Definition of Chern-Simons Terms in Thermal QED_3 Revisited

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We present two compact derivations of the correct definition of the Chern-Simons term in the topologically non trivial context of thermal QED_3. One is based on a transgression descent from a $D = 4$ background connection, the other on embedding the abelian model in $SU(2)$. The results agree with earlier cohomology conclusions and can be also used to justify a recent simple heuristic approach. The correction to the naive Chern-Simons term, and its behavior under large gauge transformations are displayed.

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Chern-Simons (CS) terms, defined in odd dimension, contain gauge information not accessible through the field strength alone. We have in mind large gauge transformations, ones not shrinkable to the identity, to which CS (but not $F_{\mu\nu}$) is sensitive. However it is also known [1–4] that, as normally expressed, the abelian CS term (in $D = 3$, say)

$$I_{CS} = \frac{1}{8\pi^2} \int A \wedge F \tag{1}$$

is not always well-defined, but requires corrections [1]. Our twofold aim is to obtain the correct form, in two complementary, compact, ways and to show explicitly that "improvement" of I_{CS} is already needed in simple but quite physical contexts, such as abelian $U(1)$ gauge fields in $D = 3$ with non-trivial topology. A generic example is finite temperature QED_3, where t ranges over a finite circle S^1 of perimeter $\beta = 1/\kappa T$ and space is a closed $2-$manifold Σ^2 with associated non-vanishing magnetic flux $\Phi = \int dx^2 B$, $B \equiv \frac{1}{2}\epsilon^{ij} F_{ij} = F_{12}$. [We recall that the flux is a necessarily quantized topological invariant, $\Phi = 2\pi k$; see *e.g.* [6].] The need to improve the naive I_{CS} is due to the fact that it explicitly involves the vector potential $\mathbf{A}$, "modulated" by the field strength. But presence of magnetic flux implies that $\mathbf{A}$ will depend on the patches needed to cover the closed manifold Σ^2; hence the integral in (1) as it stands will be, unacceptably, patch-dependent. This difficulty has been recognized and cured long ago both in cohomological $D = 3$ calculations [1,4] and by descent from $D = 4$ [2]; recently we have given a heuristic approach to the solution [3]. Improvement of I_{CS} is not a merely a mathematical nicety, but has direct bearing on real QED_3 questions such as the necessity and amount of quantization of its coefficient when I_{CS} is viewed as a dynamical field action. Our present interest in I_{CS} was aroused by calculations of effective QED actions induced by charged fermions, and the complex of questions raised there about the seeming appearance of induced CS terms and their coefficients [3]. Here we will first present a different route to the (same) correct definition of I_{CS}, based on the Chern-Weil theorem using the transgression formula involving a background connection $\hat{A}_\mu$ on the non-trivial bundle, that compactly

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[1] The non-abelian case differs in a number of respects; it will addressed separately [5].

replaces the patch-dependence and the associated boundary "counter-terms", by a simpler $\hat{A}_\mu$−dependent addition. We will then compare this method with the two earlier approaches and use it also to justify the simple-minded "derivation". Finally, we give what is perhaps a still easier definition by use of nonabelian embedding to take advantage of the simpler (!) cohomological properties avaible there.

We begin our analysis from the usual 4−dimensional identity that leads to the introduction of the abelian CS form,

$$F \wedge F \equiv d(A \wedge F) \ . \tag{2}$$

One cannot apply the Poincaré lemma to this identity when A_μ is nontrivial, as when it carries a nontrivial magnetic flux through Σ^2. For then $F \wedge F$, while closed, is not exact; equivalently A_μ is globally defined on $\mathcal{M}_4$ as a connection, but not as a 1−form. We circumvent this obstacle through the Chern–Weil theorem (see *e.g.*, [6]) which states (for our case) that, if (F, $\hat{F}$) are field strengths corresponding to two different connections (A, $\hat{A}$) on some bundle, then $(F \wedge F - \hat{F} \wedge \hat{F})$ is exact as well. A corollary, the transgression formula, provides the explicit 3−form whose divergence it is:

$$F \wedge F - \hat{F} \wedge \hat{F} = d\left[(A - \hat{A}) \wedge (F + \hat{F})\right] , \tag{3}$$

as is easily verified since the cross-terms on the r.h.s. cancel. We can therefore define I_{CS}, also on non-trivial bundles, to be[2]

$$\bar{I}_{CS} \equiv \frac{1}{8\pi^2} \int_{\mathcal{M}_4} \hat{F} \wedge \hat{F} + \frac{1}{8\pi^2} \int_{\partial\mathcal{M}_4} (A - \hat{A}) \wedge (F + \hat{F}) \ . \tag{4}$$

