Rapid Communications

Time-quasiperiodic topological superconductors with Majorana multiplexing

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Time-quasiperiodic Majoranas are generalizations of Floquet Majoranas in time-quasiperiodic superconducting systems. We show that in a system driven at d mutually irrational frequencies, there are up to 2^d types of such Majoranas, coexisting despite spatial overlap and lack of time-translational invariance. Although the quasienergy spectrum is dense in such systems, the time-quasiperiodic Majoranas can be stable and robust against resonances due to localization in periodic-drive-induced synthetic dimensions. This is demonstrated in a time-quasiperiodic Kitaev chain driven at two frequencies. We further relate the existence of multiple Majoranas in a time-quasiperiodic system to the time-quasicrystal phase introduced recently. These time-quasiperiodic Majoranas open a possibility for braiding which will be pursued in the future.

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Introduction. Majorana bound states, also referred to as Majoranas, are zero-energy excitations in topological superconductors invariant under particle-hole transformation [1–3]. Their zero-energy nature gives rise to degenerate ground states, which can be used as nonlocal qubits and memory [4–6]. Therefore, Majorana engineering in a variety of platforms has been a simmering field of study both theoretically [7–17] and experimentally [18–29].

Topological phases, however, also exist under nonequilibrium conditions and can be realized by time-periodic driving, known as Floquet engineering. Floquet topological superconductors and superfluids were proposed to be realized in either periodically driven cold atom systems [13] or proximitized nanowires [30,31]. Floquet topological phases have also been explored experimentally [32–36].

Interestingly, Floquet topological superconductors (or superfluids) host a dynamical version of Majoranas, dubbed Floquet Majoranas [13,37]. Rather than sitting at zero energy, Floquet Majoranas have quasienergies $\epsilon = 0$ or $\epsilon = \omega/2$, where ω is the driving frequency. Because energy is only defined modulo ω, $\omega/2$ is a particle-hole symmetric point in the spectrum just as $\epsilon = 0$ is, and the particle-hole symmetric nature of these Majoranas holds in a time-dependent fashion at all times. Indeed, Floquet Majoranas can form topological qubits and store quantum information, just as their equilibrium counterparts do [37]. Floquet Majoranas may therefore open a new route for topological quantum computation using the time domain as a resource [38].

A natural question arises: Could topological behavior also arise when a drive contains multiple frequencies, without any time-translational invariance? If so, could we obtain multiple Majorana modes associated with these frequencies? This would be similar to frequency multiplexing to enhance the hardware channel capacity in optical fibers [39]. For concreteness, let us consider a time-quasiperiodic superconductor driven at two frequencies ω_1 and ω_2, where ω_1/ω_2 is an irrational number, otherwise the system is time periodic. We assume the concept of quasienergy (as we will introduce it later) also exists in this context, which is defined up to $n_1\omega_1 + n_2\omega_2$ with $n_1, n_2 \in \mathbb{Z}$. Thus, there are four inequivalent particle-hole symmetric quasienergies: 0, $\omega_1/2$, $\omega_2/2$, and $(\omega_1 + \omega_2)/2$. This means one can at most have four types of Majoranas, as shown in Fig. 1. On the other hand, from a naive point of view, since $n_1\omega_1 + n_2\omega_2$ could be made to yield arbitrary energy increments, as long as $|n_1|, |n_2|$ are large enough, the quasienergy spectrum will be everywhere dense, with multiphoton energy arbitrarily small near resonances, and these Majoranas appear fully unstable.

In this Rapid Communication, we demonstrate that multifrequency-driven systems can give rise to a different class of time-quasiperiodic topological phases. Furthermore, such time-quasiperiodic topological superconductors give rise to Majorana edge states appearing simultaneously at several frequencies. These multiple Majoranas are stable and can coexist due to localization in the drive-induced synthetic n_1 and n_2 dimensions, which also suppresses the hybridization between the Majorana edge states, and bulk extended states. This renders the Majorana edge modes as stable spatially localized edge states. We confirm this by simulating a Kitaev chain driven at two incommensurate frequencies, and show the existence of Majorana edge states with half-frequency quasienergies. Furthermore, we use our simulations to demonstrate that time-quasiperiodic Majoranas are related to the "time-quasicrystal" phases introduced recently in time-quasiperiodic spin chains [40] (see also Refs. [41–45]); the half-frequency Majoranas are essentially the single-particle degrees of freedom characterizing the time-quasicrystal phase, in the same vein that the Floquet Majoranas are underlying the time-crystal period doubling of Refs. [46–49].

