Mon. Not. R. Astron. Soc. **302,** 723–730 (1999)

On particle energization in accretion flows

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Accepted 1998 September 18. Received 1998 September 11; in original form 1998 July 20

ABSTRACT

Two-temperature advection-dominated accretion flow (ADAF) or hot ion tori (HIT) models help to explain low-luminosity stellar and galactic accreting sources and may complement observational support for black holes in nature. However, low radiative efficiencies demand that ions receive a fraction $\eta \gtrsim 99$ per cent of energy dissipated in the turbulent accretion. The η value depends on the ratio of particle to magnetic pressure. If modes of dissipation involving compressions at least perpendicular to the magnetic field (like magnetic mirroring) dominate, then even when the pressure ratio is O(1), the required large η can be attained. However, the relative importance of compressible versus incompressible modes is hard to estimate. The plasma is more compressible on larger scales and the relevant length-scale for particle energization can be estimated by equating the longest eddy turnover time (which corresponds to the energy-dominating scale) to the time for which an energy equal to that in the turbulence can be drained into particles. Based on the large scales resulting from this estimate, it is suggested that magnetic mirroring may be important. Also, regardless of the precise η or dissipation mechanism, non-thermal protons seem natural in two-temperature discs because all dissipation mechanisms, and the use of an isotropic pressure, require wave–particle resonances that operate only on a subset of the particles. Finally, it is briefly mentioned how mirroring may help to generate an ADAF or HIT in the first place.

Key words: acceleration of particles – accretion, accretion discs – turbulence – binaries: general – Galaxy: centre – galaxies: general.

1 INTRODUCTION

Magnetized accretion discs have become the most convincing physical paradigm to explain emission from the central engines of active galactic nuclei (AGN) and X-ray binary sources (Frank, King & Raine 1992). The observed radiation comes from the energy dissipation required to maintain steady accretion of material on to the central object. As molecular viscosity is incapable of providing the required accretion rates, turbulent viscosity is necessary. For thin discs, this can be generated by shear and magnetic fields (Balbus & Hawley 1991). For thick discs, something similar may occur, though in this case angular momentum transport may ultimately require a global approach.

Nevertheless, as a complement to thin-disc solutions for sources requiring high radiation efficiency accretion (e.g. Frank et al. 1992), two-temperature thick advection-dominated accretion flows (ADAFs) or hot ion tori (HIT) (e.g. Shapiro, Lightman & Eardley 1976; Ichimaru 1977; Paczyński & Bisnovatyi-Kogan 1981; Rees et al. 1982; Narayan & Yi 1995) have received much attention in an effort to explain sources requiring a low radiation efficiency. Here the ions are assumed to receive the energy dissipated by the steady accretion without having enough time to transfer their energy to the cooler electrons before falling on to the central object. Some or most of the dissipated energy is advected, not radiated, as it would have been if electrons received all of the dissipated energy. Such models have been at least partially successful in explaining quiescent galactic centres (Rees 1982; Narayan, Yi & Mahadevan 1995; Mahadevan 1998; Fabian & Rees 1995, but see DiMatteo et al. 1998) and stellar X-ray binary systems (Narayan, McClintock & Yi 1996) with radiative efficiencies $\leq$1/100 that of thin-disc solutions. When the central object is a black hole, the advected energy is lost forever rather than reradiated as it would be for a neutron star. Precisely such observed differences between corresponding X-ray binary systems have been purported to provide evidence for black hole horizons (Narayan, Garcia & McClintock 1997).

There has been only a handful of work addressing how the viscous dissipation might energize particles in accretion flows (Gruzinov 1997; Quataert 1997; Quataert & Gruzinov 1998) and little work on what spectrum is produced by the dissipation (see also Gruzinov & Quataert 1998). Both the species and spectra of energized particles are extremely important for ADAFs/HIT because: (1) a two-temperature solution is insufficient to explain a low radiation efficiency and (2) interpretation of observations of the Galactic centre suggests that the protons are non-thermal when an ADAF model is employed (Mahadevan 1998). The potential catastrophe for ADAF/HIT models, if electrons are preferentially

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energized over protons, was partially explored in Bisnovatyi-Kogan & Lovelace (1997). Because electrons cool much faster than ions, even if one third of the dissipated energy went into electrons, a two-temperature solution would still result. But in this case, one third of the dissipated energy would be radiated, far too much to explain low-luminosity sources. More explicitly, I define q_t as the magnitude of the energy density input rate into particles. Then

$$q_t = q_p^+ + q_e^+ = \eta q_t + (1-\eta)q_t,$$

where q_p^+ and q_e^+ are the magnitudes of energy input rate into protons and electrons, and η is the fraction of q_t that goes into protons. In the steady state, energy loss rates are equal to energy gain rates so that, when advection is included, we have for the protons

$$q_p^+ = \eta q_t = q_a^- - q_{pe} = f\eta q_t + (1-f)\eta q_t,$$

where $q_a = f\eta q_t$ is the rate associated with advection, q_{pe} is the rate of transfer from ions to electrons, and f is the fraction of the proton energy loss rate associated with advection. For the electrons, we thus have

$$q_e^+ = (1-\eta)q_t + q_{pe} = q_t - q_a = q_t(1-f\eta).$$

Since $q_e^+ = q_e^-$, where q_e^- is the luminosity density, the quantity $(1-f\eta) \ll 1$ must hold to explain quiescent sources. Standard treatments (e.g. Rees et al. 1982; Narayan & Yi 1995) assume $\eta = 1$ so that 1 per cent radiative efficiency would correspond to $f = 0.99$. There are three important questions: (a) When can $\eta \gtrsim 0.99$ be justified? (b) Are protons non-thermal? (c) Is there a faster than Coulomb coupling between electrons and protons (e.g. Begelman & Chiueh 1988) that destroys the ADAF solution? I will address (a) and (b) here.

Proton energization by incompressible (Quataert 1997; Gruzinov 1997; Quataert & Gruzinov 1998) and/or compressible modes of dissipation both depend on the ratio of particle to magnetic pressure. The magnetic mirroring mode of dissipation, discussed herein, can occur in either a compressible or incompressible plasma, as long as there is compression across the field lines. I call this condition '$\perp$-compressible.' It simply means that regions of the plasma can have a non-zero $\nabla_\perp \cdot \boldsymbol{v}_\perp$, where $\perp$ indicates the direction to the local magnetic field and $\boldsymbol{v}$ is the local fluid velocity. Note that $\nabla \cdot \boldsymbol{v}$ can still be zero when $\nabla_\perp \cdot \boldsymbol{v}_\perp \neq 0$ (e.g. pseudo-Alfvén wave of Goldreich & Sridhar 1994; Goldreich & Sridhar 1997).