The explicit dependence of $\bar{I}_{CS}$ on $A - \hat{A}$ insures that it is globally well defined: recall that a bundle is defined by the gauge transition functions between patches, all connections on the bundle having the same patch behavior. In particular, A and $\hat{A}$ carry the same flux through Σ^2; see also discussion after (6).

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[2] In general a $D = 3$ bundle is not a boundary of a 4−bundle but cochains may be required [2]. This complication does not occur in our explicit examples, but can be handled as well.

We digress for a moment to note that appearance of an intrinsic reference background is common in connection with (gauge or gravitational) anomalies in non trivial topologies. What makes $\hat{A}$ unusual is that while it transforms as a connection when changing patches – so as to neutralize the same behavior in A – we may (and do) choose it not to transform under gauge tranformations that affect A only; this too is not unknown, for example in background field expansion of QFT. These different roles for A and $\hat{A}$ can be justified in terms of the usual BRST analysis (see *e.g.*, [7]).

Returning now to $\bar{I}_{CS}$, the elegant aspect of (4) is its "covariance" (no patch dependence), paid for by its apparent dependence on the $D = 4$ background $\hat{A}$. In the other approaches there is no $\hat{A}$, but covariance is lost. One gratifying property of (4) is that it immediately reproduces the correct gauge variation of $\bar{I}_{CS}$ at finite temperature. Under the large gauge transformation $A_0 \to A_0 + 2\pi n/\beta$, $\mathbf{A} \to \mathbf{A}$, $\bar{I}_{CS}$ changes proportionally to the flux,

$$\bar{I}_{CS} \to \bar{I}_{CS} + \frac{1}{8\pi^2} 2\pi n[\Phi(B) + \Phi(\hat{B})] = \bar{I}_{CS} + \frac{1}{8\pi^2} 2\pi n(2\pi k + 2\pi k) = \bar{I}_{CS} + nk. \tag{5}$$

The variation is double what would be naively expected from (1), where the background contribution $\phi(\hat{B})$ is absent (see also [1]). A related physical issue involves the requirement that the coefficient μ in μI_{CS}, viewed as a quantum action, be quantized. The usual argument (using μI_{CS}) is that its phase exponential (the relevant quantum path integral object) must also be (large-)invariant, so that μI_{CS} must vary by $2\pi m,\ m \in \mathbb{Z}$, requiring $\mu/2\pi$ to be even. Instead, (4,5) imply that the parameter $\mu/2\pi$ is *any* integer [1]. This choice leads to a manifestly invariant complete set of states with all possible (of course integer) fluxes.[3]

Let us next compare this definition of $\bar{I}_{CS}$ with the direct way of computing the integral $\int F \wedge F$. Here we must specify the embedding space; for simplicity, we take it to be $\mathcal{M}_4 =$

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[3]It has also been argued that consistency is preserved with a less stringent (but still based on (5)) quantization: specifically, if $\mu/2\pi$ is merely rational, (only) states with the corresponding flux values are allowed [8,9]; states with vanishing flux are compatible with any value of μ [8]. Here, the Hilbert spaces are required to carry projective representations of the large gauge group.