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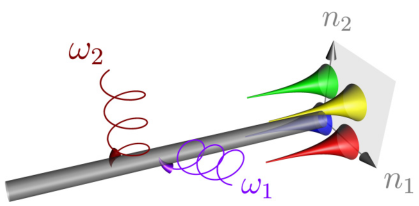

FIG. 1. Schematic representation of time-quasiperiodic Majoranas localized at the end of a 1D topological superconductor (in gray) driven at two frequencies ω_1 and ω_2. These Majoranas are localized in both real space and the two synthetic dimensions with coordinates n_1 and n_2.

Floquet recap. Let us start by briefly reviewing Floquet states. Consider a time-periodic Hamiltonian $H(t) = H(t + T)$, with driving angular frequency ω, and period $T = 2\pi/\omega$. The solutions to the time-dependent Schrödinger equation are characterized by the Floquet states, given by $|\Psi_\alpha(t)\rangle = e^{-i\epsilon_\alpha t}|\Phi_\alpha(t)\rangle$, where $|\Phi_\alpha(t)\rangle$ is a periodic function with the same period as the Hamiltonian, which satisfies the eigenvalue equation $[H(t) - i\partial_t]|\Phi_\alpha(t)\rangle = \epsilon_\alpha|\Phi_\alpha(t)\rangle$ with eigenvalue ϵ_α. Here, $K(t) = H(t) - i\partial_t$ and ϵ_α are called the quasienergy operator and quasienergy, respectively.

It is important to note that quasienergies are not uniquely defined. Indeed, ϵ_α and $\epsilon_{\alpha,n} = \epsilon_\alpha + n\omega$ with $n \in \mathbb{Z}$ actually describe the same physical state $|\Psi_\alpha(t)\rangle = e^{-i\epsilon_\alpha t}|\Phi_\alpha(t)\rangle = e^{-i\epsilon_{\alpha,n}t}|\Phi_{\alpha,n}(t)\rangle$, where $|\Phi_{\alpha,n}(t)\rangle = e^{in\omega t}|\Phi_\alpha(t)\rangle$ is also an eigenfunction of the quasienergy operator at quasienergy $\epsilon_{\alpha,n}$. Thus, the quasienergy ϵ_α is only uniquely defined modulo ω, e.g., in the range $-\omega/2 \leqslant \epsilon < \omega/2$.

Floquet synthetic dimensions and Wannier-Stark localization. Our construction of time-quasiperiodic Majoranas requires recasting the driven Hamiltonian in a time-independent way. Let us write out the Hamiltonian and Floquet states using their Fourier expansion of $H(t) = \sum_n e^{-im\omega t}h_n$ and $|\Phi_\alpha(t)\rangle = \sum_m e^{-im\omega t}\phi_m^\alpha$. The eigenvalue equation for the quasienergies then becomes

$$\sum_m h_{n-m}\phi_m^\alpha - n\omega\phi_n^\alpha = \epsilon_\alpha\phi_n^\alpha, \tag{1}$$

which describes particles hopping in a one-dimensional (1D) synthetic lattice, spanned by the coordinate n, with ω playing the role of a uniform force field. This is precisely the Hamiltonian for a Wannier-Stark ladder, with energy difference ω between neighboring rungs. We will restrict ourselves to nearest-neighbor-hopping models, i.e., $h_n = 0$ for $|n| \geqslant 2$.