Section 2 addresses the relation between magnetic, turbulent and particle energy densities in ADAFs, relating them to the viscosity parameter α. Section 3 discusses the threat of electron runaway. Section 4 employs a very physical approach to acceleration by magnetic mirroring and derives the time-scale for particle energy doubling for two distinct limits of the average particle speed. Section 5 discusses energization in ADAFs, first addressing why protons are likely to be non-thermal, regardless of the acceleration mode. The mirroring results are then specifically applied to ADAFs and the scale in the turbulent cascade where mirroring is favoured is estimated. Because larger scales are significantly compressible, and the resulting derived scale can be large, mirroring may be most important on these scales. Magnetic mirroring-type processes can favour protons to the extent required for a wider range of average particle to magnetic pressure ratios than found by Quataert & Gruzinov (1998) from dissipation of $\perp$-incompressible Alfvén waves, but the relative fractions of $\perp$-compressible versus $\perp$-incompressible modes of dissipation are hard to determine. The possibility that mirroring may help to provide a thermal instability that initially forms an ADAF is briefly addressed.

2 RELATION BETWEEN VISCOSITY PARAMETER AND PRESSURE RATIO

The standard parametrization of accretion disc turbulent viscosity for thin discs is (Shakura & Sunyaev 1973)

$$\nu_T = \alpha C_s H \sim V_T l_T/3, \tag{1}$$

where α is the viscosity parameter, C_s and H are the sound speed and disc height, and V_T and l_T are the outer (i.e. dominant energy-containing) random (turbulent) flow speed and scale. For thin discs, the scale of the turbulence is always much less than the radius of the disc. The magneto-shearing instability and the subsequent field line stretching make the turbulent kinetic and magnetic energy densities nearly equal, with the magnetic energy density slightly dominant (e.g. Brandenburg et al. 1995; Stone et al. 1996). (In thick discs, large-scale magneto-convective instabilities rather than local magnetic shear instabilities may drive turbulence.)

In a steady state, dissipation of the turbulence into particles combats the symbiotic growth of magnetic and kinetic turbulent energy. Both magnetic and kinetic turbulent energies incur a decaying power-law energy spectrum (like Kolmogorov 1941 or Kraichnan 1965) with the largest scales of the turbulence containing the most energy. Since the sound speed is constant on all scales, the largest scales are the most compressible. For thin discs, the outer turbulent scale is significantly smaller than the disc radius, but for standard ADAFs, such a strong scale separation is absent (Blackman 1998).

For thin discs, we can derive a one-to-one link between α and $\beta_p^{-1} \equiv V_A^2/C_s^2 \equiv 6(1-\beta_a)$, where V_A is the Alfvén speed, and β_a is used in ADAF modelling. In the steady state, the largest eddy turnover time $t_T = l_T/V_T$ must equal the shearing instability growth time driving the turbulence, that is $t_T \simeq R/V_\phi$, where the rotation speed $V_\phi \sim V_K$, the Keplerian speed. Since $V_A \gtrsim 2^{1/2} V_T$ in the saturated state from magneto-shearing and turbulent stretching (e.g. Parker 1979; Brandenburg et al. 1995; Stone et al. 1996; Blackman 1998) and $C_s/V_K = H/R$ from hydrostatic equilibrium, we have

$$\nu = \alpha C_s H \simeq V_T l_T/3 \sim V_A^2 (R/6V_\phi) = V_A^2 (H/6C_s),$$

which implies that

$$\alpha \simeq (1-\beta_a)(V_K/V_\phi) \simeq (1-\beta_a) \tag{2}$$

for thin discs. This result basically agrees with numerical simulations (e.g. Stone et al. 1996).

For ADAFs, the ratio of $V_\phi/V_{K]}$ can be so low that (2) is inappropriate: in this case the resulting eddy scale implied by the relation would be larger than $H \sim R$. We can instead obtain an upper limit on α for ADAFs that comes from the constraint

$$l_T < H \sim R. \tag{3}$$

Then from (1), the definition of β_a and $V_A \sim 2^{1/2}$, we have

$$\alpha \leq (1/3^{1/2})(1-\beta_a)^{1/2}, \tag{4}$$

showing that β_a and α are not independent.

3 ON ELECTRON RUNAWAY

Bisnovatyi-Kogan & Lovelace (1997) pose an interesting question: Why can direct acceleration from electric fields not drain energy into electrons, destroying ADAFs? I address this here. First, note the generalized Ohm's law (e.g. Scudder et al. 1986)

$$\boldsymbol{E} = -\boldsymbol{V}_e/c \times \boldsymbol{B} + \sigma^{-1}\boldsymbol{J} - m_e(\boldsymbol{V}_e \cdot \nabla \boldsymbol{V}_e)/e - \nabla P_e/(en_e)$$

where $\boldsymbol{B}$ is the magnetic field, $\boldsymbol{J}$ is the current density, P_e is the electron pressure, $\boldsymbol{V}_e$ is the bulk electron velocity, σ^{-1} is the resistivity, m_e is the electron mass, n_e is the density and e is the electric charge. For the plasmas of interest, a characteristic magnitude of $\boldsymbol{E}$ parallel to $\boldsymbol{B}$ is given by the last term. This gives

$$|E_{||}| \sim k_B T_e/(\delta l\ e) \sim 2\times10^{-14}(T_e/10^9\ \mathrm{K})(\delta l/10^{13}\ \mathrm{cm})^{-1},$$

where δl is the gradient length, T_e is the electron temperature and k_B is the Boltzmann constant.

For $E_{||}$ to produce electron runaway (ER), it would have to exceed the Dreicer electric field (Dreicer 1959; Holman 1985; Bisnovatyi-Kogan & Lovelace 1997)

$$E_D = e^2\ \ln\Gamma/\lambda_D^2 = 1.8\times10^{-7}(\ln\Gamma/20)(n_e/10^{12}\ \mathrm{cm}^{-3})^{1/2}$$
$$(T_e/10^9\ \mathrm{K})^{-1}\ \mathrm{St\,V\,cm}^{-1},$$

using $\lambda_D \sim 6.65(T_e^{1/2}/n_e)$ cm for the Debye length. Whether $E_{||} > E_D$ depends on the size of δl. For AGN, E_D is only exceeded on scales 10^6 times smaller than the turbulent outer scale. For stellar-size X-ray binary ADAF systems, the outer scale is $\sim10^7$ cm, so in principle ER is possible throughout the flow. But the accelerated electrons can never produce a current that induces a magnetic field in excess of the inferred ambient field. This gives an upper limit (Holman 1985) to the size of field gradients that generate ER, namely

$$\delta l \le 8(B/10^4\ \mathrm{G})(n_e/10^{12}\ \mathrm{cm}^{-3})^{-1}(T_e/10^9\ \mathrm{K})^{-1/2}(E_D/E_{||})\ \mathrm{cm}.$$

For all relevant accretion discs, this scale in the cascade is always way below that at which magnetic mirroring, employed in the next section, could have already drained most of the energy in the cascade. Nevertheless, if mirroring is not important, or if a significant component of the turbulence cascades to $\perp$-incompressible scales before draining, then the cascade may proceed down to this scale where ER or other (e.g. Quataert & Gruzinov 1998) electron energization processes may be important.