$D^2 \times S^2$. [The apparent ambiguity in choice of embedding as well as of connection A, but keeping the desired boundary ∂M_4 and the desired values of A on it, does not affect $\bar{I}_{CS}$, the differences being at most integer-valued. For example, different embeddings differ by the Chern class of the manifold obtained by gluing them together [2].] The angles $(\theta,\ \phi)$ span the 2−sphere S^2 while (r, t) are the polar (radial and angular) coordinates parameterizing the disc D^2. Our desired 3−space is the boundary $S^1 \times S^2$. A nontrivial gauge connection A on this manifold is then realized by requiring its (integer) flux $\int_{S^2} F$ through S^2 to be nonvanishing, entailing nontrivial transition functions between the different charts covering the sphere. At the simplest level, we use two charts, splitting S^2 into two cups $H_\pm$ intersecting at some latitude $\theta = \theta_0$ and assign $U(1)$ connection 1-forms to each,

$$A = A_\pm + d\psi_\pm \ \text{ on } \ H_\pm. \tag{6}$$

The transition function corresponding to nontrivial flux corresponds to $\exp(i\psi_+) = \exp(ik\phi)\exp(\psi_-)$, which implies $A_+ - A_- = kd\phi$, (*i.e.* $\Phi = 2\pi k$.). Regularity also requires all fields to be periodic in the angular variable t, with period β. We are ready now to perform the integration, for which we revert to index notation.

$$\begin{aligned}
&4\int_{D^2\times S^2} F\wedge F = \int_{D^2\times S^2} drdtd\theta d\phi\ \epsilon^{\lambda\mu\nu\rho}F_{\lambda\mu}F_{\nu\rho} = 2\int_{D^2\times S^2} drdtd\theta d\phi\ \epsilon^{\lambda\mu\nu\rho}\partial_\lambda\left(A_\mu F_{\nu\rho}\right) = \\
&2\int_{S^1\times S^2} dtd\theta d\phi \int_0^1 dr\ \epsilon^{r\mu\nu\rho}\partial_r\left(A_\mu F_{\nu\rho}\right) + 2\int_{S^2} d\theta d\phi \int_0^1 dr \int_0^\beta dt \epsilon^{t\mu\nu\rho}\partial_t\left(A_\mu F_{\nu\rho}\right) \\
&+2\int_{D^2} dtdr \int_{S^2} d\theta d\phi\ \epsilon^{i\mu\nu\rho}\partial_i\left(A_\mu F_{\nu\rho}\right),
\end{aligned} \tag{7}$$

where $i \equiv (\theta, \phi)$. The first integral in the final equality produces (upon restoring the required normalization) the naive CS action of (1),

$$I_{CS} \equiv \frac{1}{16\pi^2}\int_{S^1\times S^2} dtd\theta d\phi\ \epsilon^{r\mu\nu\rho}A_\mu F_{\nu\rho}(r=1); \tag{8}$$

the contribution at $r = 0$ vanishes since A is a regular connection on the disc. The second integral is zero since the integrand is periodic in t. However, the last term

$$2\int_{D^2} dtdr \int_{S^2} d\theta d\phi \epsilon^{i\mu\nu\rho}\partial_i\left(A_\mu F_{\nu\rho}\right) = 2\int_{D^2} dtdr \int_{S^2} d\theta d\phi \epsilon^{ir\nu\rho}\partial_i\left(A_r F_{\nu\rho}\right) +$$

$$2\int_{D^2} dtdr \int_{S^2} d\theta d\phi \epsilon^{it\nu\rho}\partial_i\left(A_t F_{\nu\rho}\right) + \ 2\int_{D^2} dtdr \int_{S^2} d\theta d\phi \ \epsilon^{ij\nu\rho}\partial_i\left(A_j F_{\nu\rho}\right) =$$
$$4\int_{D^2} dtdr \int_{S^2} d\theta d\phi \epsilon^{ij}\partial_i\left(A_j F_{rt}\right) \equiv \Delta, \qquad \epsilon^{ij} \equiv \epsilon^{rtij}, \tag{9}$$

requires a more careful analysis. The first two integrals in the second equality can be dropped, since A_r and A_t define two regular scalar function on the $2-$sphere. The surviving term carries all the non-trivial information. In fact, with our above choice of patches, we have

$$\Delta = 4\int_{D^2} dtdr \left[\int_{H_+} d\theta d\phi \ \epsilon^{ij}\partial_i\left(A_j F_{rt}\right) + \int_{H_-} d\theta d\phi \ \epsilon^{ij}\partial_i\left(A_j F_{rt}\right)\right]. \tag{10}$$

