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Thirty Meter Telescope Astrometry Error Budget

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ABSTRACT

The Thirty Meter Telescope (TMT) with its first-light multi-conjugate adaptive optics system, NFIRAOS, and high-resolution imager, IRIS, is expected to take differential astrometric measurements with an accuracy on the order of tens of micro arcsec. This requires the control, correction, characterization and calibration of a large number of error sources and uncertainties, many of which have magnitudes much in excess of this level of accuracy. In addition to designing the observatory such that very high precision and accuracy astrometric observations are enabled, satisfying the TMT requirements can only be achieved by a careful calibration, observation and data reduction strategy. In this paper, we present descriptions of the individual errors sources, how and when they apply to different astrometry science cases and the mitigation methods required for each of them, as well as example results for individual error terms and the overall error budgets for a variety of different science cases.

Keywords: Astrometry, adaptive optics, extremely large telescopes, Thirty Meter Telescope

1. INTRODUCTION

The Thirty Meter Telescope (TMT) has very stringent astrometric requirements in order to enable us to push the limits of astrometric measurements to new levels in many different science cases. In multi-conjugate adaptive optics (MCAO) mode using TMT's first-light AO system, NFIRAOS, and high-resolution imager, IRIS, differential astrometry is supposed to yield errors no larger than 50 micro arcsec (μas) in a 100-second exposure in H band. This error is supposed to fall as the square root of the integration time, T, to a systematic floor of 10 μas. Absolute astrometric accuracy is required with an error no larger than 2 mas.

One of the principle challenges to overcome is that many of the raw astrometric errors present in IRIS images will be larger than tens of μas, some of them by several orders of magnitudes, even under the best of circumstances. Achieving the required astrometric precision and accuracy therefore requires not only the control and correction of physical error sources to the maximum extent, but also their characterization and calibration with the highest possible accuracy. A careful observing and data reduction methodology, parts of which may be substantially different from those currently used at 8–10 m class telescopes, will be needed as well.

The Thirty Meter Telescope has therefore undergone a substantial effort over the last couple years to establish reliable astrometry error budgets for TMT. A large number a trade studies, involving observations, analytical analyses and simulations have been done, and are still in the process of being done. Additionally, we have also developed a mathematical formalism that can be used to assess the effects of individual error sources as well as correlations between two or more error sources. This formalism covers the error propagation through the entire astrometric observation process, that is, including calibration and data reduction and is, in its most general form, independent of the science case. We are currently in the process of collecting all this information and describing it in a top-level error budget report, together with a large number of background documents. In this paper, we present high-level summary descriptions of the individual error sources and overall error budgets for a number of general science cases. The full information will be made available elsewhere in the near future.

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Table 1. Example of astrometry error terms due to catalog errors in μas. Here, N is the number of available reference sources, f is the field of view of interest in arcsec, and t is the time the entire set of observations spans in years. See text for assumptions made.

References [μas]	Position	Proper Motion	Other	Color	Aberration
Section	2.1	2.2	2.3	2.4	2.5
Absolute Astr. accuracy	$1000N^{-0.5}$	$800N^{-0.5}t$	1	5	0
Differential Astr. accuracy	$1000N^{-0.5}f/30$	$800N^{-0.5}tf/30$	1	5	0
Differential Astr. precision	n/a	n/a	n/a	n/a	n/a

It must be understood that there is no such thing as *the* astrometry error budget for an observatory, or even for any specific *type* of astrometry observation, such as differential movement between a set of science objects. Many terms in the error budget depend on the exact details of the astrometric observations, such as whether absolute or differential astrometry is the goal, whether one observes a sparse or crowded field, the time scales of interest, if there are reference sources available in the field and how many of them there are and at what distances from the science objects, the zenith angle of observation etc. Some of these observation characteristics cause only quantitative changes in the magnitude of the errors that can be presented in parametric form, but many of them result in qualitative differences as well that require fundamentally different terms in the error budget.

The TMT astrometry error budget is therefore not only divided into terms describing the different errors, but also into a variety of science cases. This includes both general science cases, such as 'differential astrometry in sparse fields,' as well as very specific science cases such as measuring star orbits around Sgr A* at the Galactic center.[1–3] Only summary results for some of the general science cases are presented here. The terminology we use has been explained previously,[4] but in brief: 'absolute astrometry' refers to the measurement of the science object positions in the sky coordinate system. 'Differential astrometry' is the measurement of the relative positions of science objects with respect to each other.

Differential astrometry cases are furthermore split up into fields with many objects for which removal of low order distortion via coordinate transformation is possible during data reduction, and 'sparse field' science cases for which that is not possible. For the most part, we do not distinguish between the accuracy and the precision of the differential measurement in this paper and only point out potential differences in a couple places. For the differential astrometry cases presented here, we generally assume that the science objects of interest cover only a small fraction of the entire field of view, such that the precision of the measurement is the limiting factor rather than its absolute calibration.