It has been known that the electronic wave functions in the Wannier-Stark ladder are localized, with a localization length $\sim 1/\ln(\omega/V)$ when $V < \omega$, with V being the nearest-neighbor hopping amplitude, known as the Wannier-Stark localization [50,51]. Likewise we expect that the Floquet states will be localized to the vicinity of a particular n, which is a manifestation of energy conservation.

Floquet particle-hole symmetry in superconductors. The Hamiltonians of superconductors possess a unitary matrix U_P such that $U_P H(t)^* = -H(t)U_P$ for all times, with "$*$" denoting complex conjugation. This particle-hole symmetry dictates that $U_P K(t)^* U_P^\dagger = -K(t)$, and that the Floquet states appear in pairs as $|\Phi_\alpha(t)\rangle$ and $U_P|\Phi_\alpha(t)^*\rangle$, with quasienergies $\pm\epsilon_\alpha$, respectively.

Majoranas are special states that are particle-hole symmetric. Namely, with $|\psi(t)\rangle$ a Majorana state,

$$e^{-i\varepsilon t}|\phi(t)\rangle = |\psi(t)\rangle = U_P|\psi(t)^*\rangle = e^{i\varepsilon t}U_P|\phi(t)^*\rangle, \tag{2}$$

which works if $(U_P|\phi(t)^*\rangle) = e^{-ip\omega}|\phi(t)\rangle = e^{-2i\varepsilon t}|\phi(t)\rangle$ with some $p \in \mathbb{Z}$. Therefore, the Majorana quasienergies are restricted to $\varepsilon = p\omega/2$ with some $p \in \mathbb{Z}$. And because shifts by ω are just a gauge choice, there are only two inequivalent Floquet Majoranas [13,37], with $p \in \{0, 1\}$ reduced to a $\mathbb{Z}_2$ variable.

Floquet Majoranas. Next, consider a 1D Floquet topological superconductor, with Hamiltonian $H(t) = H_K + M(\omega t)$. The first term describes a static Kitaev chain,

$$H_K = -\mu\sum_{j=1}^{N}c_j^\dagger c_j - \sum_{j=1}^{N-1}[(Jc_j^\dagger c_{j+1} + i\Delta c_j c_{j+1}) + \text{H.c.}]. \tag{3}$$

with c_j ($c_j^\dagger$) annihilation (creation) operators at site j, μ is the chemical potential, J is the hopping amplitude, and Δ is the p-wave pairing potential. The second term,

$$M(\omega t) = -i\Delta'\sum_{j=1}^{N-1}(e^{-i\omega t}c_j c_{j+1} - e^{i\omega t}c_{j+1}^\dagger c_j^\dagger), \tag{4}$$

corresponds to a periodic drive. Introducing Nambu spinors in momentum (k) space $\Psi_k^\dagger = (c_k^\dagger, c_{-k})$, with $c_k = \sum_{j=1}^N c_j e^{-ikj}/\sqrt{N}$. For periodic boundary conditions, we get the Bogoliubov–de Gennes Hamiltonian,

$$H = \sum_{k>0}\Psi_k^\dagger[\mathcal{H}_K(k) + \mathcal{M}(k,\omega t)]\Psi(k),$$
$$\mathcal{H}_K(k) = \tau_z\xi_k + \tau_x\Delta\sin k, \quad \mathcal{M}(k,\omega t) = \tau_x\Delta'\sin k e^{i\omega t\tau_z}, \tag{5}$$

where $\tau_{x,y,z}$ are the Pauli matrices in Nambu space, and $\xi_k = -J\cos k - \mu/2$ is the normal state dispersion.

The spectrum of the driven Kitaev model can be interpreted using the synthetic dimension and Wannier-Stark ladder approach of Eq. (1). For each k there are two orbitals for each harmonic n. Thus, in the absence of a pairing potential, the system has two groups of equally spaced spectra $\epsilon_{n,e/h} = \pm\xi_k + n\omega$, with $n \in \mathbb{Z}$. The $+$ or $-$ signs indicate electronlike (e) or holelike (h) states. The static pairing potential Δ opens a topological gap at $n\omega$, when $\epsilon_{n,e} = \epsilon_{n,h}$, while the dynamical pairing Δ' opens a topological gap at $(n+1)\omega/2$ when $\epsilon_{n+1,h} = \epsilon_{n,e}$, i.e., at the edge of the "Floquet zone." In Fig. 2(a), we show the spectrum of the ladder as a function of k in a window between $-\omega$ and ω, with a set of parameters producing the two topological gaps. An open chain, then, supports two types of Floquet Majoranas at quasienergies $0, \omega/2$, with same-rung equal superposition of electron and hole states [Fig. 2(b)], and between neighboring rungs [see Fig. 2(c)], respectively.

