Model Independent Naturalness Bounds on Magnetic Moments of Majorana Neutrinos

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Abstract. We analyze the implications of neutrino masses for the magnitude of neutrino magnetic moments. By considering electroweak radiative corrections to the neutrino mass, we derive model-independent naturalness upper bounds on neutrino magnetic moments, generated by physics above the electroweak scale. For Majorana neutrinos, these bounds are weaker than present experimental limits if μ_ν if generated by new physics at ~ 1 TeV, and surpass current experimental sensitivity only for new physics scales $> 10 - 100$ TeV. The discovery of a neutrino magnetic moment near present limits would thus signify that neutrinos are Majorana particles.

Keywords: neutrino magnetic moment, majorana
PACS: 12.15.-y 12.15.Lk 12.60.-i 13.15.+g 14.60.Pq

In the Standard Model (minimally extended to include non-zero neutrino mass) the neutrino magnetic moment is given by $\mu_\nu \approx 3 \times 10^{-19} \left(\frac{m_\nu}{1\text{eV}}\right) \mu_B$ [1]. An experimental observation of a larger magnetic moment would be an unequivocal indication of physics beyond the minimally extended Standard Model. Current laboratory limits, $\mu_\nu < 1.5 \times 10^{-10} \mu_B$ [2] and $\mu_\nu < 0.7 \times 10^{-10} \mu_B$ [3], are obtained from solar and reactor experiments, respectively. Slightly stronger bounds are obtained from astrophysics, $\mu_\nu < 3 \times 10^{-12}$ [4]. In the case of Dirac neutrinos, the neutrino mass constrains the magnetic moments to be [5] $\mu_\nu \leq 10^{-15} \mu_B$, which is several orders of magnitude more stringent than current experimental constraints.

Following Refs. [5, 6] we assume that the magnetic moment is generated by unspecified physics beyond the SM at an energy scale Λ above the electroweak scale. We shall work exclusively with dimension $D > 4$ operators that involve only SM fields, obtained by integrating out the physics above the scale Λ, and respect the $SU(2)_L \times U(1)_Y$ symmetry of the SM, and contains only SM fields charged under these gauge groups. The lowest order contributions to the neutrino (Majorana) mass arise from the usual five and seven dimensional operators containing Higgs and left-handed lepton doublet fields, L and H, respectively, $\left[\mathbf{O_M^{5D}}\right]_{\alpha\beta} = \left(\overline{\mathbf{L_\alpha^c}}\varepsilon\mathbf{H}\right)\left(\mathbf{H^T}\varepsilon\mathbf{L_\beta}\right)$ the five dimensional, and $\left[\mathbf{O_M^{7D}}\right]_{\alpha\beta} = \left(\overline{\mathbf{L_\alpha^c}}\varepsilon\mathbf{H}\right)\left(\mathbf{H^T}\varepsilon\mathbf{L_\beta}\right)\left(\mathbf{H^\dagger H}\right)$ the seven dimensional one. We use the notation $\varepsilon = -i\tau_2$, $\overline{L^c} = L^T C$, C denotes charge conjugation, and α, β are flavor indices. The neutrino magnetic moment operator is generated by dimension 7 operators involving the $SU(2)_L$ and $U(1)_Y$ gauge fields, W_μ^a and B_μ, respectively, $[\mathbf{O_B}]_{\alpha\beta} = \mathbf{g'}\left(\overline{\mathbf{L^c}}_\alpha\varepsilon\mathbf{H}\right)\sigma^{\mu\nu}\left(\mathbf{H^T}\varepsilon\mathbf{L_\beta}\right)\mathbf{B}_{\mu\nu}$, and $[\mathbf{O_W}]_{\alpha\beta} = \mathbf{g}\left(\overline{\mathbf{L_\alpha^c}}\varepsilon\mathbf{H}\right)\sigma^{\mu\nu}\left(\mathbf{H^T}\varepsilon\tau^a\mathbf{L_\beta}\right)\mathbf{W_{\mu\nu}^a}$. In the above definitions, $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - g\varepsilon_{abc}W_\mu^b W_\nu^c$ are the U(1)$_Y$ and SU(2)$_L$ are the

field strength tensors, respectively, and g' and g are the corresponding couplings. The three 7D operators defined above do not form a basis under renormalization. The full basis of 7D operators may be found in [7].