There are many different ways to slice the TMT astrometry error budget, because many error sources affect several aspects of the astrometric measurements. As the error budgeting process was started in order to ensure that TMT is designed and build in a way that enables the highest possible precision and accuracy astrometric measurements, our error budget is organized by cause of the errors, rather than by effect in the focal plane. The error budget terms are divided into five categories, (roughly) following the light path: reference and source catalog errors, atmospheric refraction correction and modeling errors, other atmospheric effects, opto-mechanical errors and focal-plane measurement errors. Each of these categories is described in one of the following sections.

2. REFERENCE SOURCE AND CATALOG ERRORS

This section describes astrometry error budget terms caused by differences between the real and assumed properties of the reference sources. 'Reference sources' here refers to astronomical objects that are used to determine the absolute separation and/or absolute sky coordinates of the science objects. In most cases, reference sources are objects in the science fields whose positions are known from other observations. They might be some of the science objects themselves, or other sources that are also visible in the science field but are not physically associated with the science objects. Distant background galaxies/quasars are examples of such reference sources.

In some cases, no known reference sources are available in the science field. The NFIRAOS natural guide stars (NGSs) are then the best available references.

2.1 Position errors

The most obvious cause of reference source error is the position of the sources themselves as taken from reference catalogs. These cause direct errors in the reconstructed positions and absolute pixel scale, although their effect is different depending on the science case. For the general astrometry cases shown in Table 1, we assume that the reference sources are catalog stars (HIPPARCOS, GAIA, ...). The absolute astrometry accuracy is then given directly by the catalog position error divided by $\sqrt{N}$, where N is the number of available reference sources. This even applies when no references are available in the field, by using the catalog errors of the NGSs and $N = 3$.

For differential astrometry accuracy, Trippe et al.[5] show that a 1 mas error in position (available today from the HIPPARCOS catalog) is sufficient to achieve the required accuracy of 10 μas for the relevant E-ELT/MICADO science cases when three reference stars are available. This is due to the fact that the error scales with the ratio of the size of the science field of interest to the separation of the reference stars. For many science cases the spatial scales of interest range from a few tens of μas (e.g. parallaxes of globular clusters) to a few hundreds of mas (e.g. stellar orbits around Sgr A*) while the separation of the reference objects is on the order of tens of arcsec. This error will be further reduced once GAIA measurements become available, in part due to their improved accuracy and also because many more reference stars will become usable. Obviously, this error term is strongly dependent on the individual science case (not only on the type of science case). As an example, Table 1 shows the errors based on reference source errors of 1 mas and separation of 30 arcsec.

2.2 Proper motion errors

Errors in our knowledge of the proper motions of the reference sources cause the same kinds of errors as do catalog position errors, with the difference being that they also change from epoch to epoch. For the purpose of this error budget, we use 0.8 mas/year in Table 1, as given by Trippe et al.[5] for the HIPPARCOS catalog. As with reference source positions, the respective values for the GAIA catalog should be significantly smaller than those given in the table.

2.3 Other or unknown motion

More subtle effects are introduced through other, generally unknown motions of the reference sources, such as binary star motion or gravitational lensing. They cause errors of the position measurements of individual sources and can further complicate data processing in crowded fields. This error term can be assumed to be small as both undetected periodic motions and sporadic individual events average out when sufficient amounts of data are taken. For the purpose of error budgeting we therefore allocate a value of 1 μas to this term, but it must be understood that, in individual exposure or epochs, it can take on more or less any value. Of course, if they are large enough, these effects will be considered signal rather than noise.

2.4 Color errors and variability

Uncertainties of the colors of the reference sources and their variability also cause errors, most notably through their coupling with differential atmospheric refraction. We have done detailed simulations of this effect and found that it can be a significant source of astrometric error, in particular at high zenith angles, depending on what is known about the star colors or spectra.

If we assume that our only knowledge of the star spectrum is the J-K color, known with an uncertainty of 0.2 mag, then this error term in K band (after correction for the expected displacement due to the star color) is smaller than 10 μas for most star types and observing conditions, and significantly so in many cases. However, it can be significantly larger than 10 μas for some star types at large zenith angles, in particular for M stars. This error can be reduced through a variety of means, for example, observations closer to the zenith, observations with narrow(er)-band filters, determination of the star color with errors smaller than 0.2 mag, determination of stellar spectra, knowledge that all stars in the field or of similar spectral type, etc. Not all of these might be possible for any given science case, but at least some of them should be doable. Thus, a very small color error should be achievable, albeit potentially at the expense of requiring supporting observations, for astrometry science cases requiring this. The color error term in the error budget is therefore a top-down allocation, set to 5 μas, that can be relaxed for science cases not requiring this sort of precision.