Time-quasiperiodic Majoranas. Our main result is that Majoranas also emerge due to multifrequency drive. Consider a time-quasiperiodic Hamiltonian $H(t)$ characterized by d

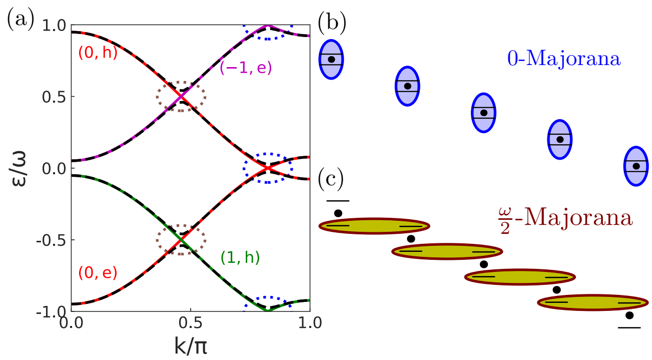

FIG. 2. (a) Quasienergy spectrum as a function of k between $-\omega$ and ω, for the model defined in Eqs. (3) and (4). The black dashed lines are obtained with $J/\omega = 0.51$, $\mu/\omega = 0.87$, $\Delta/\omega = 0.051$, and $\Delta'/\omega = 0.038$. The solid red, green, and magenta lines corresponding to the quasienergies $\epsilon_{n,e/h}$ when setting $\Delta = \Delta' = 0$, for a certain $(n, e/h)$, as indicated in the figure with the same color. The two types of topological gaps are indicated in the blue and brown dotted circles. (b) and (c) are the Wannier-Stark ladders with two orbitals (black lines) per rung (black dot), when $\epsilon_{n,e} \simeq \epsilon_{n,h}$ and $\epsilon_{n+1,h} \simeq \epsilon_{n,e}$ respectively. The 0-Majoranas are formed from equal superposition between states (n, e) and (n, h) (blue ellipses), while the $\omega/2$-Majoranas are formed from equal superposition between states $(n+1, h)$ and (n, e) (green ellipses).

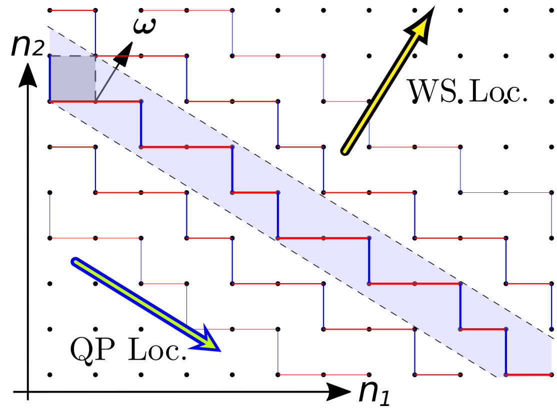

FIG. 3. 2D synthetic lattice with an electric field vector $\boldsymbol{\omega} = (\omega_1, \omega_2)$ consisting of the driven frequencies. The equipotential lines perpendicular to $\boldsymbol{\omega}$ are denoted as black dashed lines. One obtains a 1D quasicrystal in between the two dashed lines denoted as the blue region. The nearest-neighbor couplings within the quasicrystal are denoted as solid red or blue lines, corresponding to the original horizontal and vertical couplings. The two big arrows denote the directions along which there are localizations: Wannier-Stark (WS) vs quasiperiodic (QP).