However, the more extreme ER of Bisnovatyi-Kogan & Lovelace (1997) is not likely. They employ the mean electric field, obtained by coarsely averaging $\boldsymbol{E} = \langle\boldsymbol{E}\rangle + \boldsymbol{E}_T$ over the turbulent scales l_T. The dominant terms in this mean Ohm's law are then

$$\langle\boldsymbol{E}\rangle \simeq -\langle\boldsymbol{V}_T\times\boldsymbol{B}_T\rangle - \langle\boldsymbol{V}\rangle\times\langle\boldsymbol{B}\rangle,$$

where the turbulent electromotive force (EMF) (Parker 1979) is, in kinematic theory,

$$\langle\boldsymbol{V}_T\times\boldsymbol{B}_T\rangle = (\alpha_d/c)\langle\boldsymbol{B}\rangle - \beta_d\nabla\times\langle\boldsymbol{B}\rangle,$$

where α_d is a pseudo-scalar helicity, and β_d is the turbulent diffusivity. A representive magnitude of $\langle E\rangle_{||}$ using $|\alpha_d| \sim V_T/3$ is $\langle E_{||}\rangle \sim V_T\langle B\rangle/3c$. Assuming that $\langle B\rangle \sim B$, then

$$\langle E_{||}\rangle \sim 3\times10^3(V_T/10^{10}\ \mathrm{cm/s}^{-1})(B/10^4\ \mathrm{G})\ \mathrm{StV\,cm}^{-1}.$$

Thus $\langle E\rangle \gg E_D$ and one might be tempted to conclude that runaway electron acceleration is extreme. But particles do not actually see $\langle\boldsymbol{E}\rangle$, since the average is only defined on scales larger than l_T. Thus extreme ER should not occur.

4 ENERGIZATION BY MAGNETIC MIRRORING

In this section I show that $\perp$-compressible modes of dissipation (i.e. non-zero $\nabla_\perp\cdot\boldsymbol{v}_\perp$, as defined in section 1) modelled by magnetic mirroring, can favour protons to the extent required by ADAFs when the thermal speeds of electrons and protons are comparable as in ADAFs/HIT.

4.1 Basic physical picture

Fermi energization, or magnetic mirroring (Fermi 1949; Spitzer 1962), of particles off magnetic compressions (compressions perpendicular to the ambient field in the local region) provides a means of dissipation of turbulent energy into particles. Consider a field, B_T, that represents the field on the largest turbulent scale, superimposed on which is a smaller-scale magnetic compression. The total field in the compression is $B = B_T + \delta B$. The compressions travel along field lines at speeds $\sim V_A$, and can transfer energy to particles. Consider what happens as a particle travelling along B_T interacts with a compression $B_T + \delta B$. Since the magnetic force is perpendicular to the particle velocity, as long as the magnetic gradient scale is much greater than the particle gyro-radius (adiabatic approximation), the angular momentum and energy of the particle are conserved in the frame of a magnetic compression at rest. Denoting quantities in this frame by a prime and working in the non-relativistic limit, the energy and angular momentum magnitudes are given by $u' = mv'^2/2$ and $j' = mv'_\perp r_g = m^2cv'^2_\perp/(eB)$, where m is the particle mass, v' and $v'_\perp$ are the total speed and speed perpendicular to the field, and r_g is the gyro-radius. The constancy of both u' and j' implies that $v'^2_\perp/B = v'^2\sin^2\phi'/B$ is also constant. Thus, because v' is constant, $\sin^2\phi' \propto B = B_T + \delta B$, or

$$\sin^2\phi' = \sin^2\phi'_T\ (B_T + \delta B)/B_T. \tag{5}$$

When $\sin\phi' = 1$, the particle reflects. Thus there exists a minimum pitch angle the particle must have with respect to the ambient B_T such that it can reflect upon entering the compression. This is given by

$$\sin^2\phi'_{T,\min} \equiv \sin^2\phi'_{\min} = B_T/(B_T + \delta B) \tag{6}$$

(see also Fig. 1). In the laboratory frame, the moving compression then boosts the velocity component of a given particle parallel to B_T. For energy to be gained from repeated reflections, the boost must be rapidly isotropized by particle-generated waves (Eilek & Hughes 1991; Larosa et al. 1996) as discussed further in Section 5. Assuming isotropy in the laboratory frame, in the frame of a moving magnetic fluctuation the velocity distribution is centred around $\sim V_A$. The minimum angle for mirroring by a magnetic compression then gives a minimum speed that particles need to reflect:

$$v_{\min} = V_A\sin\phi'_{\min} = V_A B_T^{1/2}/(B_T + \delta B)^{1/2}\ \sim V_A \quad \text{for } \delta B/B_T < 1, \tag{7}$$

as seen in Fig. 1.

4.2 Time-scale for energy doubling

Different regions will have B_T aligned in different directions, but consider B_T in one region of size l_T in which B_T is assumed constant. Following Larosa et al. (1996), define $\tau_r \equiv U_r(\mathrm{d}U_r/\mathrm{d}t)^{-1} = N\delta t$ as the time-scale for the average *reflected* particle energy U_r to double, where N is the number of required reflections and δt is the time between reflections. Since only a fraction F are reflected, the energization time averaged for *all* particles is then

$$\tau = U/(\mathrm{d}U/\mathrm{d}t) = (\delta t/F)(NU/U_r), \tag{8}$$

where U is the average particle energy averaged over *all* particles. We need to estimate $N, \delta t, F$ and U/U_r.