Table 2. Astrometry error terms due to atmspheric refraction correction errors in μas. Here, f is the field of view of interest in μas and r is the separation of the binary stars in arcsec. See text for assumptions made.

Refraction [μas]	Achromatic diff refr.	Dispersion	Variability
Section	3.1	3.2	3.3
Absolute Astrometry	$1000f/30$	15	2
Differential Astrometry	2	15	2
Differential Astrometry sparse	$1000f/30$	15	2

2.5 Differential aberration and gravitational light deflection

Aberration is the compression of the field (in one direction) due to the relative directions of the Earth's motion and the velocity vector of the light coming from the reference sources. This effect is epoch dependent in that the velocity vector of the Earth changes throughout the year. It is proportional to the field of view and independent of integration time and wavelength. It can be shown that this effect is deterministic and negligible once the lowest order distortion terms have been removed in post processing. The same is true for the gravitational deflection of light due to solar system objects, most notably the Sun and Jupiter.

2.6 Extra-galactic references

If extra-galactic references (for example, globular clusters in/around distant galaxies) need to be used as reference sources for absolute astrometry, their faintness and the background of the host galaxy cause astrometric errors. These errors average out in time, although not necessarily as fast as $T^{-0.5}$ so that very long integration times and thousands of such references may be needed to reduce the residual error to the level of tens of μas.[5] As this is a specialized case, it is not part of our standard astrometry error budget, but an extra science case in which this is used might be included at some point.

3. ATMOSPHERIC REFRACTION CORRECTION AND MODELING ERRORS

Atmospheric refraction (see also Table 2) is separated out from other atmospheric effects because it can be one of the largest astrometry error sources, at least for some astrometry science cases. It causes differences between the observed and physical zenith angles of the science objects and contributes an absolute, achromatic term as well as two differential terms, one achromatic and one chromatic, to the astrometry error budget. The absolute term, corresponding to a vertical shift of the entire field, is irrelevant to NFIRAOS/IRIS astrometry, as all fields are always measured relative to the 3 NGS which are shifted by the same amount (except for the differential terms).

3.1 Achromatic differential atmospheric refraction

Achromatic differential refraction is due to the fact that different objects in the field are at different zenith angles. This term is large, as much as tens of mas, but consists mostly of low-order terms that can be taken out with coordinate transformations for differential astrometry. Trippe et al.[5] show that the residual is on the order of 2 μas even if only the first-order terms are removed, which is the value we adopt for the TMT error budget.

For astrometry in very sparse fields, the low-order terms cannot be removed through coordinate transformations. However, an atmospheric model can be applied to correct for the majority of the effect. Its accuracy depends on our knowledge of the atmospheric temperature, humidity and pressure profiles. Simulations, supported by observations at Subaru, show that it is possible to reduce the residual after application of such a model to values on the order of 1 mas under most conditions, which is the value we show in Table 2 together with its dependence on the size of the science field.

3.2 Chromatic differential atmospheric refraction (dispersion)

Chromatic differential refraction (dispersion) is due to the index of refraction being a function of the wavelength of light. The change of vertical position as a function of wavelength can be tens of mas even across an individual broad-band filter. Thus, stars of different spectral type exhibit different offsets due to atmospheric refraction. In order to reduce dispersion errors to acceptable values, it has been shown that atmospheric dispersion correctors

Table 3. Astrometry error terms due to residual atmospheric effects other than refraction in μas. Here, T is the exposure time in seconds and f is the field of view of interest in arcsec. See text for assumptions made.

Atmosphere [μas]	Abs. T/T	Diff. T/T	Higher Order	PSF Shape	Halo	Variability
Section	4.1	4.2	4.3	4.4	4.5	4.6
Absolute Astrometry	0	$150T^{-0.5}$	$105T^{-0.5}$	$55T^{-0.5}$	0	5
Differential Astrometry	0	0	$105T^{-0.5}$	$50T^{-0.5}$	10	5
Differential Astr. sparse	0	$150T^{-0.5}f/30$	$105T^{-0.5}$	$55T^{-0.5}$	0	5

(ADCs) are necessary.[5,6] However, even with an ADC there remain residual errors because 1. even the perfectly built ADC has a wavelength-dependent residual and 2. our knowledge of the atmospheric profiles is incomplete and has uncertainties. Our simulations have shown that the residual dispersion can be on the order of ± 15 μas for large zenith angles.

The effect of dispersion residuals on astrometric errors is very different depending on the science case/field. First, the error only affects the elevation position, not azimuth. Thus, horizontal (on the sky) positions do not have a dispersion error. Second, in fields with many stars of similar colors, a large fraction of the error is the same for all stars and therefore does not have an effect on differential astrometry. By contrast, if star colors vary significantly in the field the residual might not be reducible at all in post-processing. Finally, this error averages out if observations are taken under many different observing conditions, resulting in its effect being smaller for science cases involving frequent and/or multiple observations.