The operator O_W is asymmetric in flavor, and it is useful to express it in terms of operators with explicit flavor symmetry, $[O_W^\pm]_{\alpha\beta} = \frac{1}{2}([O_W]_{\alpha\beta} \pm [O_W]_{\beta\alpha})$. Operators O_M^{5D}, O_M^{7D} and O_W^+ are flavor symmetric, while O_B and O_W^- are antisymmetric. Our effective Lagrangian is therefore

$$\mathscr{L} = \frac{C_M^{5D}}{\Lambda} O_M^{5D} + \frac{C_M^{7D}}{\Lambda^3} O_M^{7D} + \frac{C_B}{\Lambda^3} O_B + \frac{C_W^+}{\Lambda^3} O_W^+ + \frac{C_W^-}{\Lambda^3} O_W^- + \ldots, \tag{1}$$

where the "$+\cdots$" denote other terms that are not relevant to the present analysis. After spontaneous symmetry breaking, the above operators reduce to the flavor symmetric Majorana mass term $\frac{1}{2}[\mathbf{m}_\nu]_{\alpha\beta}\,\overline{\nu^c}_\alpha \nu_\beta$, and flavor antisymmetric Majorana magnetic moment interaction $\frac{1}{2}[\mu_\nu]_{\alpha\beta}\,\overline{\nu^c}_\alpha \sigma^{\mu\nu} \nu_\beta \mathbf{F}_{\mu\nu}$, where $F_{\mu\nu}$ is the electromagnetic field strength tensor. At the EW scale, we have

$$\frac{[\mu_\nu]_{\alpha\beta}}{\mu_B} = \frac{2m_e v^2}{\Lambda^3}\left([C_B(M_W)]_{\alpha\beta} + [C_W^-(M_W)]_{\alpha\beta}\right), \tag{2}$$

$$\frac{1}{2}[m_\nu]_{\alpha\beta} = \frac{v^2}{2\Lambda}\left[C_M^{5D}(M_W)\right] + \frac{v^4}{4\Lambda^3}\left[C_M^{7D}(M_W)\right], \tag{3}$$

where the Higgs VEV is $<H^T> = (0, v/\sqrt{2})$.

Below, we calculate radiative corrections to the neutrino mass operators (O_M^{5D} and O_M^{7D}) generated by the magnetic moment operators O_W^- and O_B and put constraints on the size of the magnetic moment in terms of the neutrino mass, using Eqs.(2) and (3).

- **5D mass term - 1 loop**

One-loop radiative corrections only yield a contribution to O_M^{5D} associated with the flavor symmetric operator O_W^+. No model independent bound can be obtained from the one-loop radiative corrections from magnetic momentto the 5D mass operator.

- **7D mass term - one loop**

As the operator O_W^- is flavor antisymmetric, it must be multiplied by another flavor antisymmetric contribution in order to produce a flavor symmetric mass term. This can be accomplished through insertion of Yukawa couplings in the diagram shown in Fig. 1 [6]. This diagram provides a logarithmically divergent contribution to the 7D
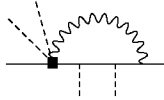

FIGURE 1. Contribution of O_W^- to the 7D neutrino mass operator.

mass term, given by

$$\left[C_M^{7D}(M_W)\right]_{\alpha\beta} \simeq \frac{3g^2}{16\pi^2} \frac{m_\alpha^2 - m_\beta^2}{v^2} \ln\frac{\Lambda^2}{M_W^2} \left[C_W^-(\Lambda)\right]_{\alpha\beta} \tag{4}$$

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where m_α are the charged lepton masses, and the exact coefficient has been computed using dimensional regularization and renormalized with modified minimal subtraction. Using this result, as well as Eqs. (2) and (3), leads to bound (i) in Table 1.

If we insert O_B in the diagram in Fig. 1, the contribution vanishes. A non-zero contribution to O_M^{7D} from O_B can arise at two-loop level, for instance, from a virtual W boson loop as in Fig. 2 [6]. This mechanism gives the leading contribution of the operator
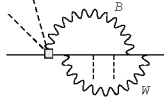

FIGURE 2. Representative contribution of O_B to the 7D neutrino mass operator at two loop order.

