

Simon *et al.* Reply: In our Letter¹ the electric fields of both the incident beam and the beam reflected from the mirror m_1 are described in the *same* transverse coordinate system (x, y) , by the pair of complex electric field components E_x, E_y arranged as a column. The projective space defined by the ratio $\xi = E_x/E_y$, with the points at infinity identified, is isomorphic to the Poincaré sphere; R and L correspond, respectively, to $\xi = \pm i$. The advantage of the single coordinate system is that the mirror leaves ξ unaffected and one indeed gets the closed circuit $ALBRA$, shown in Fig. 1. Bhandari's difficulty² originates partly from a possible lack of clarity in the specification of polarization states in our Letter and partly from a misconception that the return beam going through L will necessarily imply retracing of the circuit yielding a net zero geometric phase.

The argument presented by Bhandari amounts to choosing for the reflected beam a different coordinate system $(x_r, y_r) = (x \cos \theta + y \sin \theta, x \sin \theta - y \cos \theta)$ which has the same sense to the return propagation direction as (x, y) has to the forward propagation direction. Let us choose $\theta = 0$, so that $(x_r, y_r) = (x, -y)$. Since the incident beam at the mirror is linearly polarized at angle ϕ , the reflected beam is linearly polarized at $-\phi$ in (x_r, y_r) , and hence is represented by the point B' . Now the slow axis of the quarter-wave plate QWP2 makes an angle $\pi/4 - \phi$ in (x_r, y_r) and rotates B' about OQ , rather than about ON , and takes it to L . The slow axis of QWP1 makes an angle $3\pi/4$ rather than $\pi/4$ and rotates L about OM' and takes it to A . We thus get the circuit $ALBB'LA$ having the same sense and solid angle as $ALBRA$. Thus, while the return circuit goes through L and ends up at A , it does not imply that the circuit is retraced as naively concluded by Bhandari. Since the mirror maps B to B' , the return circuit starts from B' rather than B . This pinpoints the misconception leading to Bhandari's difficulty.

It is clear that insisting on two transverse coordinate systems which are pseudo-orthogonally related with arbitrary θ introduces two effects: a jump on the state space produced by the mirror action, and the circuit becoming an open one. The use of a single coordinate system as in our Letter results in a continuous closed circuit rendering the mirror action an identify transformation.

Finally, it is possible to associate not just two Poincaré spheres, one each with the incident and return beams, but a whole series of spheres, one for each transverse section of the beam. The point is that for any problem involving any number of *coplanar* propagation directions all the spheres can be mapped into just one sphere. If the propagation directions are not coplanar, then and only then does an additional geometric phase of the type

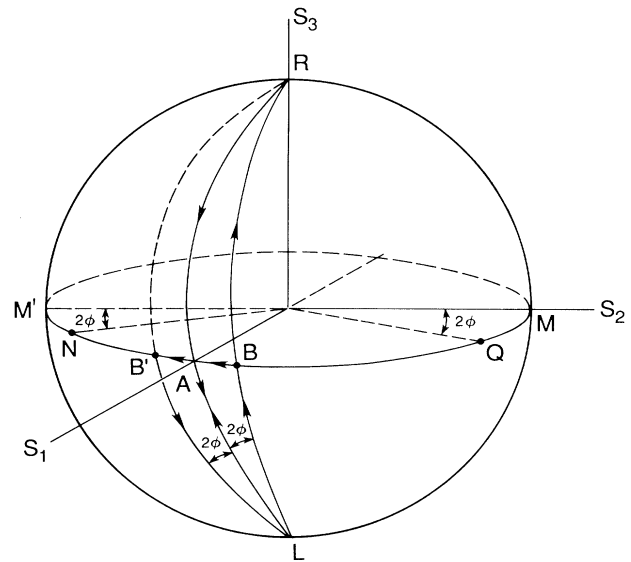


FIG. 1. The unit sphere acting as the projective space. The north pole R and south pole L correspond, respectively, to $\xi = \pm i$.

observed by Chiao *et al.*³ come into the picture. In this case the (local) direct product of the k space and the space of polarization states of the field takes the form of a fiber bundle with base space S^2 (k space) and fiber the Poincaré sphere. However, for the present problem it is unlikely that better insight is obtained by Bhandari's analysis which is restricted in validity to the 1D case and which employs a pair of spheres and open circuits for a problem which is more simply understood on a single sphere in terms of a closed circuit.

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³R. Y. Chiao, A. Antaramian, K. M. Ganga, H. Jiao, S. R. Wilkinson, and H. Nathel, Phys. Rev. Lett. **60**, 1315 (1988).