Table 2 therefore uses the full 15 μas for the general science cases error budget, but we note that it might be possible to reduce this error further for specific science cases such as stellar orbits around the Galactic center where many stars have similar spectral types, if narrow-band filters are being used or if observations close to the zenith are possible.

Note that, for this error term, we assume perfect knowledge of the objects' spectral types, as astrometric errors due to color uncertainties are included in the error budget in Section 2.4. Similarly, errors caused by imperfect manufacturing and positioning (rotation angle) of the ADC are accounted for in Section 5.

3.3 Coupling of AR errors with image motion and rotation

Atmospheric refraction and its changes also produce an error term (uncertainty) through their coupling with image motion and rotation, in a way similar to transparency and Strehl ratio variations (see Section 4.6). We have not yet investigated this separately, but based on our investigation of the static errors, we expect this term to be smaller than those caused by transparency and Strehl variations. We thus put a top-down allocation of 2 μas into the error budget. As described in Section 4.6, the main effect of this allocation is that it puts a limit on the allowable integration time for certain observations.

4. OTHER RESIDUAL ATMOSPHERIC EFFECTS

Other (non-refraction) atmospheric effects include residual atmospheric distortions after AO correction, the "halo effect", PSF elongation due to anisoplanatism and variable atmospheric effects such as transparency or the seeing. Many of the terms described in this section have been analyzed in detailed NFIRAOS, IRIS and Galactic center astrometry simulations resulting in reliable estimates of their magnitude that are summarized in Table 3. These studies will be made available together with the TMT astrometry error budget, some of their main results are summarized here.

4.1 Residual atmospheric tip/tilt

As is the case for absolute atmospheric refraction, the aperture-wide residual tip/tilt after AO correction has only second order effects on absolute astrometry as its first-order term is the same for the science field and the NGSs. Second order effects enter through chromatic effects and tilt anisoplanatism, but their magnitudes are small compared to refraction effects and indistinguishable from them in practice. We therefore mention this effect here, but assign no value to it in the error budget.

The absolute residual tip/tilt also has an effect on differential astrometry through the reduction of the signal-to-noise ratio and Strehl ratio of the observations. This is generally a small contribution to the error budget and is negligible compared to other error sources having the same effect. As for absolute astrometry, we therefore assign no value to this term for differential astrometry.

4.2 Differential residual atmospheric tip/tilt – plate scale modes

Differential residual tip/tilt, as used in this section, refers to the difference in residual T/T between the 3 NGSs. This residual is large, but it only causes plate scale variations which are taken out entirely by coordinate transformations in differential astrometry with many reference sources. Only a very small residual due to noise remains after that, which was shown in Galacitc center astrometry simulations to be a small fraction of a μas. Thus, it mostly affects absolute astrometry and differential astrometry in very sparse fields. It affects differential astrometry in fields with many stars only through the reduction of SNR and Strehl ratio and is in magnitude even smaller than the aperture-wise tip/tilt.

This error averages out with integration time as $T^{-0.5}$. It is proportional to the size of the field of view of interest. Its magnitude under typical conditions was found to be 150 μas in a 1-second exposure averaged over a 30 arcsec field in NFIRAOS simulations. Thus, this is the relevant value for absolute astrometry in sparse fields, while for differential astrometry in sparse fields it is multiplied by the field size of interest divided by 30 arcsec.

4.3 Higher-order residual atmospheric distortions

Higher-order residual atmospheric turbulence causes local distortions in the image plane. These are smaller in magnitude than low-order modes, but can be one of the dominant error sources, especially in shorter-exposure images, if they are not well corrected by the AO system. The higher-order the AO correction is, and the more uniform over the science field, the smaller is the residual astrometric error. Conversely, better correction translates into a shorter integration time to average out these errors to the same level. This is one of the reasons why MCAO was chosen as first-light AO mode for TMT.

Note that all types of AO correction are preferable over no correction for this, and the smaller the residual the better, as the corrected wavefront always has a lower residual than that caused by atmospheric turbulence without correction. The possible exception to this are quasi-static effects, but we consider these an opto-mechanical error term rather than an atmospheric residual. They are therefore not included here and are dealt with in Section 5.6.

The residual distortion error averages out with integration time no slower than $T^{-0.5}$ (there are $T^{-0.5}$ and $T^{-1.5}$ terms, depending on the wind direction, the former being dominant in most realistic cases). NFIRAOS simulations determined its magnitude to be 105 μas in a 1-second exposure under typical conditions. This term applies equally to all types of astrometry, except if coordinate transformations of orders higher than plate scale modes are done for differential astrometry in fields with many sources. Such higher-order transformations are not assumed to be necessary for the time being (and their effect is therefore not included in the current error budget), but they remain as an option to improve the accuracy of the differential measurement in cases when such terms become dominant.