To understand the role of F, two limits *not* usually distinguished *must* be considered separately: For particles with speeds $v_{\min} < v < V_A$, all reflections are 'head-on' since the particles can never catch up to the fluctuations, which move at speeds along the

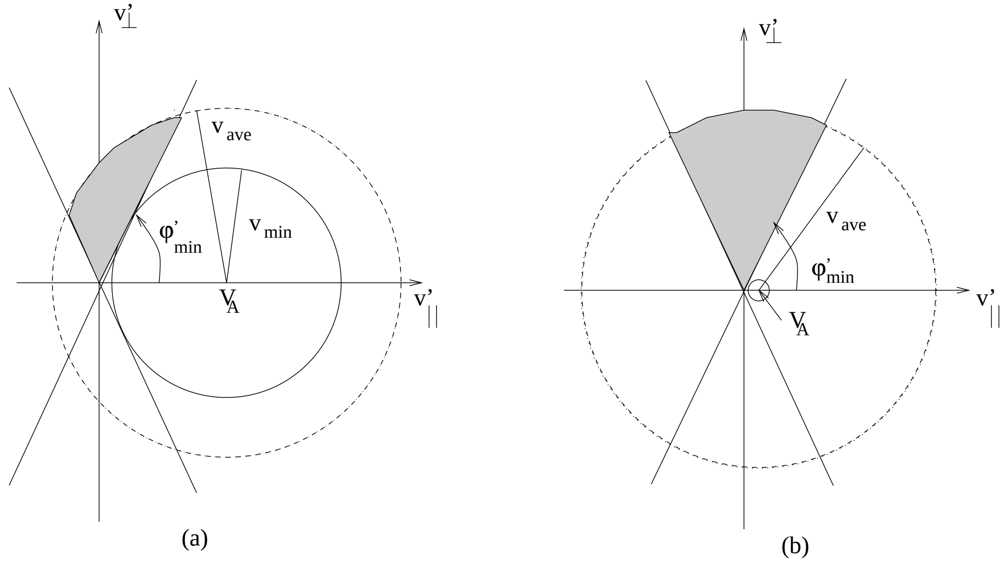

Figure 1. Particle speed diagram for magnetic mirroring. The magnetic compression is assumed to move at velocity $\sim -V_{\mathrm{A}}$ along the field line and so an isotropically distributed population of velocities in the laboratory frame has a spherical distribution centred around $v'_{||}=V_{\mathrm{A}}$. The angle $\phi'_{\min}$ and the speed $v_{\min}$ bound the respective minima needed for a particle to reflect at the magnetic compression. The weaker the compression, the larger these minima. The area inside the shaded region between the two circles represents the particle speed region that can be reflected. Approximate schematics of the regimes (a) L2 ($v_{\mathrm{ave}}\sim V_{\mathrm{A}}$) and (b) L1 ($v_{\mathrm{ave}}\gg V_{\mathrm{A}}$) of the text are shown. L2 is relevant for $\beta_{\mathrm{a}}\sim 1$ ADAFs. Note that only the region L1 strictly corresponds to stochastic energization since in this case the number of 'catch-up' and head-on reflections are about equal, whereas region L2 has primarily 'head-on' (albeit fewer total) reflections.

field lines $\sim V_{\mathrm{A}}$ (see Fig. 1). For $v\gg V_{\mathrm{A}}$, there are both 'catch-up' and head-on reflections. Let L1 and L2 label separate regimes where the average particle speed, v_{ave}, satisfies for L1: $v_{\mathrm{ave}}\gg V_{\mathrm{A}}\sim v_{\min}$ and for L2: $v_{\mathrm{ave}}\sim v_{\min}\sim V_{\mathrm{A}}$. That $v_{\min}\sim V_{\mathrm{A}}$ follows from the assumption that $\delta B<B_{\mathrm{T}}$ in (7). L1 is appropriate for electrons in a thermal equilibrium system whose magnetic pressure is not dominant. L2 is appropriate for protons in a thermal equilibrium system, or for both protons and electrons in plasma of $\beta_{\mathrm{p}}\sim 1$ with the ratio of proton to electron temperature $T_{\mathrm{p}}/T_{\mathrm{e}}\gtrsim 1000$, like ADAFs or HIT. Define the corresponding energization times τ_{L1} and τ_{L2} for the two limits. Limit L1 is the standard 'stochastic Fermi acceleration' limit, and the energization time for L1, derived below simply, agrees with other treatments (e.g. Miller 1991; Melrose 1986). The limit L2 produces a different formula.

To proceed further, I compute velocity moments of reflected particles in the two cases L1 and L2 for thermal and non-thermal distributions. This is necessary for computing F. For a power-law distribution,

$$\mathrm{d}g_{\mathrm{nt}}=(\lambda-1)(v/v_0)^{-\lambda}\,\mathrm{d}(v/v_0),$$

where $\mathrm{d}g_{\mathrm{nt}}/\mathrm{d}(v/v_0)$ is the distribution function, v_0 is the lower cut-off on the power law, and $\lambda>3$ will be assumed (to avoid the appearance of logarithms). Integrating over the reflected particles gives

$$\int_{v_{\min}}^{\infty}\mathrm{d}g_{\mathrm{nt}}=(v_{\min}/v_0)^{1-\lambda}, \tag{9}$$

if $v_0<v_{\min}$. If $v_0\geq v_{\min}$, then v_0 replaces $v_{\min}$ as the lower integral bound. The average velocity of reflected particles is then

$$v_{\mathrm{r,ave}}|_{\mathrm{nt}}=(\lambda-1)v_0(v_{\min}/v_0)^{2-\lambda}/(\lambda-2),$$

when $v_0<v_{\min}$. In the thermal case,

$$\mathrm{d}g_{\mathrm{th}}=(4/\pi^{1/2})(v^2/v_{\mathrm{ave}}^2)\exp(-v^2/v_{\mathrm{ave}}^2)\mathrm{d}(v/v_{\mathrm{ave}}),$$

so that

$$\int_{v_{\min}}^{\infty}\mathrm{d}g_{\mathrm{th}}=1-\operatorname{erf}(v_{\min}/v_{\mathrm{ave}})+(v_{\min}/v_{\mathrm{ave}})\exp(-v_{\min}^2/v_{\mathrm{ave}}^2). \tag{10}$$