O_B to the 7D mass term. The O_B and O_W contributions to the 7D mass term are thus related by $\frac{(\delta m_\nu)^B}{(\delta m_\nu)^W} \approx \frac{\alpha}{4\pi} \frac{1}{\cos^2 \theta_W}$, where θ_W is the weak mixing angle. The corresponding limit is shown as bound (iii) in Table 1.

- **5D mass term - two loop**

The neutrino magnetic moment operator O_W^- will also contribute to the 5D mass operator *via* two-loop matching of the effective theory onto the full theory at at the scale Λ, as shown in Fig. 3. This diagram contributes to the 5D mass operator, and we provide a
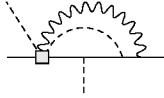

FIGURE 3. Representative contribution of O_W^- and O_B to the 5D neutrino mass operator.

naïve dimensional analysis [8] (NDA) estimate for the O_B and O_W insertions:

$$\left[C_M^{5D}(\Lambda) \right]_{\alpha\beta} \simeq \frac{g^2}{(16\pi^2)^2} \frac{m_\alpha^2 - m_\beta^2}{v^2} \left[C_W^-(\Lambda) \right]_{\alpha\beta} . \tag{5}$$

Using Eqs.(2) and (3), we obtain the bound (ii) in Table 1. The O_B contribution to the 5D mass term is the same as that for O_W, except for a factor of $(g'/g)^2 = \tan^2 \theta_W$, and it corresponds to the bound (iv) in Table 1. In doing so, we have neglected the running of the operator coefficients from the scale Λ to M_W since the effects are higher order in the gauge couplings and have a negligible numerical impact on our analysis. Our conclusions can be summarized according to the scale of the new physics, Λ:

I) $\Lambda \leq 10$ TeV

- No conflict with experimental limits.
- Both O_W and O_B contributions to μ_ν are possible, though O_W contributions are more tightly constrained.

II) $\Lambda \geq 10$ TeV

- $\mu_{\tau\mu}$, $\mu_{\tau e}$ bounds stronger than experimental limits
- $\mu_{\mu e}$ bound weaker than experimental limits

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TABLE 1. Summary of constraints on the magnitude of the magnetic moment of Majorana neutrinos. The upper two lines correspond to a magnetic moment generated by the O_W operator, while the lower two lines correspond to the O_B^- operator. $R_{\alpha\beta} = \frac{m_\tau^2}{|m_\alpha^2 - m_\beta^2|}$, with m_α being the masses of charged lepton masses. Numerically, $R_{\tau e} \simeq R_{\tau\mu} \simeq 1$ and $R_{\mu e} \simeq 283$.

i) 1-loop, 7D	$\mu_{\alpha\beta}^W$	$\leq 1 \times 10^{-10} \mu_B \left(\frac{[m_\nu]_{\alpha\beta}}{1\,\mathrm{eV}} \right) \ln^{-1} \frac{\Lambda^2}{M_W^2} R_{\alpha\beta}$
ii) 2-loop, 5D	$\mu_{\alpha\beta}^W$	$\leq 1 \times 10^{-9} \mu_B \left(\frac{[m_\nu]_{\alpha\beta}}{1\,\mathrm{eV}} \right) \left(\frac{1\,\mathrm{TeV}}{\Lambda} \right)^2 R_{\alpha\beta}$
iii) 2-loop, 7D	$\mu_{\alpha\beta}^B$	$\leq 1 \times 10^{-7} \mu_B \left(\frac{[m_\nu]_{\alpha\beta}}{1\,\mathrm{eV}} \right) \ln^{-1} \frac{\Lambda^2}{M_W^2} R_{\alpha\beta}$
iv) 2-loop, 5D	$\mu_{\alpha\beta}^B$	$\leq 4 \times 10^{-9} \mu_B \left(\frac{[m_\nu]_{\alpha\beta}}{1\,\mathrm{eV}} \right) \left(\frac{1\,\mathrm{TeV}}{\Lambda} \right)^2 R_{\alpha\beta}$

- Same limit irrespective of whether μ_ν generated by O_W and O_B

III) $\Lambda \geq 100$ TeV

- The $\mu_{\alpha\beta}$ bound becomes stronger than current experimental constraints, for all α, β.

We emphasize that the bound (iv) in Table 1 is the most general bound on the magnetic moment.

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