4.4 PSF irregularities and elongation after MCAO correction

Atmospheric residuals also cause PSF irregularities. In addition, the PSF is elongated in the direction away from the tip/tilt guide stars, both of which cause errors and uncertainties in the determination of the center of the PSF. The integration time dependence of this error term cannot be calculated analytically, but we have shown with simulations that it averages out as $T^{-0.5}$ just like the previous terms. The combination of PSF irregularities and elongation was investigated in NFIRAOS simulations and found to range from 35 to 55 μas in a 1-second exposure, depending on wavelength (from K to J band). There is also a small improvement if plate scale modes are removed. As this is significantly smaller than the previous error (and they add in quadrature), we are using 55 μas for all generic spares-field science cases and 50 μas for fields with many sources for the general science cases listed in Table 3.

4.5 Halo effect

The so-called halo effect is caused by stars being located on the halos of other stars in crowded fields. Uncertainties in our knowledge of the PSF, local irregularities of the halo and changes thereof in time then cause (variable) background gradients and thus astrometric errors. As with the errors above, the best counter measure is the highest possible AO correction and stability, both in time and across the field.

The value of this error term is highly variable from science field to science field, and even from exposure to exposure. It is negligible (or even non-existent) in sparse fields and can be one of the dominant terms in crowded fields such as the Galactic center.[2] We have not investigated the halo effect in detail yet. It is the topic of future Galactic center simulations as well as other semi-analytical analyses. For now, we assign a top-down value of 10 μas in Table 3 for the generic differential astrometry science case in fields with many stars which, by definition, have enough objects to reduce residual distortions through coordinate transformations effectively, but not so many that the halo effect and confusion (see Section 6.6) become the dominant terms.

4.6 Coupling of turbulence and transparency variations with other effects

Variable properties of the atmosphere such as overall transparency that, on first look, don't seem to have any effect on image position can also cause astrometric errors. Transparency variations cause the flux received by the detector to vary. If, at the same time, image motion is present (for example due to variable opto-mechanical distortions or changes in atmospheric dispersion due to zenith angle or atmospheric profile changes), we get astrometric errors. These errors are random if either the transparency variations or the image motion are random, but can be systematic if both vary in a systematic way during an exposure. Strehl ratio variations due to turbulence strength changes have a similar effect.

The magnitude of this effect depends strongly on the residual image motion. In general terms, we have shown the following using temporal power spectra of transparency and turbulence:

- Transparency fluctuations contribute astrometric errors on the order of 10 μas whenever the image drifts by 1 mas during short exposures and 0.2 mas during long exposures.
- Seeing fluctuations typically contribute astrometric errors on the order of 10 μas whenever the image drifts by 1 mas during short exposures at wavelengths shorter than 1.6 μm and/or at zenith angles greater than 30° and 0.2 mas during long exposures.
- Astrometric errors increase linearly with image drift, for a constant exposure time.
- Unintuitively, astrometric errors due to transparency and seeing fluctuations *increase* with exposure time. This is due of the shape of the temporal spectra of these fluctuation and the fact that the drift increases with increasing integration time.

For the error budget, we allocate a value of 5 μas. In practice this results in a limit for the maximum allowable integration time when such accuracies are required.

4.7 Atmospheric scintillation

Atmospheric scintillation is a minor effect for astrometry. Its overall (aperture-wide) value is extremely small for large-aperture telescopes (and in any case much smaller than transparency variations) and local scintillation is smaller than effects such as Strehl ratio variations for the time and size scales of interest to TMT astrometry. We therefore do not assign a term to scintillation in the error budget.

Table 4. Astrometry error terms due to opto-mechanical errors in μas. Here, T is the exposure time in seconds and f is the field of view of interest in arcsec. See text for assumptions made.

Opto-mechanics [μas]	Plate Scale	IRIS	Surfaces	Tel.	Rotator	QS Effects	Diff. Spikes	Vibr.
Section	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8
Absolute Astr.	$2000f/30$	8	7	5	5	25	1	2
Differential Astr.	1	8	7	5	5	5	1	2
Diff. Astr. sparse	$2000f/30$	8	7	5	5	25	1	2

5. OPTO-MECHANICAL ERRORS

Distortions in the opto-mechanics cause astrometric errors that can be major contributors to the astrometry error budget.[7] They manifest themselves (and can be dealt with) in a variety of ways. Distortions that are stable in time can be calibrated by observing a reference grid, such as a pinhole mask inserted into the focal plane or a dense star field. A large fraction of the low-order distortions can then be eliminated in post-processing and (mostly) only the high spatial frequency distortions remain. This calibration might have to be done for multiple telescope/NFIRAOS/IRIS configurations, such as different elevation and rotator angles.

I addition to the high spatial frequency distortions, low-order distortions contribute to the astrometric error after calibration because they are 1. not entirely stable in time, 2. the configuration is not entirely repeatable and 3. the interpolation between different configurations to the conditions of the science observation is not exact.

