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Optimization of Q factor in optical nanocavities based on free-standing membranes

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ABSTRACT

We express the quality factor of a mode in terms of the Fourier transforms of its field components, and prove that the reduction in radiation loss can be achieved by suppressing the mode's wave-vector components within the light cone. Although this is intuitively clear, our analytical proof gives us insight into how to achieve the Q factor optimization, without the mode delocalization. We focus on the dipole defect mode in free standing membrane and achieve $Q > 10^4$, while preserving the mode volume of the order of one half of cubic wavelength in material. The derived expressions and conclusions can be used in optimization of Q factor for any type of defect in planar photonic crystals.

Keywords: Q factor, optical resonators, integrated optics, optoelectronic devices, optics, FDTD methods, Fourier transforms, microcavity, photonic crystals, photonic bandgap.

1. INTRODUCTION

One of the most important properties of photonic crystals is their ability to localize light into small mode volumes. Even the simplest single defect microcavities in planar photonic crystals with triangular lattice, produced by changing the radius or refractive index of a single hole or rod, can localize light into the volumes as small as one half of cubic wavelength in material. Unfortunately, these most obvious microcavity designs have maximum quality factors of the order of only a few thousand.[1–4] Our group at Caltech has recently proposed the design and fabrication of optical microcavities based on free-standing membranes, with $Q > 10^4$ and mode volumes of the order of one half of cubic wavelength of light in material.[2,5] We have also demonstrated the experimental Q factor of 2800 in this type of structure, for which the theoretically predicted Q was around 4000.[6] In our earlier work,[2] we have only briefly addressed the mechanism behind the reduction of radiation loss in our structures: the suppression of wave-vector components of the defect mode that are positioned within the light cone. In this article, we discuss this phenomenon in details and derive the analytic expression relating the Q factor of a mode to the Fourier transform of the mode pattern. We also show how to suppress the wave-vector components within the light cone, without delocalizing a mode. Although our study focuses only on the dipole mode, the derived relations are universal and conclusions can be used in the optimization of Q factor for any type of mode and defect in planar photonic crystals.

2. EXPRESSING THE Q FACTOR OF A MODE IN TERMS OF THE FOURIER TRANSFORM OF ITS FIELD PATTERN

The detailed derivation of the relation between the Q factor and the Fourier transform of a mode will appear in our forthcoming article.[7] Briefly, the 3D FDTD analysis can provide us with the near field distribution of the analyzed microcavity. The FDTD analysis of the far field would require large amounts of computer memory, and would be very computationally inefficient. However, we can compute the far field starting from the known near field distribution. Any wavefront can be considered as a source of secondary waves that add to produce distant wavefronts, according to Huyggens principle. Let us assume that we know the field distribution across the surface S, positioned in the near field and above the free standing membrane. For example, let S be the plane positioned at $z = 0$, in parallel to the surface of the membrane, and at a small distance Δz from it. This choice of surface S will allow us to relate the Q factor of a mode to the Fourier transform of its field pattern. We also assume that the medium above S is

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homogeneous and isotropic. The far fields can be considered as arising from the equivalent current sheets at the surface S, and we can calculate the far field and the total averaged radiated power into the half-space above the plane S:[8]

$$
\begin{aligned}
P_1 &= \frac{\eta}{8\lambda^2} \int_0^{\pi/2} \int_0^{2\pi} d_\theta d_\phi sin(\theta) K(\theta, \phi) \\
K(\theta, \phi) &= \left| N_\theta + \frac{L_\phi}{\eta} \right|^2 + \left| N_\phi - \frac{L_\theta}{\eta} \right|^2 \\
\eta &= \sqrt{\frac{\mu_o}{\epsilon_o}},
\end{aligned}
\tag{1}
$$

where $\vec{N}$ and $\vec{L}$ represent radiation vectors. It can be proved that components of radiation vectors are proportional to Fourier transforms of tangential field components at the plane S:[7]

$$N_x = -FT_2(H_y)\Big|_{\vec{k}_{||}} \tag{2}$$

$$N_y = FT_2(H_x)\Big|_{\vec{k}_{||}} \tag{3}$$

$$L_x = FT_2(E_y)\Big|_{\vec{k}_{||}} \tag{4}$$

$$L_y = -FT_2(E_x)\Big|_{\vec{k}_{||}} \tag{5}$$

$$\vec{k}_{||} = \frac{2\pi}{\lambda} sin\theta(\hat{x} cos\phi + \hat{y} sin\phi) \tag{6}$$