Using the Poincaré lemma in each cup then yields

$$\Delta = 4\int_{D^2} dtdr \int_0^{2\pi} d\phi \ \left[\left(A_\phi^+ - A_\phi^-\right) F_{rt}\right](\theta=\theta_0) = 4k\int_0^{2\pi} d\phi \int_0^\beta dt A_t(\theta=\theta_0, r=1), \tag{11}$$

where we have used $A_\phi^+ - A_\phi^- = k$, as required by (6). The final result for $\bar{I}_{CS}$ in this procedure thus reads

$$\bar{I}_{CS} = \frac{1}{16\pi^2}\int_{S^1\times S^2} dtd\theta d\phi \ \epsilon^{r\mu\nu\rho} A_\mu F_{\nu\rho} + \frac{k}{8\pi^2}\int_0^{2\pi} d\phi \int_0^\beta dt A_t(\theta=\theta_0) \equiv I_{CS} + \frac{\Delta}{32\pi^2}. \tag{12}$$

We have dropped the $r = 1$ argument because henceforth all fields in (12) are the $4-$dimensional ones computed on the $r = 1$ boundary, that is, the $3-$dimensional ones. The above route is a realization of the prescription of [2] as well as of the procedures of [4]. Several comments about (12) are in order: (a) it is "small" gauge invariant: in fact the new contribution depends only on the integral of A_t over S^1 and this quantity, like the naive CS, is small (but not large) invariant[4]. (b) As advertised previously, the final result

[4]That the final result is not large invariant even though the original 4D integral is manifestly unchanged by all gauge transformations, is traceable to the fact that three-dimensional fields differing by a large transformation are not gauge equivalent as components of four-dimensional fields. In our case one need merely notice that a $3D$ large transformation affecting the integral of A_t over S^1 must alter the flux of the $4D$ field through the disc. Recall that under $U_n^{\rm large} = \exp(2\pi n t/\beta)$,

is fundamentally dependent on the patches or more precisely on the specific intersection between different charts. (c) Finally, although (12) seems quite different from (4), the two are actually the same (and of course (12) varies exactly as in (5)). The equivalence can be easily shown by an appropriate choice of the reference connection in (4). Take, for example, $\hat{A}$ to be any four dimensional connection that reduces on $\partial\mathcal{M}_4$ to $(0, \hat{\mathbf{A}})$ where $\hat{\mathbf{A}}$ is the usual instanton of topological charge k on S^2; then (4) can be shown to reduce to (12). (d) This also shows that, in (4), the $4D$ dependent parts of $\hat{\mathbf{A}}$ cancel between the two terms there; its residual lack of invariance under large transformations is purely 3−dimensional.

We have just noticed in the descent from $D = 4$ that the correction Δ required by the naive I_{CS} is an integral over the intersection of the patches of the transition function modulated by A_t. This correction was derived in [1] entirely within $D = 3$ (and also related there to the above descent method) using the machinery presented in [4]. To accomplish this "intrinsic" process, the cohomological aspects is carried here by the various transition functions of the (generally complicated) overlaps. The extra contributions beyond the sum over the patches α of $\int A_\alpha \wedge F$ in

$$\int A \wedge F = \sum_\alpha \int A_\alpha \wedge F + \sum_\alpha T_\alpha \tag{13}$$

stand for the various transition region overlap terms required cohomologically to give the improved $\bar{I}_{CS}$. They in turn are specified by the flux Φ. From this redefinition it is then also possible to read the desired variation of $\bar{I}_{CS}$ on a large transformation.

All the above routes for defining a correct CS action rely heavily on cohomological machinery. We now relate them to the heuristic, "physical" approach [3] that recasts the naive I_{CS} of (1) into a "maximally" gauge invariant, discarding any "ill-defined" contribution in the process (specifically in the integrations by part). To simplify our analysis, we confine ourselves again to the case of $S^1 \times S^2$. It can be shown that there is a gauge reachable by

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$A_t \to A_t + 2\pi n/\beta$. In $4D$ language, this corresponds to sending $A_t \to A_t + 2\pi rn/\beta, \mathbf{A} \to \mathbf{A}$, which is not a gauge transformation ($\int drdt \Delta F_{rt} = 2\pi n$).