While significant progress has been made in our analysis of distortion-caused astrometric errors, the distortion category is still the portion of the error budget with the largest uncertainties. In particular temporal variability of distortions has not yet been investigated in depth. Nevertheless, we have found no show stoppers for very high accuracy astrometry so far and do not expect to find any in the remaining analyses.

In the following sections and in Table 4, we describe distortion-caused astrometry error terms due to different parts of the TMT opto-mechanics, the status of the current analyses and the results found to date.

5.1 Plate scale variations due to guide probe positioning and other distortions

The natural guide star (NGS) probe arm positioning errors cause variations of the plate scale. The positioning errors are large (2 mas rms over the separation of the NGSs) and are one of the dominant terms for astrometry in sparse fields. For astrometry in fields with many stars, they can, in theory, be corrected for entirely in post-processing with only a small residual remaining due to noise. That this does indeed work in practice was demonstrated in our Galactic center astrometry simulations, where this term was reduced to a small fraction of 1 μas in the analysis of simulated images. We thus use 1 μas as error budget term for this science case, which is likely an upper limit in most cases.

For astrometry in sparse fields, the error is 2 mas times the field of view of interest divided by the average separation of the NGSs. Assuming 30 arcsec as a representative value for the separation, we get the results in Table 4.

Overall plate scale variations due to other distortions have the same (non)effect as those due to guide probe positioning errors. They should, however, be small compared to the guide probe positioning error and are therefore not given a separate allocation.

5.2 IRIS imager optics calibration residuals

The current analysis of IRIS imager distortions shows that a pinhole grid calibration can reduce the static distortions to approximately 8 μas on average across the IRIS field of view, which is the value we adopt for the error budget. The astrometric error will be somewhat smaller than the distortions themselves, with its exact magnitude depending strongly on the science case (size of field of interest, number of stars in the field, positions of the stars in the field, etc.).

The locations of the holes in the pinhole grid do not have to be known to this accuracy (which is not achievable with current technology), as long as they are stable on this level. It is then, however, necessary that the pinhole grid is put on an xy and/or rotation stage. The exact requirements and their trade-offs are still under investigation but several feasible solutions, all with their own advantages and disadvantages, have already been demonstrated.

Note that distortions of the IRIS imager filters are equivalent to other optical elements as far as the distortion analysis is concerned. We only mention them separately here because they introduce a wavelength dependence, whereas all other distortions are independent of wavelength.

5.3 NFIRAOS/IRIS optical surface error calibration residuals

Irregularities of the optical surfaces in NFIRAOS and IRIS (on the order of a few to tens of nanometers rms) cause distortions that are large compared to the astrometry error budget requirements in absolute terms. They can, however, be calibrated with a combination of on-sky and pinhole grid measurements, depending on the location of the optics in the optical train. Our analysis shows that the two NFIRAOS entrance windows are the largest contributors to the error budget, followed by the instrument selection mirror. The entrance windows are ahead of focus and therefore need to be calibrated on the sky, which means that:

1. Star density is not a problem as these observations can be done using globular clusters. The achievable accuracy then comes down to the stability of the distortions and repeatability of the setup. Our current results show that these likely are not limiting factors in the astrometry error budget.
2. The observing procedure and overall on-sky calibration time become the main driver in the achievable accuracy and need to be balanced against the higher cost of having smoother surface (lower distortion) optics in the first place. It is, at least in principle, always possible to lower the residual error for a given surface quality by longer on-sky calibration.

Our results to date indicate that 7 μas residuals should be achievable with realistic requirements for the surface qualities and on-sky calibration time.

5.4 Telescope optics calibration residuals

The shapes, alignment and optical surface errors of the three telescope mirrors all cause distortions. Alignment errors are mostly low order and will be either taken care of by NFIRAOS or can be calibrated out for differential astrometry. Shape and surface errors of the telescope mirrors have not yet been investigated separately, but based on the result found for NFIRAOS and IRIS, we expect that 5 μas should be achievable and assign this as error budget value for the time being.

5.5 Rotator errors

Rotator errors enter through two effects, the difference between the assumed and real rotator angle, and the (small, but non-negligible) misalignment error between the rotation and optical axes, which causes the beam footprint to wander on the optics. The latter term is similar to the errors discussed in previous sections and is currently work in progress as part of the same analyses used to evaluate those.

The rotation angle error needs to be treated differently as it does not produce its own distortion of the field but a rotation of the already existing distortion, and therefore an additional calibration error. This rotation can be eliminated, for low-order distortions, through coordinate transformations in fields with many sources, but might need to be considered for absolute astrometry and in sparse fields and its high spatial frequency distortion content needs to be investigated. Also, if the rotation error varies during an exposure, this causes image motion with all the effects that go along with it (reduction of Strehl, PSF elongation and coupling with variable effects).