I now use equations (9) and (10) to determine F, the fraction of particles that reflect. Generally,

$$F=\int_{v_{\min}}^{\infty}f\mathrm{d}g, \tag{11}$$

where f is the fraction of particles that reflect at a given speed. The f is the 'area' of the sphere (see Fig. 1) corresponding to particles that can be reflected, divided by the total area, $4\pi v^2$:

$$f=2\pi\int_{v_{||-}}^{v_{||+}}v_{\perp}[1+(\mathrm{d}v_{\perp}/\mathrm{d}v_{||})^2]^{1/2}\,\mathrm{d}v_{||}/4\pi v^2=2\pi v[v_{||}]_{v_{||-}}^{v_{||+}}/4\pi v^2, \tag{12}$$

where $||$ ($\perp$) indicates parallel (perpendicular) to B_{T}, and the second equality comes from using the equation for the circle centred at V_{A} for the particle speed, $v_{\perp}^2=v^2-(v_{||}-V_{\mathrm{A}})^2$. The bounds $v_{||\pm}$ are determined by finding the abscissa values at which the line defining $\phi_{\min}$ intersects (Fig. 1) the circle defined by v_{ave}. Setting the equation for the lines, $v_{\perp}^2=v_{||}^2\tan^2\phi_{\min}$, equal to that of the circle gives

$$v_{||\pm}=V_{\mathrm{A}}\cos\phi'_{\min}\pm\cos\phi'_{\min}(v_{\mathrm{ave}}^2-V_{\mathrm{A}}\sin^2\phi'_{\min})^{1/2}. \tag{13}$$

For L1 (i.e. $v_{\mathrm{ave}}\gg V_{\mathrm{A}}$), using (13) in (12) gives

$$f\sim\cos\phi'_{\min}\sim(\delta B/B_{\mathrm{T}})^{1/2}\sim 2(2\pi v^2\cos\phi'_{\min})/4\pi v^2=\cos\phi'_{\min}.$$

This can be pulled out of the integral in (11). Then because $v_{\min}/v_{\mathrm{ave}}\ll 1$ in this limit,

$$F_{\mathrm{L1}}=f=\cos\phi'_{\min}, \tag{14}$$

for both the non-thermal and thermal cases.

For L2 (i.e. $v_{\mathrm{ave}}\sim v_{\min}$), equations (12) and (13) give $f\lesssim\cos\phi'_{\min}$. This can be pulled out of the integral in (11). For the non-thermal case, using (9) for the remaining integrand, I obtain

$$F_{L2} \lesssim \cos\phi'_{\min}\,[(v_{\min}/v_{\mathrm{ave}})(\lambda-1)/(\lambda-2)]^{1-\lambda}. \tag{15}$$

For the L2 thermal case, I use (10) instead of (9), noting that the first two terms in (10) approximately cancel, giving

$$F_{L2} \sim \cos\phi'_{\min}\,(v_{\min}/v_{\mathrm{ave}})\exp(-v_{\min}^2/v_{\mathrm{ave}}^2). \tag{16}$$

Now consider $\delta t = \delta l/\langle|v_{||}|\rangle$, the time between reflections, where δl is the length-scale of the fluctuation δB and $\langle|v_{||}|\rangle$ is the average magnitude of the velocity of the reflected particle parallel to B_{T}. For L1,

$$\langle|v_{||}|\rangle = v_{||+}/2 = (v_{\mathrm{ave}}/2)\cos\phi'_{\min} \sim (v_{\mathrm{ave}}/2)(\delta B/B_{\mathrm{T}})^{1/2},$$

where the latter similarity follows for $\delta B/B_{\mathrm{T}} < 1$ in (7). Thus

$$\delta t_{L1} = (2\delta l/v_{\mathrm{ave}})(B_{\mathrm{T}}/\delta B)^{1/2}. \tag{17}$$

For L2

$$\langle|v_{||}|\rangle \sim v_{||+}/2 \sim (V_{\mathrm{A}}/2)\cos^2\phi_{\min} \sim (V_{\mathrm{A}}/2)(\delta B/B_{\mathrm{T}}),$$

so that

$$\delta t_{L2} = (2\delta l/V_{\mathrm{A}})(B_{\mathrm{T}}/\delta B). \tag{18}$$

Consider now $N(U/U_{\mathrm{r}})$ appearing in (8). For L1, $U/U_{\mathrm{r}} \sim 1$, as the energy of reflected particles is of the order of the average energy of all particles, but we must determine N. For L1, the energy gain is stochastic (Fermi 1949; Spitzer 1962; Eilek & Hughes 1991; Larosa et al 1996) as the particles incur random walks through momentum space and

$$N_{L1} = U^2/\langle\delta U_+\rangle^2, \tag{19}$$

where $\langle\delta U_+\rangle$ is the average energy gain by a particle from a head-on reflection. For L2, there are mainly head-on reflections, so that

$$U_{\mathrm{r}}/\langle\delta U_+\rangle \lesssim N_{L2} \lesssim U_{\mathrm{r}}^2/\langle\delta U_+\rangle^2.$$

Since a lower bound on τ_{r} suffices, I employ $N_{L2} \gtrsim U_{\mathrm{r}}/\langle\delta U_+\rangle$. This means that for L2,

$$[NU/U_{\mathrm{r}}]_{L2} \gtrsim U/\langle\delta U_+\rangle. \tag{20}$$

I now need $\langle\delta U_+\rangle$ for both L1 and L2. The $\langle\delta U_+\rangle$ is determined by energy and momentum conservation before and after a mirroring. This gives

$$\langle\delta U_+\rangle \sim 2mV_{\mathrm{A}}\langle|v_{||}|\rangle.$$

For L1, using the value of $\langle|v_{||}|\rangle$ calculated above then gives

$$\langle\delta U_+\rangle \sim mV_{\mathrm{A}}v_{\mathrm{ave}}(\delta B/B)^{1/2},$$

and thus

$$[NU/U_{\mathrm{r}}]_{L1} = N_{L1} = (v_{\mathrm{ave}}^2/4V_{\mathrm{A}}^2)(B_{\mathrm{T}}/\delta B), \tag{21}$$

while for L2, using the appropriate $\langle|v_{||}|\rangle$ calculated above (18) gives

$$[NU/U_{\mathrm{r}}]_{L2} \gtrsim U/\langle\delta U_+\rangle = (v_{\mathrm{ave}}^2/2V_{\mathrm{A}}^2)(B_{\mathrm{T}}/\delta B). \tag{22}$$

