Auctions," *Review of Economic Design*, **15**, 177-211.

[6] Maskin, Eric and John Riley (2003), "Uniqueness of Equilibrium in Sealed High-bid Auctions," *Games and Economic Behavior*, **45(2)**, 395-409.

[3]For simplicity, we assumed a continuous set of values and possible bids. We note, however, that our results do not critically depend on this assumption. Indeed, without communication, Chwe (1989) shows that symmetric equilibria in discrete settings converge to the equilibrium in the continuous setting and our results for the second-price auction remain as they are. Furthermore, the comparison of the two auction formats when communication is available remains intact.