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Broken Supersymmetry and Supergravity

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We consider the supersymmetric Higgs effect, in which a spin-$\frac{1}{2}$ Goldstone fermion is transformed away by a redefinition of the supergravity fields and the spin-$\frac{3}{2}$ gauge field acquires the degrees of freedom appropriate to finite mass. More generally we discuss the consistency and physical applicability of supergravity theories with broken local supersymmetry.

Rigorous supersymmetry implies the existence of supermultiplets made up of fermions and bosons with equal masses. If supersymmetry is to be relevant for the physical world, it must be broken, either softly or spontaneously. Spontaneous breaking of global supersymmetry gives rise to the appearance of one or more Goldstone fermions.[1] When global supersymmetry is promoted to a local invariance by coupling supersymmetric matter to supergravity, the Goldstone fermion disappears as a consequence of a phenomenon analogous to the Higgs effect of ordinary gauge theories. In this Letter we describe this supersymmetric Higgs effect,[2] and consider its possible application to the construction of realistic models.[3] In particular, the supersymmetric Higgs effect gives a possible solution to the problem of the apparent nonexistence in nature of the Goldstone fermion of spontaneously broken supersymmetry. As we know, this cannot be identified with the electron neutrino, because it would satisfy low-energy theorems which contradict observed properties of the neutrino spectrum.[4]

The Goldstone fermion is described by a Majorana spin-$\frac{1}{2}$ field λ. Irrespective of the particular field theory in which it arises, it can be characterized, following Volkov and Akulov,[5] by the nonlinear realization of global supersymmetry

$$\delta\lambda = a^{-1}\alpha + ia\bar{\alpha}\gamma^{\mu}\lambda\partial_{\mu}\lambda, \tag{1}$$

where α is the infinitesimal supersymmetry parameter and a is a constant which measures the strength of the spontaneous breaking of supersymmetry. The nonlinear Lagrangian for λ, invariant (up to a divergence) under (1), is given by

$$L_{\lambda} = -(2a^2)^{-1}\det(\delta_{\mu}{}^{\nu} + ia^2\bar{\lambda}\gamma^{\nu}\partial_{\mu}\lambda)$$
$$= -(2a^2)^{-1} - \tfrac{1}{2}i\bar{\lambda}\gamma\cdot\partial\lambda + \dots . \tag{2}$$

The analogy with nonlinear pion dynamics is apparent. However, the chiral group $SU(2)\otimes SU(2)$ is also broken explicitly by a pion mass term. In contrast, if we assume that supersymmetry is broken only spontaneously, the above description is expected to be rigorous and to be actually valid for a suitably defined field λ in any renormalizable model in which a Goldstone fermion emerges.

Let us now try to promote (1) to a local transformation with parameter $\alpha(x)$ and to make (2) invariant under it by coupling λ to the supergravity fields $e_\mu{}^a$ and ψ_μ. The complete Lagrangian will be rather complicated. Assuming its existence, one can easily find the first terms in an expansion in the coupling constants a and κ (gravitational constant). The Lagrangian ($e \equiv \det e_\mu{}^a$)

$$L_\lambda = -(2a^2)^{-1}e - \tfrac{1}{2}i\bar{\lambda}\gamma\cdot\partial\lambda - (i/2a)\bar{\lambda}\gamma\cdot\psi + \ldots \quad (3)$$

changes by a divergence under

$$\delta\lambda = a^{-1}\alpha(x) + \ldots,$$
$$\delta e_\mu{}^a = -i\kappa\bar{\alpha}\gamma^a\psi_\mu, \quad (4)$$
$$\delta\psi_\mu = -2\kappa^{-1}\partial_\mu\alpha + \ldots.$$

To (3) one must add the usual supergravity Lagrangian[6,7]

$$L_{sg} = -(2\kappa^2)^{-1}eR - \tfrac{1}{2}i\epsilon^{\lambda\mu\nu\rho}\bar{\psi}_\lambda\gamma_5\gamma_\mu D_\nu\psi_\rho, \quad (5)$$

where

$$D_\mu = \partial_\mu - \tfrac{1}{2}\omega_{\mu,ab}\Sigma^{ab}, \quad \Sigma^{ab} = \tfrac{1}{4}[\gamma^a, \gamma^b], \quad (6)$$

and R is the contracted Riemann tensor, taken as a function of the vierbein and of the connection ω and its derivatives. The sum $L_{sg} + L_\lambda$ is invariant under (4). The transformation law for the field λ shows that it can be transformed to zero by a suitably chosen local supersymmetry transformation. In other words, the field λ can be absorbed into a redefinition of the fields $e_\mu{}^a$ and ψ_μ. The resulting theory is described by the Lagrangian (5) of supergravity plus the cosmological term from (3) (plus possible additional terms from the supersymmetric matter part which gave rise to spontaneous symmetry breaking). This is the supersymmetric Higgs effect.[8]

At first sight, the result is puzzling and disappointing. It is puzzling because the disappearance of the Goldstone particle gave rise to a cosmological term, instead of generating a mass for the spin-$\frac{3}{2}$ field, as one would have expected from a count of degrees of freedom. It is disappointing because the empirical smallness of the cosmological constant $\sim (2a^2)^{-1}$ seems to destroy any hope that the spontaneous breaking of supersymmetry will be large enough to account for the observed mass splitting between bosons and fermions. We shall deal with both problems.