mutually irrational frequencies $\boldsymbol{\omega} = (\omega_1, \dots, \omega_d)$. The Floquet ansatz introduced previously can be generalized to the time-quasiperiodic system [52]. The function $|\Phi_\alpha(t)\rangle$, which becomes time quasiperiodic at frequencies specified by $\boldsymbol{\omega}$, satisfies the eigenvalue equation of the time-quasiperiodic quasienergy operator $K(t)$,

$$K(t)\,|\Phi_\alpha(t)\rangle = \left(H(t) - i\frac{\partial}{\partial t}\right)|\Phi_\alpha(t)\rangle = \epsilon_\alpha\,|\Phi_\alpha(t)\rangle\,, \quad (6)$$

with the quasienergy ϵ_α defined modulo $\mathbf{n}\cdot\boldsymbol{\omega}$.

Time-quasiperiodic Majoranas then emerge as particle-hole symmetric states. These must have quasienergies $\epsilon = \mathbf{p}\cdot\boldsymbol{\omega}/2$, with $\mathbf{p} \in \mathbb{Z}^d$. Furthermore, they fall into 2^d groups, reducing $\mathbf{p} \in \{0, 1\}^d$, corresponding to 2^d types of Majoranas.

Contrary to a gapped Floquet topological phase, the quasienergy spectra in a time-quasiperiodic system are dense, since $\mathbf{n}\cdot\boldsymbol{\omega}$ can approach any value. It seems, therefore, that time-quasiperiodic Majoranas do not have a gap that could protect them from hybridizing with bulk states due to local perturbations. Below, we show that these Majoranas are stable not due to a gap, but rather due to localization in the drive-induced synthetic dimensions.

Multidrive synthetic lattice and localization. Similar to the Floquet case, the time-quasiperiodic system could be posed as a time-independent problem. The quasienergy eigenvalue equation becomes a tight-binding problem on a d-dimensional lattice whose coordinates are given by $\mathbf{n} \in \mathbb{Z}^d$ embedded in the d-dimensional Euclidean space $\mathbb{R}^d$. In addition, a force field given by ω pointing into the synthetic dimensions keeps track of the energy of energy quanta absorbed from the drive [53,54].

The equipotential surface perpendicular to the synthetic electric field defines a $(d-1)$-dimensional quasicrystal [55]. Figure 3 describes the quasicrystal construction for $d = 2$, which is easily generalized to more dimensions. The lattice sites in a narrow strip (contained in the blue region) normal to the frequency vector $\boldsymbol{\omega}$ make a one-dimensional (1D) quasicrystal where the on-site energy goes up and down by ω_2 and ω_1. By shifting the strip along $\boldsymbol{\omega}$, the whole two-dimensional (2D) lattice will be covered, and every lattice site will be uniquely contained in one 1D quasicrystal. Hence, the original system is equivalent to a Wannier-Stark ladder of 1D quasicrystals. Now it is clear, however, what can protect Majoranas from bulk hybridization. Motion in a quasicrystal is fully localized if the hopping strength is smaller than the quasiperiodic modulation of the on-site potential [56,57].

Therefore, Majoranas emerge from a combination of three localization mechanisms: (1) real space localization due to the superconducting gap; (2) Wannier-Stark localization along the synthetic "electric" field $\boldsymbol{\omega}$; and (3) quasiperiodicity-induced localization perpendicular to $\boldsymbol{\omega}$. We focus on the time-quasiperiodic Kitaev chain $H(t) = H_K + M(\omega_1 t) + M(\omega_2 t)$, following Eqs. (3) and (4), with $\frac{\omega_2}{\omega_1} = \frac{\sqrt{5}+1}{2}$. In the synthetic space, n_1, n_2, of harmonics of the ω_1, ω_2 drives, the system is localized along the $\boldsymbol{\omega}$ direction due to Wannier-Stark localization. The system is localized perpendicular to $\boldsymbol{\omega}$ due to quasiperiodic localization when $\Delta' < \omega_1, \omega_2$. On a ring, there are two orbitals per rung for each quasimomentum k. Ignoring the pairing potentials Δ, Δ', the eigenvalues of this system are $\epsilon_{n_1,n_2,e/h} = \pm\xi_k + n_1\omega_1 + n_2\omega_2$. By choosing proper parameters, one has three special quasimomenta at which $\epsilon_{n_1,n_2,e} = \epsilon_{n_1,n_2,h}$, $\epsilon_{n_1+1,n_2,h} = \epsilon_{n_1,n_2,e}$, and $\epsilon_{n_1,n_2+1,h} = \epsilon_{n_1,n_2,e}$. Δ and Δ', however, open topological gaps at these crossings. In an open chain, these gaps give rise to three kinds of Majoranas, with quasienergies 0, $\omega_1/2$, and $\omega_2/2$ [Fig. 4(a)]. The existence, stability, and localization of these Majoranas are verified via numerical simulation outlined in the Supplemental Material [52]. Figure 4(b) shows these wave