Collecting the calculations of $N(U/U_{\mathrm{r}})$, δt, and F for L1 in (8) gives

$$\tau_{L1} = (\delta t_{L1}/F_{L1})[NU/U_{\mathrm{r}}]_{L1} = (\delta l/4V_{\mathrm{A}})(v_{\mathrm{ave}}/V_{\mathrm{A}})(B_{\mathrm{T}}/\delta B)^2$$
$$= (l_{\mathrm{T}}/4V_{\mathrm{A}})(v_{\mathrm{ave}}/V_{\mathrm{A}})(\delta l/l_{\mathrm{T}})^{1/2}, \tag{23}$$

where the last equality follows from assuming a Kraichnan (1965) spectrum $(\delta B/B) = (\delta l/l_{\mathrm{T}})^{1/4}$ relating the magnetic to scale fluctuations. Equation (23) describes 'stochastic Fermi' energization (Miller 1991; Melrose 1986). Similarly for L2, using the appropriate above results for $N(U/U_{\mathrm{r}})$, δt and F in (8), I obtain

$$\tau_{L2} = (\delta t_{L2}/F_{L2})[NU/U_{\mathrm{r}}]_{L2} \gtrsim (\delta l/V_{\mathrm{A}})(v_{\mathrm{ave}}/V_{\mathrm{A}})^3(B_{\mathrm{T}}/\delta B)^{3/2}$$
$$\times\exp(V_{\mathrm{A}}^2/v_{\mathrm{ave}}^2)$$
$$= (l_{\mathrm{T}}/V_{\mathrm{A}})(v_{\mathrm{ave}}/V_{\mathrm{A}})^3(\delta l/l_{\mathrm{T}})^{5/8}\exp(V_{\mathrm{A}}^2/v_{\mathrm{ave}}^2), \tag{24}$$

for the thermal case, while for the non-thermal case with $v_{\min} > v_0$ and $\lambda > 3$

$$\tau_{L2} \gtrsim (l_{\mathrm{T}}/V_{\mathrm{A}})(v_{\mathrm{ave}}/V_{\mathrm{A}})^{3-\lambda}(\delta l/l_{\mathrm{T}})^{5/8}, \tag{25}$$

where the Kraichnan (1965) relation has again been used.

We see that each of the energy doubling times (23)–(25) depends only the particle average speeds v_{ave} and not on their particle mass. But if electrons and protons have the same v_{ave}, the protons have $(m_{\mathrm{p}}/m_{\mathrm{e}})$ more energy. Thus each of (23)–(25) shows that electrons take $(m_{\mathrm{p}}/m_{\mathrm{e}})$ longer to drain the same amount of energy. When electrons and protons do not have the same v_{ave} one population could be in L2 and the other in L1, and the comparison of energy doubling times becomes more subtle. This is because although in L1 there are many more reflections possible than in L2, L1 has both energy gaining and energy losing reflections [i.e. note the smaller shaded area and absence of symmetry in Fig. 1(a) compared to Fig. 1(b)]. Thus the energization is second order as expected for stochastic acceleration. For L2 however, while there are fewer reflections, they are mainly head-on (i.e. energy gaining). These two effects (fewer reflections but mainly energy gaining = L2 versus more reflections but both energy gaining and energy losing = L1) compete, and the β_{p} regime for which protons versus electrons dominate the drain then also depends on the particle distribution of that population in the L2 limit. Another complication comes if the populations have the same v_{ave} but different distribution functions. Then one must compare (24) and (25). We will study some of these cases more specifically in the next section.

5 APPLICATION TO ACCRETION FLOWS

5.1 Why protons are probably non-thermal in ADAFs

The discussion of Section 4 is one approach to the mirroring or Fermi energization process. Others include stochastic magnetic pumping (e.g. Hall & Sturrock 1967) and transit time pumping (e.g. Stix 1962; described as the magnetic analogue of Landau damping). Achterberg (1981) showed that all small-amplitude ($\delta B \ll B_{\mathrm{T}}$) approaches in L1 to mirroring in a turbulent plasma can also be described by quasi-linear diffusion of particles in momentum space, from magnetosonic wave–particle resonances at the Cherenkov resonance $(\omega_w - k_{||}v\cos\phi) = 0$. The relevant waves have frequencies $\omega_w \ll$ particle gyro-frequencies (i.e. very long wavelengths compared to the gyro-radii), which is equivalent to the adiabatic approximation discussed in Section 4.

This resonance requires a minimum particle speed $v_{\min} \sim V_{\mathrm{A}}$ and also a minimum $\sin\phi'$, as derived in Section 4. The required minimum in $\sin\phi'$ means that, in order for particles to undergo repeated reflections and gain energy, their momentum must be rapidly isotropized on a time-scale shorter than the time between reflections, which itself must be shorter than the largest eddy turnover time. The largest eddy turnover time is in turn shorter than the ADAF infall time, given by $t_{\mathrm{in}} \sim 1.8\times10^{-5}Mr^{3/2}/\alpha$, where

M is the central mass in units of $M_\odot$, and r is the radius in Schwarzschild units. But for ADAFs, Coulomb isotropization is not fast enough: the time-scale for momentum isotropization from Coloumb collisions is of the order of the time-scale for thermalization and is given by (e.g. Spitzer 1962; Mahadevan & Quataert 1997)

$$t_{\rm pp} = (2\pi)^{1/2}(n_{\rm p}\sigma_{\rm T}c\ln\Lambda)^{-1}(m_{\rm p}/m_{\rm e})^2(kT_{\rm p}/m_{\rm p}c^2) \sim 10^{-2}\alpha(\beta_{\rm a}/0.5)^{3/2}M\dot{M}, \tag{26}$$

where $n_{\rm p}$ is the proton number density, $\sigma_{\rm T}$ is the Thomson cross-section, $\ln\Lambda$ is the Coulomb logarithm, and $\dot{M}$ is the accretion rate is units of the Eddington value, $1.4\times10^{18}M$ g s^{-1}. Setting (26) equal to $t_{\rm in}$ shows that protons can only be Coulomb thermalized/isotropized well outside of the dominant energy emission location, i.e. for $r \gtrsim 100$ (Mahadevan & Quataert 1997). (In fact, this feature is fundamental to enabling an ADAF solution.) Thus the isotropization requires an additional kind of wave–particle resonance.

Unlike the mirroring waves, the required isotropizing waves have wavelengths of order the particle gyro-radius. Some or all of these small-wavelength (Whistler, Alfvén or magnetosonic) waves can be generated by the particles themselves and then they do not transfer energy to the particles. Some fraction may also be generated directly from the turbulence, in which case they *can* transfer energy to the particles. This latter possibility is explored in e.g. Quataert & Gruzinov (1998) as the primary means by which the turbulence dissipates into particles. The resonances occur when the wave frequency in the particle frame is an integer multiple of the particle gyro-frequency, that is $\omega - k_{||}v\cos\phi - N\Omega^* = 0$, where $\Omega^* \equiv eBc/E_{\rm p}$ and $E_{\rm p}$ is the total proton rest + kinetic energy. Quataert & Gruzinov (1998) show that Whistlers are not damped by protons. The short-wavelength Alfvén waves [$\omega = k_{||}V_{\rm A} \lesssim \Omega_{\rm g} \equiv eB/(m_{\rm i}c) < \Omega^*$, where $m_{\rm i}$ is the ion mass] are the most relevant for isotropization and have the approximate resonance condition $-eBc/E_{\rm p} - k_{||}v\cos\phi = 0$. The condition $|\cos\phi| < 1$ then leads to the injection condition $E_{\rm p} > (V_{\rm A}/v)m_{\rm i}c^2(\Omega_{\rm g}/\omega)$. For $\omega \sim \Omega_{\rm g}$, this leads to $v_{\rm min} \sim V_{\rm A}$ for protons, similar to the requirement of the mirroring waves derived earlier.