$$FT_2(f(x,y)) = \iint_S d_x d_y f(x,y) e^{i(k_x x + k_y y)}, \tag{7}$$

Therefore, just by knowing the Fourier transforms of the tangential field components at the plane S, we can evaluate the time averaged radiated power. From the previous expressions it is clear that the wave-vector of interest $\vec{k}_{||}$ lies within the light cone for any values of angles θ and ϕ in the circular polar coordinate system (i.e. $|\vec{k}_{||}| \leq \frac{2\pi}{\lambda}$). This implies that the radiated power P depends only on the wave-vector components located within the light cone. Hence, the reduction in radiation loss and improvement in Q factor can be achieved by suppressing the Fourier components within the light cone, or by redistributing them outside the light cone.
In the case when most of the radiated power is collected at vertical incidence (i.e., at small θ), the expression 1 can be simplified as follows:

$$
\begin{aligned}
P_2 &= \frac{\eta}{8\lambda^2 k^2} \iint_{|\vec{k}_{||}| \leq k} d\vec{k} \left(\left| N_x + \frac{L_y}{\eta} \right|^2 + \left| N_y - \frac{L_x}{\eta} \right|^2 \right) \\
&= \frac{\eta}{8\lambda^2 k^2} \iint_{|\vec{k}_{||}| \leq k} d\vec{k} \left(\left| FT_2(H_y) + \frac{1}{\eta} FT_2(E_x) \right|^2 + \left| FT_2(H_x) - \frac{1}{\eta} FT_2(E_y) \right|^2 \right)
\end{aligned}
\tag{8}
$$

The integral of the cross-terms in the equation 8 gives approximately one half of the radiated power. This can be proved easily starting from the expansion of fields in terms of the Fourier components and the expression for the radiated power as the integral of the z component of the Poynting vector $\vec{\Gamma}$ over the surface S. This leads to the following expression for the averaged radiated power:

$$P_3 = 2\frac{\eta}{8\lambda^2 k^2} \iint\limits_{|\vec{k}_{||}|\leq k} d\vec{k}\left(\left|FT_2(H_x)\right|^2 + |FT_2(H_y)|^2 + \frac{1}{\eta^2}|FT_2(E_x)|^2 + \frac{1}{\eta^2}|FT_2(E_y)|^2\right) \tag{9}$$

It is important to note that if some field component $u(x,y)$ is odd with respect to the x coordinate (i.e., $u(x,y) = -u(-x,y)$), then its Fourier transform must be equal to zero for any point in the Fourier space with $k_x = 0$. Similarly, any field component which is odd with respect to the y coordinate has a Fourier transform which is zero for any point with $k_y = 0$.

Let us introduce the radiation factor RF which is directly proportional to the radiated power P:

$$RF_i = \frac{P_i}{W}, i = 1, 2, 3, ... \tag{10}$$

where W represents the total energy of a mode in the half-space above the middle of the membrane. The radiation Q factor of a mode (which is a measure of the radiation, out-of-plane loss) can be expressed as:

$$Q = \omega\frac{W}{P} = \frac{\omega}{RF}. \tag{11}$$

3. HIGH Q OPTICAL NANOCAVITIES IN FREE STANDING MEMBRANES

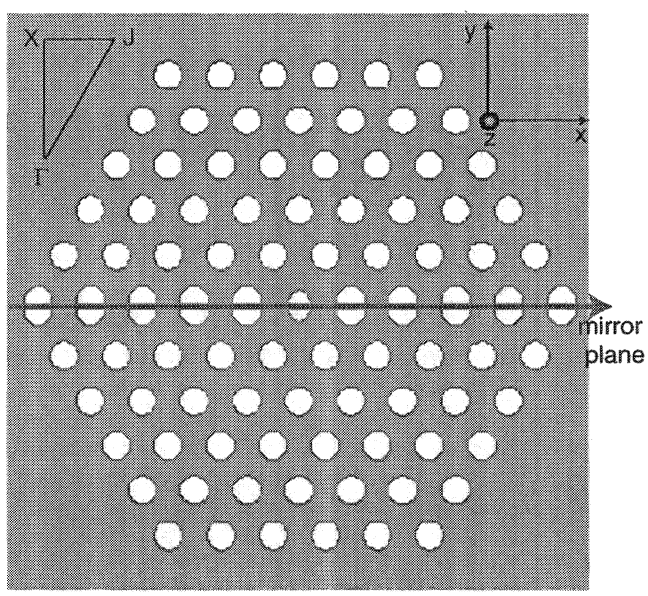

Figure 1. Microcavity structure consisting of a single defect (produced by reducing the radius of the central hole to $r_{def}/a = 0.2$ from $r/a = 0.275$) and a fractional edge dislocation of order $p = 4$ along the x axis. The applied discretization is 20 pixels per periodicity a.