To resolve the puzzle we first note that, in the presence of a cosmological term, one cannot quantize in a Minkowski background, but must take instead as background space a solution of the Einstein equations with cosmological term. The simplest and most natural is the corresponding de Sitter space and one knows that the concept of mass is rather delicate there.[9] We next recall the recent observations[10-12] that one can add to the supergravity Lagrangian (5) the sum of a cosmological term and of a spin-$\frac{3}{2}$ mass term

$$ce - \tfrac{1}{2}im\epsilon^{\lambda\mu\nu\rho}\bar{\psi}_\lambda\gamma_5\Sigma_{\mu\nu}\psi_\rho. \quad (7)$$

Local supersymmetry is valid provided that the two parameters are related by

$$c\kappa^2 = 3m^2. \quad (8)$$

Indeed, the sum of (5) and (7) is then invariant under a modified supersymmetry transformation, in which the usual transformation law for the spin-$\frac{3}{2}$ field, $\delta\psi_\mu = -2\kappa^{-1}D_\mu\alpha$, is replaced by

$$\delta\psi_\mu = -2\kappa^{-1}\mathfrak{D}_\mu\alpha, \quad (9)$$

where

$$\mathfrak{D}_\mu \equiv D_\mu + \tfrac{1}{2}m\gamma_\mu \quad (10)$$

(there is a corresponding change in $\delta\omega_{\mu,ab}$).

The existence of this local supersymmetry[13] shows that, in spite of the apparent mass term in (7), the spin-$\frac{3}{2}$ field has the number of degrees of freedom appropriate to the massless case, namely two. For example, in an external de Sitter space satisfying (8) the covariant derivatives (10) commute, i.e., $[\mathfrak{D}_\mu, \mathfrak{D}_\nu] = 0$, and the quantities $\psi_{\mu\nu} = \mathfrak{D}_\mu\psi_\nu - \mathfrak{D}_\nu\psi_\mu$ and $*\psi^{\mu\nu} = \frac{1}{2}e^{-1}\epsilon^{\mu\nu\lambda\rho}\psi_{\lambda\rho}$ are gauge invariant under (9), as is the Rarita-Schwinger equation (11). The latter is equivalent to $*\psi_{\mu\nu} - \gamma_5\psi_{\mu\nu} = 0$, $\gamma_\mu\psi^{\mu\nu} = 0$, or $\gamma_\mu *\psi^{\mu\nu} = 0$, and implies $\mathfrak{D}_\mu(e\psi^{\mu\nu}) = \mathfrak{D}_\mu(e*\psi^{\mu\nu}) = 0$. Therefore also $D_\mu(e\psi^{\mu\nu}) = D_\mu(e*\psi^{\mu\nu}) = 0$. These are the correct equations for a massless spin-$\frac{3}{2}$ particle. On the other hand, if (8) is violated, the constraints $\gamma\cdot\psi = 0 = D_\mu(e\psi^\mu)$ replace the gauge invariance and the field is massive, with four degrees of freedom. Similar conclusions follow when gravity is dynamical. Thus, when the cosmological term and the spin-$\frac{3}{2}$ mass term are *not* related as in (8), there is no local supersymmetry and the spin-$\frac{3}{2}$ field is effectively massive. This is true in particular when there is *only* a cosmological term, which finally resolves the paradox of the Goldstone fermion's degrees of freedom.

We now show that despite loss of supersymmetry, at least the classical equations of motion remain consistent in that no anomalous propagation hypersurfaces occur. To see this we follow a method similar to that which led Velo and Zwanziger[14] to discover the anomalous propagation of a spin-$\frac{3}{2}$ field coupled minimally to an external electromagnetic field. Let us consider the equation of motion for ψ_μ which follows from (5) plus (7), without assuming (8). It can be written, including the mass term, as

$$eR^\mu \equiv \epsilon^{\mu\nu\lambda\rho}\gamma_5\gamma_\nu\mathfrak{D}_\lambda\psi_\rho = 0. \tag{11}$$