Along similar lines, ADC positioning (rotation) errors also need to be considered. This has been analyzed and was found not to be critical.

We expect rotator errors to be less severe than the combination of the other opto-mechanical errors and assign a value of 5 μas for the error budget term with a more detailed analysis to follow.

Table 5. Astrometry error terms due to focal plane measurement errors in μas. Here, λ is the observing wavelength in μm and SNR is the signal-to-noise ratio. See text for assumptions made.

Focal Plane [μas]	Photon Noise	Det. Noise	Pixel Size	Pixel Irreg.	Detector Non-Lin.	Confusion	Mosaic
Section	6.1	6.2	6.3	6.4	6.5	6.6	6.7
Absolute Astrometry	$22\lambda 100/\mathrm{SNR}$	4	2	2	1	0	n/a
Differential Astrometry	$22\lambda 100/\mathrm{SNR}$	4	2	2	1	5	n/a
Differential Astrometry sparse	$22\lambda 100/\mathrm{SNR}$	4	2	2	1	0	n/a

5.6 Interaction between finite pupil distance and quasi-static errors (speckles, distortions)

Quasi-static errors originate from the fact that variable non-common path aberrations couple with the finite pupil distance of the second deformable mirror in NFIRAOS (DM11.2) to produce time-varying distortions. Our current results obtained through both simulations and a Fourier domain aberration analysis indicate that 5 μas accuracy should be achievable in fields with many sources, and 25 μas in sparse fields.

5.7 Effects due to stuck actuators and diffraction spikes

Secondary astrometry errors might enter the error budget through the effects of diffraction spikes or stuck actuators. While this has not been analyzed in detail yet, preliminary results suggest that these will not be major contributors to the error budget and a top-down value of 1 μas is allocated for all science cases.

5.8 Vibrations

The vibration error budget for TMT is not fully understood yet. It is expected that vibrations matter more for other science cases than for astrometry and that they will therefore not be a major contributor to the TMT astrometry error budget. We assign a placeholder value of 2 μas to this term for the time being.

6. FOCAL PLANE MEASUREMENT ERRORS

This section as well as Table 5 summarize errors and uncertainties that are caused by the accuracy with which we can measure the location of a source PSF on the detector in the absence of other error sources. Some of them are caused by the detector properties themselves, while others are due to the nature of of observed sources (such as star magnitudes and confusion), but manifest themselves in the focal plane.

6.1 Photon noise

The fundamental statistical one-dimensional position measurement uncertainty (in radians) caused by photon noise for an Airy function of infinite extent is given by

$$\sigma_{stat} = \frac{\lambda}{\pi D} \frac{1}{\mathrm{SNR}}, \tag{1}$$

where λ is the observation wavelength, D is the telescope diameter and SNR is the signal-to-noise ratio of the observation.[8] For $D = 30$ m, we get

$$\sigma_{stat}[\mu\mathrm{as}] \approx 22 \frac{\lambda}{1\mu\mathrm{m}} \frac{100}{\mathrm{SNR}} \tag{2}$$

which is the error shown in Table 5. In other words, for TMT, a SNR of 100 will give a statistical error of 22 μas at $\lambda = 1\mu$m and 48 μas in the K band.

6.2 Detector noise, flatfield and dark current calibration errors

Detector noise, flat fielding and dark current also need to be taken into account. Detector noise is a random error, while flat-fielding and dark current correction can cause random as well as systematic errors for astrometry. This term was investigated through several types of simulations. It was determined that it should be easy to keep this noise term below 4 μas.

6.3 Pixel size effect

The pixelization of the detector causes a random error, as determination of the center of the PSF for a discrete data set causes an uncertainty in addition to that of Eq. (1), which is derived using a continuous PSF. Trippe et al.[5] estimate this error to be 1 μas for MICADO on the E-ELT with 3 mas pixels and investigate its dependence on wavelength and pixel size. We have confirmed the behavior they found in our own simulations and adopt a value of 2 μas for IRIS on TMT (4 mas pixels). Note that this error increases up in crowded fields due to confusion. That effect is captured in Section 6.6.

6.4 Geometric stability of detector pixels

Pixel irregularities also contribute to the astrometry error, through both their shapes and intra-pixel sensitivity variations. These errors need to be assessed through careful, sub-pixel characterization of the detector. For the HAWAII-4RG detectors, Trippe et al.[5] estimate this error to be 2 μas but note that "any distortion calibration scheme must provide the ability to catch potential inaccuracies of the detectors." Lacking additional information at this time, we adopt this value for the TMT error budget as well.