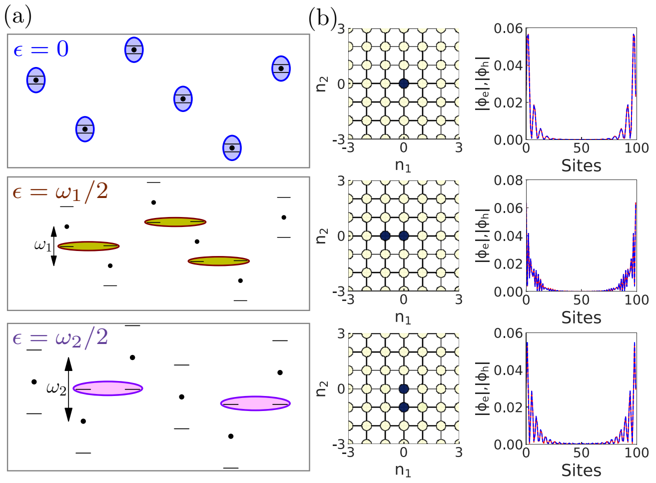

FIG. 4. (a) The quasiperiodic ladder perpendicular to $\boldsymbol{\omega}$ in the 2D synthetic lattice, with each rung corresponding to a Kitaev chain. For a periodic chain, when k is close to three special quasimomenta such that $\epsilon_{n_1,n_2,e} \simeq \epsilon_{n_1,n_2,h}$ (top), $\epsilon_{n_1+1,n_2,h} \simeq \epsilon_{n_1,n_2,e}$ (middle), and $\epsilon_{n_1,n_2+1,h} \simeq \epsilon_{n_1,n_2,e}$ (bottom), topological gaps are induced. The three types of topological gaps give rise to three types of Majoranas in an open chain. (b) Numerical solution of the zero-frequency and time-quasiperiodic Majorana states on the 2D synthetic lattice of size 15×15. Each site of the lattice corresponding to a Kitaev chain of length $N = 100$. Left: $|\phi_{n_1,n_2}|^2$ for the 0, $\frac{\omega_1}{2}$, and $\frac{\omega_2}{2}$ Majoranas on the 2D synthetic lattice, where the darker color corresponds to a larger magnitude. Right: The absolute value of the corresponding Majorana wave function, summed over the 2D synthetic lattice. The electron and hole components ϕ_e, ϕ_h are plotted as red solid and blue dashed curves. The other parameters are $\omega_2/\omega_1 = (\sqrt{5}+1)/2$, $J/\omega_1 = 0.51$, $\mu/\omega_1 = 0.87$, $\Delta/\omega_1 = 0.051$, and $\Delta'/\omega_1 = 0.038$.

functions $\phi_{n_1,n_2} = (\phi_{n_1,n_2,e}, \phi_{n_1,n_2,h})$ in the synthetic and real spaces. Indeed, the wave function, which is identical for the hole and electron components, is localized at a single, or two neighboring sites, in the synthetic directions, and near the edges in real space.