This equation does not contain $\partial_0\psi_0$. Therefore we transform it in a standard way. First we observe that, using the equations of motion for the vierbein and for the torsion, one finds

$$2\mathfrak{D}_\mu(eR^\mu) = (c\kappa^2 - 3m^2)e\gamma\cdot\psi. \tag{12}$$

This can be either verified directly, by the same arguments as in Ref. 7, or simply deduced from the fact that the Lagrangian (5) plus (7) can be written as the sum of a part invariant under (9) plus a noninvariant term $(c - 3m^2\kappa^{-2})e$. Therefore Eq. (11) implies the validity of the constraints

$$\chi \equiv \gamma\cdot\psi = 0\,, \tag{13}$$

and

$$N \equiv -\gamma\cdot R + 2\gamma\cdot\mathfrak{D}\chi - 2m\chi = 0 \tag{14}$$

(as will become clear, one cannot simply use $\gamma\cdot R$ instead of N). We now replace (11) by the equation

$$\mathfrak{R}_\mu \equiv R_\mu + \tfrac{1}{2}\gamma_\mu N + \mathfrak{D}_\mu\chi - \gamma_\mu\gamma\cdot\mathfrak{D}\chi = 0. \tag{15}$$

With use of the simple identity

$$R_\mu - \tfrac{1}{2}\gamma_\mu\gamma\cdot R \equiv \gamma^\lambda(\mathfrak{D}_\lambda\psi_\mu - \mathfrak{D}_\mu\psi_\lambda), \tag{16}$$

it follows that

$$\mathfrak{R}_\mu = \gamma^\lambda(\mathfrak{D}_\lambda\psi_\mu - \mathfrak{D}_\mu\psi_\lambda) + (\mathfrak{D}_\mu - m\gamma_\mu)\gamma\cdot\psi$$
$$= (\gamma\cdot\partial + m)\psi_\mu + \dots, \tag{17}$$

where the dots denote terms with no derivatives of ψ_μ. Similarly,

$$N = 2\partial_\mu\psi^\mu + \dots\,. \tag{18}$$

Finally we verify that, if (15) is satisfied, the initial validity of the constraints (13) and (14) implies the vanishing of $\partial_0\chi$ and $\partial_0 N$. Indeed one finds, using (14),

$$\gamma\cdot\mathfrak{R} = N - (\gamma\cdot\mathfrak{D} + 2m)\chi, \tag{19}$$

and, using (12),

$$2\mathfrak{D}_\mu(e\mathfrak{R}^\mu) = (c\kappa^2 - 3m^2)e\chi + \mathfrak{D}_\mu(e\gamma^\mu N)$$
$$+ 2\mathfrak{D}_\mu e(\mathfrak{D}^\mu - \gamma^\mu\gamma\cdot\mathfrak{D})\chi. \tag{20}$$

In the last term in (20) the second derivatives of χ cancel. This proves our statement. We conclude that (15), together with (13) and (14), is equivalent to (11). The characteristic surfaces of (13), (14), and (15) are obviously the local light cones.

The argument just given can be applied essentially without change to the O(2) and O(3) supergravity theories with additional couplings. It has been shown by Freedman and Das[10] that these theories are locally supersymmetric, provided certain relations are satisfied among the gravitational constant, the cosmological constant, the spin-$\frac{3}{2}$ mass, the minimal vector coupling, and the magnetic-moment coupling. One finds by the present methods that, if local supersymmetry is broken by changing only the value of the cosmological constants, the theory is still consistent and propagation occurs along the local light cones. In particular, the cosmological constant can be set equal to zero. This latter model then provides an example of consistent coupling of a massive spin-$\frac{3}{2}$ field to both electromagnetism (or Yang-Mills) and gravitation, which can be quantized in Minkowski space. It is remarkable that gravity is necessary (and suffices!) to compensate the inconsistencies of flat-space spin-$\frac{3}{2}$–electromagnetic coupling.

We now return to the Higgs effect and the apparently disappointing magnitude of the accompanying cosmological constant. Existence of an invariant action including (7) that satisfies (8) can be used to resolve this difficulty as well. The sign of the cosmological term in (7) is fixed by (8), and corresponds to a de Sitter space with O(3, 2) invariance. On the other hand, that of the cosmological term in (3) is also fixed, but opposite. Adding (3), (5), and (7), one can adjust the constants so that the cosmological terms cancel. [One must, of course, modify (3) so as to make it invariant under (9), which, to the order considered here, requires adding to (3) a term[15] $-im\bar{\lambda}\lambda + \dots$.] Now one can clearly cancel the cosmological term between (3) and (7) with (8) by setting

$$m^2 = \tfrac{1}{6}(\kappa/a)^2. \tag{21}$$

If we assume that the spontaneous supersymmetry breaking is responsible for the observed mass splittings between mesons and baryons, the order of magnitude of the constant a must be given by a hadronic mass, say the proton mass, in which case we find

$$m \sim (\kappa m_p)m_p\,, \tag{22}$$

with $\kappa m_p \sim 10^{-19}$. The mass of the spin-$\frac{3}{2}$ field is very small, which is not unacceptable, but we now have hadronic mass splittings of reasonable magnitude and zero cosmological constant.[16]