6.5 Detector non-linearity calibration and saturation errors

The detector sensitivity is non-linear, especially as the fluxes get close to saturation. This non-linearity is usually carefully calibrated, but even a small calibration error can cause astrometric errors if the PSF is not centered exactly on a pixel or exactly between two adjacent pixels (which it usually will not be). It is interesting to note that this error increases with star brightness, contrary to most other errors. It should generally be small and we allocate a top-down budget value of 1 μas for it. As this is a calibration error of a non-variable quantity, this imposes a requirement on the accuracy of the detector response and/or on the maximum allowable flux in any pixel.

6.6 Confusion

In dense star fields, there often exist many faint stars on the cores and halos of the brighter stars that cannot be detected or resolved. This is known as confusion. Their fluxes are small compared to the brighter sources, but they can cause significant astrometric errors and uncertainties nevertheless. This error is systematic in the same field during the same epoch, but not necessarily of constant magnitude if, for example, atmospheric conditions or distortions vary between exposures. It is random between different fields and can become random for the same field if the sources move with respect to each other from epoch to epoch. As with many other errors, the best way to deal with this error is through high-order stable AO correction, such that as many of these background stars as possible can be resolved.

This error is generally negligible for absolute astrometry measurements (unless the reference sources are in a crowded part of the field) and differential astrometry in sparse fields. It is highly variable for dense fields. It was determined in simulations of astrometry of the Galactic center that it is smaller than 10 μas for bright stars (K < 15), but can become the dominant term for this science case for fainter stars.[3] As the Galactic center is an extremely crowded field, we adopt a value of 5 μas for our general (by definition not too crowded) science cases, with the understanding that it can be significantly larger for some other science cases.

6.7 Mosaicing errors

In some cases, either the science field or the field covering the reference sources is larger than the IRIS field. An example are the masers needed to get absolute values for stellar orbits at the Galactic center, as these masers span a field larger than the IRIS field of view. This requires individual images to be combined into a mosaic, causing an additional astrometric error. This error does not apply to our general astrometry cases in Table 5 which are assumed to use fields no larger than the IRIS detector and has therefore not been investigated yet, but it needs to be included for some specific science cases.

6.8 Focal surface tilt

A focal surface tilt would, among other things, cause Strehl ratios to vary across the field and thus change in time with rotation and image motion. This is, however, a secondary effect compared to other causes of Strehl ratio variation and is therefore not included as a separate term in our error budget.

	Differential Astrometry	
N_reference_sources	0	many
N_science_objects	2	many
	[uas]	[uas]
Reference catalog errors		
Position errors	33	3
Proper motion errors	27	3
Other or unknown motion	1	1
Color errors + variability	5	5
Differential aberration	0	0
	43.0	**6.6**
Atm. refraction correction		
Achromatic diff. refraction	33	2
Dispersion	15	15
Coupling with other effects	2	2
	36.6	**15.3**
Other atmospheric effects		
Residual atmospheric tip/tilt	0	0
Differential residual tip/tilt	1	0
Higher order residuals	11	11
PSF irregularity	6	5
Halo effect	0	10
Coupling with other effects	5	5
	12.9	**16.1**
Opto-mechanical errors		
Guide probe pos. (plate scale)	67	1
Imager optics calibration residual	8	8
Optical surface calibration resid.	7	7
Telescope optics calibration resid.	5	5
Rotator calibration errors	5	5
Pupil distance and QS errors	25	5
Stuck actuators, difraction spikes	1	1
Vibrations	2	2
	72.4	**13.9**
Focal-plane errors		
Photon noise etc.	24	24
Flatfield/dark current	4	4
Pixel size effect	2	2
Geometric stability of pixels	2	2
Detector non-linearity, saturation	1	1
Confusion	0	5
Mosaicing	n/a	n/a
	24.6	**25.1**
Total	**96**	**37**

Figure 1. Examples of possible TMT astrometry error budgets for two general differential astrometry science cases, one with a large number of science objects and reference sources, and one with only two science objects and no reference sources other than the three natural guide stars. Note that these values are very dependent on the exact type of observation, even within the same category. This table should only be used for getting an impression of the dominant error terms and their general magnitudes. See text for assumptions made.

7. TOTAL ASTROMETRY ERROR AND SUMMARY

The individual TMT astrometry error terms are captured in a spreadsheet and added in quadrature first for each error category, and then for the combination of all errors. Examples of possible error budgets for two differential science cases are shown in Fig. 1. The parameters used for these specific examples were: observation in the K band, a signal-to-noise ratio of 200, science field of view of 1 arcsec diameter, 100 references stars (for the middle column), an individual integration time of 100 seconds and an overall time span of 1 year from the first to the last epoch.

This demonstrates that there are indeed science cases for which, to the best of our knowledge at this time, TMT will be able to satisfy the astrometry requirements specified above. It should, however, also have become clear that these requirements cannot be met for all science cases. A detailed understanding of the conditions under which the smallest astrometric errors can be achieved and of the calibration, observation and data reduction procedures necessary for this is therefore essential for an astrometry program to achieve its maximum measurement accuracy.

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