From Majorana multiplexing to time quasicrystal. The different types of Majoranas, give rise to a quasiperiodic oscillating pattern distinct from the driving pattern in the correlation function $\langle \hat{O}(t)\hat{O}(0)\rangle$ of a local observable $\hat{O}$, resembling the time quasicrystal of Ref. [40]. Take, for instance, $\hat{O}$ to be $\gamma_1 = (c_1 + c_1^\dagger)/\sqrt{2}$, with $c_1, c_1^\dagger$ the electron creation and annihilation operators at the first site. The correlation function is then closely related to the local spectral function, and is dominated by the boundary modes, namely, the Majorana operators,

$$\gamma_1(t) = c_0\psi_0(t) + c_1\psi_1(t) + c_2\psi_2(t) + \cdots, \qquad (7)$$

where $\psi_{0,1,2}$ are the time-quasiperiodic Majorana operators at quasienergies 0, $\omega_1/2$, and $\omega_2/2$. Hence, $\langle\gamma_1(t)\gamma_1(0)\rangle$ generically contains peaks at frequencies, 0, $\omega_1/2$, and $\omega_2/2$ (see Fig. 5), where the average is with respect to the BCS vacuum at $t = 0$. In fact, the spectral peaks at half frequencies persist even when we include temporal disorders or take commensurate frequencies (see the Supplemental Material [52] for details).

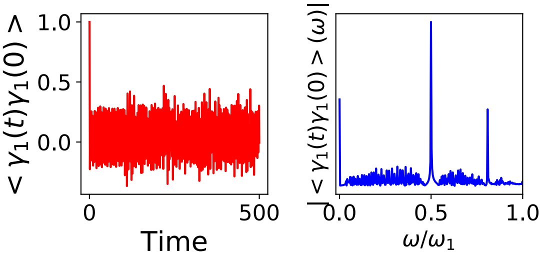

FIG. 5. Left: Time evolution of $\langle\gamma_1(t)\gamma_1(0)\rangle$ simulated on the time-quasiperiodic Kitaev chain, with the same parameters as in Fig. 4. Right: The Fourier transform of $\langle\gamma_1(t)\gamma_1(0)\rangle$ in the frequency domain. There are three dominant peaks at 0, $\omega_1/2$, and $\omega_2/2 \simeq 0.81\omega_1$.

If one applies a Jordan-Wigner transform of the time-quasiperiodic Kitaev chain, we get a time-quasiperiodic Heisenberg model. $\langle\gamma_1(t)\gamma_1(0)\rangle$ becomes the spin correlation function $\langle\sigma_1^x(t)\sigma_1(0)\rangle$. This shows that the time-quasiperiodic Majoranas in a fermionic system are indeed the single-particle degrees of freedom which are responsible for the formation of the time-quasicrystal correlations discussed in Ref. [40].

Conclusion. In this Rapid Communication, we establish the existence of time-quasiperiodic topological phases, and generalize the concept of Floquet Majoranas to time-quasiperiodic systems. We show that there are at most 2^d types of Majoranas at quasienergies $\mathbf{p}\cdot\boldsymbol{\omega}/2$, with $\mathbf{p} \in \{0,1\}^d$ with $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_d)$ consisting of d mutually irrational frequencies. Furthermore, we show that these Majorana states are stable, fully localized, edge states. We study the time-quasiperiodic Kitaev chain with $d = 2$, and find coexisting stable and robust Majoranas at quasienergies 0, $\omega_1/2$, and $\omega_2/2$. The localization in synthetic dimensions emerges as a resource that allows these localized Majorana edge modes despite a dense quasienergy spectrum. These Majoranas are also the single-particle degrees of freedom which are relevant to the formation of time quasicrystals [40].

The existence of time-quasiperiodic Majoranas opens a direction for performing and controlling topological quantum computations using the time domain as a resource for topological anscilla qubits, for instance. Instead of using multiple static topological superconducting wires, one can dynamically generate multiple Majoranas at different locations for manipulation, by driving a single superconductor at different frequencies in different regions. While this raises issues of equilibration and heating, protocols for finite time manipulation may keep such problems at bay, even if these may be experimentally challenging at present.

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