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QThN — 4:45 pm–6:30 pm
Room 327/328

Quantum Information

Rainer Blatt, *Univ. Innsbruck, Austria, Presider*

QThN1 (Invited) 4:45 pm

Quantum feedback and adaptive measurements

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Although real-time feedback of measured signals is an essential component of sensing and control in classical settings, models for *quantum feedback* that are rigorous yet useful[1] have only become possible since the advent of measurement-based quantum trajectory theory.[2] The quantum feedback scenario introduces new concerns of coherence and measurement backaction, but recent work has shown that these can be treated properly in a formal integration of quantum trajectory theory with standard state-space formulations of filtering and control theory.[3] Pioneering studies by H. M. Wiseman have shown that such models can be used to design and to analyze realistic schemes for adaptive homodyne measurement[4] and for feedback control of atomic motion.[5]

Much of the ongoing research in our group focuses on the experimental implementation of such schemes. For a broad range of quantum feedback scenarios, certain recurring technical issues arise out of the need to perform complex, high-bandwidth processing of measured signals. We are developing a "rapid-prototyping" approach to refining signal processing and feedback algorithms via quantum trajectory simulation on a PC, followed by translation of the algorithms into Hardware Description Language (HDL). The HDL code can be directly synthesized into a netlist that can be efficiently implemented on Field Programmable Gate Arrays (FPGA's) with clock speeds ≥ 100 MHz and highly parallel dataflow. In this talk we will give an overview of this methodology, in the context of our laboratory implementation of Wiseman's scheme for adaptive homodyne measurement.

We will also discuss the concept of quantum Kalman filtering, which plays a central role in establishing the connection between quantum measurement theory and state-space formulations of feedback control. We will describe the performance of Kalman state estimators, which have been derived using methods from the theory of open quantum systems and implemented on FPGA's, for tracking the motion of individual trapped atoms in cavity QED.[6]

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QThN2 5:15 pm

Quantum State Reconstruction: A comparison of Maximum Likelihood and Tomographic Schemes

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For some time now experimental techniques for the measurement of the quantum state of light have been the subject of intensive investigation.[1]. Tomographic techniques have been applied to experiments such as the homodyne measurement of the Wigner function of a single mode of light[2] and of the density matrix of the polarization degrees of freedom of a pair of entangled photons.[3] In this technique the density matrix (or Wigner function) which characterizes the quantum state of the system being measured is found from a linear transformation of experimental data. There are a number of drawbacks to the method, principally in that the recovered state might not, because of experimental noise, correspond to a physical state. For example, density matrices for any quantum state must be Hermitian, positive semi-definite matrices with unit trace. The tomographically measured matrices often fail to be positive semi-definite.

To avoid this problem the "maximum likelihood" approach to the estimation of quantum states has been developed.[4–9] In this approach the density matrix that is "mostly likely" to have produced a measured data set is determined by a numerical optimization routine. For example, a density matrix for a physical state of a two qubit system can *always be put in the following form*

$$\hat{\rho}(t) = \hat{T}^{\dagger}\hat{T}/\mathrm{Tr}\,\{\hat{T}^{\dagger}\hat{T}\}, \tag{1}$$

where $\hat{T}$ is a 4×4 matrix as follows

$$\hat{T}(t) = \begin{pmatrix} t_1 & 0 & 0 & 0 \\ t_5 + it_6 & t_2 & 0 & 0 \\ t_{11} + it_{12} & t_7 + it_8 & t_3 & 0 \\ t_{15} + it_{16} & t_{13} + it_{14} & t_9 + it_{10} & t_4 \end{pmatrix}, \tag{2}$$

and t stands for the set of 16 real variables $\{t_1, t_2, \ldots, t_{16}\}$. Then if we can define some likelihood function $f(t)$, which is some measure of the probability that the state $\hat{\rho}(t)$ would produce a given set of experimental data generated by measurements of our two photon system, then we can apply a numerical optimization routine to find the set of values t_{max} for which f has its maximum value.

We will evaluate two different approaches to the maximum likelihood technique, namely defining the likelihood function f as a weighted sum of squared variances from measured data, and defining it in terms of the information content. These will be compared with results of standard tomographical schemes. We numerically generated a physical density matrix, then simulated sets of experimental data by incorporating random errors appropriate to our experiment. Based on this "pseudo-data", we used both maximum likelihood routines to generate an approximate density matrix, which we compared the original density matrix. This allows us to compare the accuracy of conceptually different techniques.

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QThN3 5:30 pm

Quantum tomography of the single-photon Fock state

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The single photon Fock state is one of the most fundamental states of the light field. It is highly non-classical and reveals the wave-particle duality of light most strikingly. Its marginal distributions are of non-Gaussian shape and its Wigner function exhibits a strong negativity around the origin of phase space. Although the reconstruction of the Fock state has already been performed for the motional state of trapped beryllium ions,[1] in the optical domain this task has not been resolved so far, the main difficulty being the lack of coherent single-photon sources.