So both types of resonant waves – long-wavelength mirroring waves and short-wavelength Alfvén waves (whether they accelerate or just isotropize) – have a proton minimum speed requirement of order $V_{\rm A}$. The inefficiency of Coulomb thermalization, and the need for wave–particle resonances to dissipate the turbulence, means that a significant non-thermal particle population should be produced. Since ADAFs are most commonly modelled with $\beta_{\rm a} \sim 0.5$, non-thermal protons will likely be a fraction ~O(1) of the population. The inefficiency of Coulomb collisions in ensuring a non-thermal population is fundamental. Even when stochastic Fermi energization (the L1 limit) can be shown to lead rigorously to a power-law distribution in the energized particles (e.g. Eilek & Hughes 1991) efficient Coulomb scattering would thermalize the distribution. The fact that Coulomb collisions are inefficient, as shown above, precludes redistribution of energy over the full population of protons.

Note that at least the isotropizing waves are also *implicitly* built into ADAFs because the standard models presume isotropic pressure and this would be impossible without wave–particle resonances. In fact, the plasma must be 'collisional' in the sense of wave–particle interactions, even though it is 'collisionless' with respect to Coulomb collisions. In short, non-thermal protons should be a generic prediction of ADAFs. This is consistent with observations (Mahadevan 1998), which can distinguish between thermal and non-thermal proton distributions in an ADAF framework (and so far are not too sensitive to the proton power-law index.) In principle, similar arguments could be applied to electrons with more stringent resonance conditions. However Mahadevan & Quataert (1997) and Ghisselini, Haardt & Svensson (1998) argue that synchrotron self-absorption can thermalize weakly relativistic and non-relativistic electrons (at least those not produced from pion decay) under ADAF conditions during an infall time. Thus I assume (25) applies to ADAF/HIT protons in the steady state, and (24) to electrons.

5.2 Reflecting waves, scales of dissipation, and when mirroring preferentially energizes protons versus electrons

Slow or fast magnetosonic waves are the dominant mirrorers in the low-amplitude limit ($\delta B \ll B_{\rm T}$), since here the Alfvén waves are $\perp$-incompressible and magnetic compression is required for mirroring, though the relevant compression speed along the field lines is always $\sim V_{\rm A}$ regardless of the wave mode. Achterberg (1981) considered a magnetically dominated plasma at a single temperature, and focused only on electrons. Here we are interested in a two-temperature plasma and consider both electrons and ions. In general, both slow and fast waves may participate in the mirroring.

Though slow waves and fast magnetosonic waves dominate in the low-amplitude limit, this is not necessarily true in the large-amplitude limit ($\delta B \sim B_{\rm T}$). Because the largest scales of turbulence in discs are the most compressible, the large-amplitude limit is relevant when the scale on which the mirroring can compete with the cascade of energy from larger to smaller turbulent scales is a large fraction of the outer turbulent scale $l_{\rm T}$. In this case, the energy could be drained into particles before it reaches smaller scales in the cascade where $\perp$-incompressible modes of dissipation dominate. Since large-amplitude Alfvén waves are $\perp$-compressible (Alfvén & Falthammar 1963), even they could then contribute to the mirroring. Such Alfvén waves could even steepen to form shocks and perhaps shock-Fermi acceleration would be relevant. This must be considered in future work, as we will see that in fact the relevant mirroring scales can be large.

I now proceed to estimate the scales on which the favoured particles are energized and when protons versus electrons are favoured. For low-luminosity sources, ADAFs require $1 - f\eta \leq 0.01$, implying an accretion efficiency $\leq$1 per cent of that for thin discs (e.g. Rees et al. 1982; Narayan & Yi 1995). The respective energization times then need to satisfy

$$\tau_{\rm p}/\tau_{\rm e} \leq \zeta \equiv T_{\rm p}(1 - f\eta)/T_{\rm e} \lesssim 10, \tag{27}$$

where the subscript p (e) indicates ions (electrons). Since the turbulence cascades from large to small scales, I compare the scales of energy drain for protons, $(\delta l)_{\rm p}$, and electrons, $(\delta l)_{\rm e}$, for which (27) is satisfied. The larger of the two length-scales then determines the dominant drain. Using (25) for ADAF protons and setting it equal to the eddy turnover time $l_{\rm T}/V_{\rm A}$ gives

$$(\delta l)_{\rm p}/l_{\rm T} \sim (V_{\rm A}/v_{\rm p,ave})^{(24-8\lambda)/5}, \tag{28}$$

where $v_{\rm p,ave}$ is the average proton speed. For electrons, setting ζ times (24) equal to $l_{\rm T}/V_{\rm A}$ gives

$$(\delta l)_{\rm e}/l_{\rm T} \sim \zeta^{-8/5}(V_{\rm A}/v_{\rm e,ave})^{24/5}\exp(-8V_{\rm A}^2/5v_{\rm e,ave}^2). \tag{29}$$

Then we can see that

$$(\delta l)_{\rm p}/(\delta l)_{\rm e} = \zeta^{8/5}k^{24/5}\beta_{\rm p}^{4\lambda/5}\exp[8/(5k^2\beta_{\rm p})]. \tag{30}$$

This is greater than 1 for a range of parameters applicable to ADAFs (e.g. $\beta_p \sim k \sim O(1)$, $\zeta \sim 10$). The same conclusion results when the particle distributions are either *both* thermal or non-thermal. Protons can be favoured to the required extent when $\perp$-compressible modes dominate the turbulent dissipation.

Let us determine the scale on which the protons are dissipated. From (28) we see that when $V_A \sim v_{p,ave}$, it is not hard to have $(\delta l)_p/l_T \sim O(1)$ (recall $\lambda > 3$). This means that at least $\perp$-compressible modes may be very important and much of the energy in the turbulence may drain before approaching the $\perp$-incompressible scales where Quataert & Gruzinov (1998) is applicable. In general, the small-amplitude limit may not be fully appropriate in describing energy dissipation in ADAFs.