small transformations $U = \exp i\Omega$, $\Omega = \tilde{A}_0$ (leaving I_{CS} invariant) in which, starting from abitrary A_μ, the new A_μ become

$$A_0^U(t,\mathbf{x}) = \frac{1}{\beta}\int_0^\beta dt' A_0(t',\mathbf{x}) \equiv \mathcal{A}_0(\mathbf{x}), \tag{14a}$$

$$\mathbf{A}^U(t,\mathbf{x}) = \mathbf{A}(0,\mathbf{x}) - \tilde{\mathbf{E}}(t,\mathbf{x}), \quad \tilde{\mathbf{E}} \equiv -\left(\int_0^t dt' - \frac{t}{\beta}\int_0^\beta dt'\right)\mathbf{E}(t',\mathbf{x}) \tag{14b}$$

In terms of these variables, the naive I_{CS} has the form

$$\begin{aligned} I_{CS} &= 2\int_0^\beta dt \int d^2x \left[\mathcal{A}_0(\mathbf{x})B(t,\mathbf{x}) + \epsilon^{ij}(\tilde{E}_i(t,\mathbf{x}) + A_i(0,\mathbf{x}))E_j(t,\mathbf{x})\right] = \\ &= 2\int_0^\beta dt \int d^2x \left(\mathcal{A}_0(\mathbf{x})B(t,\mathbf{x}) + \epsilon^{ij}\tilde{E}_i(t,\mathbf{x})E_j(t,\mathbf{x}) + \epsilon^{ij}A_i(0,\mathbf{x})\partial_j\mathcal{A}_0(\mathbf{x})\right) = \\ &= 2\int_0^\beta dt \int d^2x \left[\mathcal{A}_0(\mathbf{x})(B(t,\mathbf{x}) + B(0,\mathbf{x})) + \epsilon^{ij}\tilde{E}_i(t,\mathbf{x})E_j(t,\mathbf{x})\right], \end{aligned} \tag{15}$$

where, in the last term of the second equality, we have used $\mathbf{E}(t,\mathbf{x}) = \nabla\mathcal{A}_0(\mathbf{x}) - \partial_0\mathbf{A}(t,\mathbf{x})$ and then dropped $\partial_0\mathbf{A}(t,\mathbf{x})$ by periodicity. In the last equality, we have omitted the boundary term $K \equiv \int d^3x \partial_j \epsilon^{ij}(\mathcal{A}_0(\mathbf{x})A_i(0,\mathbf{x}))$ coming from the integration by parts, which is patch-dependent. Surprisingly, the final truncated expression (15) is the correct answer. A quick way of checking this is to choose, in (4), any background $\hat{A}$ that reduces to $\mathbf{A}(0,\mathbf{x})$ on the boundary. In other words the heuristic approach implicitly promotes $\mathbf{A}(0,\mathbf{x})$ to be our reference connection $\hat{A}$. However, this simple "derivation" really involves an unjustified choice: the amount of "bad term" that we have to throw away is not uniquely defined. Before integrating by parts, the last term in the second equality of (15) involves $\partial_j\mathcal{A}_0(\mathbf{x})$ and so does not depend on the constant part a of $\mathcal{A}_0$, while this dependence is restored (by hand) after the integration. This mismatch obviously arises as a consequence of having dropped the specific boundary term K. However, since a part proportional to a is well-defined irrespective of $\mathbf{A}$'s jumps, the amount of a that goes into the action or into the boundary contribution cannot be decided merely from requiring a well-defined final result.

Our discussion so far has been enterely abelian. Our final derivation will take advantage of a simplification available in the nonabelian context of simply connected group such as $SU(N)$, where all $D = 3$ bundles are trivial. This implies that there are always gauges in which the connection has no jumps[5] and therefore the standard formula

$$I_{CS}^{NA} = \frac{1}{16\pi^2} \int \mathrm{Tr}[A \wedge dA + \frac{i}{3} A \wedge A \wedge A] \tag{16}$$

is valid without improvement. this fact is easy to understand in our $S^1 \times S^2$ context because the structure of the transition function between the caps on S^2 is necessarely trivial, $\Pi_1(SU(N)) = 0$. Hence there are always sections where A has been trivialized (no jumps) and (16) is applicable. Let us therefore embed our A of $U(1)$ in $SU(2)$, by defining the $SU(2)$-valued form $A\sigma_3$. To remove the discontinuity in A_ϕ, we have to introduce the -necessarily nonabelian- gauging U, with as usual, $A^U = U^{-1}AU - iU^{-1}dU$. For our model, we take