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[1]J. Iliopoulos and B. Zumino, Nucl. Phys. B76, 310 (1974); A. Salam and J. Strathdee, Phys. Lett. 49B, 465 (1974); P. Fayet and J. Iliopoulos, Phys. Lett. 51B, 461 (1974); L. O'Raifeartaigh, Nucl. Phys. B96, 331 (1975).

[2]D. V. Volkov and V. A. Soroka, Pisma Zh. Eksp. Teor. Fiz. 18, 529 (1973) [JETP Lett. 18, 312 (1973)], were the first to point out the possible occurrence of a supersymmetric Higgs effect. [See also D. V. Volkov, to be published.] However, in spite of its formal similarity their approach seems to be essentially different from ours.

[3]B. Zumino, in *Lectures at the 1976 Scottish Universities Summer School, August 1976* (Edinburgh Univ. Press, Edinburgh, Scotland, 1976).

[4]W. Bardeen, unpublished; B. DeWitt and D. Z. Freedman, Phys. Rev. Lett. 35, 827 (1975).

[5]D. V. Volkov and V. P. Akulov, Phys. Lett. 46B, 109 (1973).

[6]D. Z. Freedman, P. van Nieuwenhuizen, and S. Ferrara, Phys. Rev. D 13, 3214 (1976).

[7]S. Deser and B. Zumino, Phys. Lett. 62B, 335 (1976).

[8]Note that the choice of gauge implicit in the elimination of λ restores absolute meaning to space-time, just as the photon direction is specified in the corresponding vector case.

[9]P. A. M. Dirac, Ann. Math. (N.Y.) 36, 657 (1935); F. Gürsey and T. D. Lee, Proc. Nat. Acad. Sci. 49, 179 (1963); L. Halpern, to be published.

[10]D. Z. Freedman and A. Das, State University of New York at Stony Brook Report No. ITP-SB-76-64 (to be published).

[11]S. W. MacDowell and F. Mansouri, Phys. Rev. Lett. 38, 739, 1376(E) (1977).

[12]P. K. Townsend, State University of New York at Stony Brook Report No. ITP-SB-77-2 (to be published).

[13]It is also easy to see that the Lagrangian (5) plus (7) satisfying (8) admits a global supersymmetry, obtained by taking α independent of x. It is the global supersymmetry of the de Sitter space having cosmological constant given by (8) (radius m^{-1}), which has O(3,2) as the maximal Lie subalgebra. Here we disagree with Refs. 10 and 12, where the spin-$\frac{3}{2}$ mass term is interpreted as giving rise to a breaking of local supersymmetry, even though (8) is satisfied.

[14]G. Velo and D. Zwanziger, Phys. Rev. 186, 1337 (1969), and 188 (1969) 2218. See also J. Madore, Phys. Lett. 55B, 217 (1975).

[15]The meaning of this term can be understood by considering the invariant Lagrangian for a Goldstone spinor in de Sitter space of radius m^{-1}; B. Zumino, to be published.

[16]The compensation between cosmological terms of opposite sign was considered by Freedman and Das in Ref. 10.

Low-Temperature Approach to the Renormalization-Group Study of Critical Phenomena*

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A new method of exploring the contents of the renormalization-group equations for discrete spins is introduced. The equations are expanded in low-temperature series and the truncated series are used to obtain the critical exponents and critical temperature of a system. The method is demonstrated on the planar triangular Ising lattice and the critical parameters are found to be within a few percent of the exactly known values in third nonvanishing order of approximation.

The work of Kadanoff[1] and Wilson[2] on the application of scaling laws and the renormalization group to the study of lattices has inspired a wide range of activity in the field of critical phenomena.[3]

Wilson's method of continuous spins exposes the symmetries and general features of the theory, but it is not very useful for the calculation of critical exponents, critical surfaces of coupling constants, and scaling functions, because the ϵ expansion used in the calculations is only an asymptotic series.

Niemeijer and van Leeuwen have proposed a method which is based on the direct study of the discrete lattice.[4] Following the idea of Kadanoff,[1] they introduce a finite rescaling of the lattice L by dividing it into cells, each containing n spins, in such a way that the new lattice L' made up of the cells is a similar rescaled version of the original. The cell spin s_i' of the ith cell is defined and the spin variables inside a cell $s_k^{(i)}$ are replaced by the variables s_i' and $\sigma_m^{(i)}$, the latter