Now consider a thermal plasma with $T_p = T_e$, which corresponds to radii outside the ADAF region (Narayan & Yi 1995) or to a thin, precursor disc. In this case, no matter which particles initially receive the energy, Coulomb collisions redistribute this energy between electrons and protons. However, whether protons versus electrons receive the dissipated energy determines the heating rate. When $\beta_p \sim O(1)$, the relevant limits of interest are (23) for electrons and (24) for protons. Using $v_{e,ave} = (m_p/m_e)^{1/2} v_{p,ave}$, the dissipation scale ratio becomes

$$(\delta l)_p/(\delta l)_e \sim (m_p/16m_e)\beta_p^{-7/5} \exp[-8/(5\beta_p)]. \qquad (31)$$

This is less than 1 for $\beta_p \lesssim 0.25$ and greater than 1 for $\beta_p > 0.25$. For $\beta_p \gg 1$, both electrons and protons are in the limit of (23), for which $(\delta l)_p/(\delta l)_e \sim m_p/m_e$. For $\beta_p < m_e/m_p$, both electrons and protons are in the limit of (24) for which $(\delta l)_p/(\delta l)_e \sim (m_p/m_e)^{4/5}$. In sum, electrons are favoured only for the range $m_e/m_p \lesssim \beta_p \lesssim 0.25$, while for β_p outside this range protons are favoured. [The conditions of low β_p for which electrons are favoured may be found in solar flares (e.g. Larosa et al. 1996) and some thin accretion disc coronae models (Field & Rogers 1993).]

5.3 Can mirroring help form an ADAF?

It is sometimes believed that purely a low enough accretion rate is enough to form an ADAF/HIT. However, unless the disc is already thick, the critical accretion rate below which electrons and protons do not couple by Coulomb collisions on an infall time as computed for a standard thin disc is far too low to be physically relevant. For a thin disc system to evolve into an ADAF, a mechanism is needed to form a thick disc first. This may occur by thermal instability and mirroring may help. The condition (e.g. Pringle 1981) for thermal instability is

$$\mathrm{d}\ln(q_t)/\mathrm{d}T > \mathrm{d}\ln(q_e^-)/\mathrm{d}T. \qquad (32)$$

If the instability proceeds from within an optically thick disc, then we must compare blackbody emission to the heating. In the regime $\beta_p \lesssim 0.25$ for the thermal disc, electrons are favoured as shown above, and (23) is applicable. Taking the inverse of (23) for electrons, multiplying by $v_{ave}^2 \propto T$ and differentiating gives

$$\mathrm{d}\ln(q_t)/\mathrm{d}T = 1/(2T).$$

If the emission is blackbody, then

$$\mathrm{d}\ln(q_e^-)/\mathrm{d}T = 4/T, \qquad (33)$$

and the instability is not favoured. For $\beta_p \gtrsim 0.25$ protons are favoured and using (24) gives

$$\mathrm{d}\ln(q_t)/\mathrm{d}T = T^{-1}(1/\beta_p - 1/2)$$

and still, even for $0.2 < \beta_p \lesssim 0.5$, the thermal instability cannot ensue.

However, it is more likely that the formation of an ADAF would proceed by thermal instability within the very surface layer of the thin disc, and successive layers would eventually evaporate from the surface to form the thick ADAF disc. The particle distribution in the very surface layer could be non-thermal. To see how mirroring might help, in the limit where the protons dominate the energy drain ($\beta_p \gtrsim 0.2$), I invoke (25) for protons, take its reciprocal, and multiply by $v_{ave}^2 \propto T_p = T$ to obtain the quantity proportional to q_t. Then

$$\mathrm{d}\ln(q_t)/\mathrm{d}T = (\lambda - 2)/(2T).$$

For $\lambda > 8$ this can satisfy (32) when the emission is blackbody (33). For bremsstrahlung

$$\mathrm{d}\ln(q_e^-)/\mathrm{d}T = 3/(2T),$$

and (32) can be satisfied when $\lambda > 5$.

6 CONCLUSIONS

Unless the accretion rate decreases inwards, dissipation of turbulence in presumed ADAF sources must preferentially energize protons by a factor $\eta > 99$ per cent over electrons if ADAFs are to account for the observed low luminosities. Magnetic mirroring can in principle favour protons to the extent required for a ratio of particle to magnetic pressure $\beta_p > 0.25$. This is less stringent than the requirement of $\perp$-incompressible modes, which demands $\beta_p \gg 1$. The use of the word $\perp$-incompressible is employed because the required compressions for mirroring are perpendicular to the local magnetic field.

It is not easy to determine exactly what fraction of energy is dissipated by long wavelength in $\perp$-compressible by long wavelength modes. It is likely that the mirroring will be most effective the more compressible the plasma. Since the plasma is most compressible on the largest scales, one is motivated to estimate the the scale at which mirroring can compete with the transfer of energy down the turbulent cascade as a self-consistency check for the relevance of mirroring. This indicates that, for $\beta_p \sim 1$, the relevant scale can be quite a large fraction of the outer turbulent scale, which for ADAFs/HIT can be a large fraction of the disc size (Blackman 1998). Two implications result: (1) a significant fraction of the energy may be dissipated by long wavelength $\perp$-compressible modes and (2) the small-amplitude approach to dissipation may not be valid. In the small-amplitude limit, the relevant waves involved in the energy transfer to particles slow waves or fast magnetosonic waves, as Alfvén waves will not be damped by mirroring. However, on the larger scales in the turbulent cascade, the large-amplitude limit is relevant. Since large-amplitude Alfvén waves are $\perp$-compressible, they too may be involved in mirroring. This presents additional complications for future work. Magnetic reconnection may also be a complication; however, reconnection itself generates turbulence, and possibly shock or direct acceleration processes, which may also favour protons, but it is important to know on what scale the reconnection is occurring.

Regardless of the fraction of energy dissipated in $\perp$-compressible versus $\perp$-incompressible modes, the fact that ADAFs are 'Coulomb collisionless' on the radial infall time-scale seems to make a non-thermal proton population inevitable. The required dissipation of turbulence must proceed through wave–particle interactions, all of which act on only a subset of the particles. Since ADAF models presume an isotropic pressure tensor, wave–particle resonances are implicitly assumed to play a role in ADAFs because Coulomb isotropization is necessarily too slow. The presence of a non-thermal proton population seems indeed to be indicated by observations of the Galactic centre (Mahadevan 1998) when modeled with an ADAF. A remaining fundamental problem that still needs more attention is the question of a faster than Coulomb coupling between particles (Begelman & Chiueh 1988) even if the protons could receive the dissipated energy.

ACKNOWLEDGMENTS

Thanks are due to B. Chandran and M. Rees for stimulating discussions and insights and to the Institute of Astronomy, Cambridge where this work began.

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