$$U_+(\theta, \phi) = \sin f(\theta) \cos n\phi I + i \sin f(\theta) \sin n\phi \sigma_3 + i \cos f(\theta) \sigma_2 \qquad U_- = I \tag{17}$$

where $\pm$ refers to the two caps on S^2 and $f(\theta)$ is monotonic regular function so that: $f(\pi/2) = \pi/2$ and $f(0) = f'(0) = f'(\pi/2) = 0$. At this point, A^U is no longer abelian of course and we must keep both terms in (16). The standard gauge transformation rule for I_{CS}^{NA} is [11]

$$\begin{aligned} I_{CS}^{NA}[A^U] &= I[A] + \frac{i}{16\pi^2} \int \mathrm{Tr}[\, d(A \wedge dUU^{-1})] + w(U), \\ w(U) &= \frac{1}{24\pi^2} \int \mathrm{Tr}[U^{-1}dUU^{-1}dUU^{-1}dU]. \end{aligned} \tag{18}$$

For us, $I[A] = I_{CS}[A]$ is just the naive abelian form (1), so the complete, well-defined, result is to take $\bar{I}_{CS} = I_{CS}^{NA}$. Next, we observe that the winding number contribution $w(U)$ vanishes since it involves an explicit ∂_t and the U of (17) is time-independent. The equality of the remaining term in (18) with Δ of (10) is easily verified by direct computation. The difference between our "secretly abelian" and truly nonabelian configurations is also manifested by the fact that for us $w(U)$ vanished, whereas there it is its nonvanishing that

[5] A non-abelian configuration with “abelian” characteristics that lead to apparent definition difficulties for I_{CS}^{NA} is proposed in [10]

requires μ-quantization [11], rather than the effect of $I_{CS} + \Delta$ noted earlier. In term of the language of our initial analysis, the role of U is essentially that of the background $\hat{A}$.

To summarize, we have been comparing, from different points of view, a set of topological and cohomological issues encountered in the analysis of the abelian CS term at finite temperature. We have shown how transgression naturally allows us to define $\bar{I}_{CS}$ on nontrivial bundles, which are unavoidable in interesting (non-vanishing flux) configurations, and to easily reproduce its behavior under large gauge changes; we have compared this approach with previous ones given in the literature and also shown it to underline a simple but correct heuristic definition. Finally, we have availed ourselves of the cohomological properties of simply connected groups by embedding $U(1)$ in $SU(2)$; the resulting $\bar{I}_{CS}$ immediately produced the desidered improvement.

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[1] A. P. Polychronakos, *Nucl. Phys. B.* **281** (1987) 241.

[2] R. Dijkgraaf and E. Witten, *Comm. Math. Phys.* **129** (1990) 393.

[3] S. Deser, L. Griguolo and D. Seminara, *Effective QED Actions: Representations, Gauge Invariance, Anomalies and Mass Expansions*, BRX-TH-417, hep-th 9712066.

[4] O. Alvarez, *Comm. Math. Phys.* **100** (1985) 279.

[5] S. Deser, L. Griguolo and D. Seminara, *Non Abelian Effective Actions at Finite Temperature*, BRX-TH-426.

[6] M. Nakahara, *Geometry, Topology and Physics*, IOP Publishing Ltd, Bristol, chap. 11.

[7] J. Manes, R. Stora and B. Zumino, *Comm. Math. Phys.* **102** (1985) 157.

[8] A. P. Polychronakos, *Phys. Lett. B.* **241** (1990) 37.

[9] R. Iengo and K. Lechner, *Phys. Rept.* **213** (1992) 179.

[10] R. Jackiw and S.-Y. Pi , *"Reducing the Chern-Simons term by a symmetry"*, MIT-CTP-2696, BU-HEP-97-30, hep-th/9712087.

[11] S. Deser, R. Jackiw and S. Templeton, *Ann. Phys.* **140** (1982) 372.