We have recently proposed the design and fabrication of optical microcavities in free standing membranes with $Q > 10^4$ for the dipole mode, and mode volumes of the order of one half of cubic wavelength of light (measured in material).[2,5] The dramatic improvement in Q factors over single defect microcavities (without a significant increase

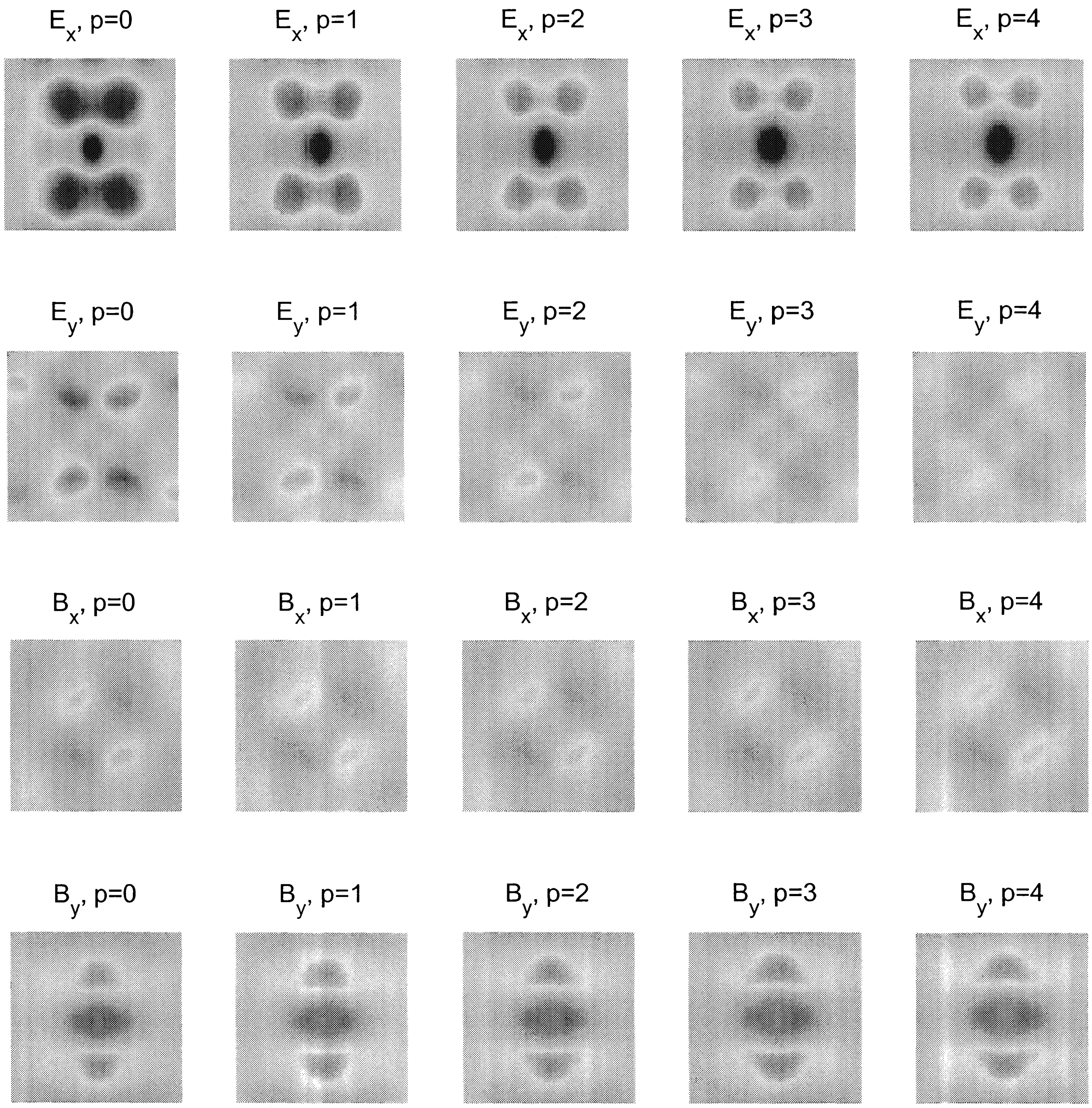

Figure 2. Field components of the x-dipole mode at the surface S positioned at approximately $d/4$ from the surface of the membrane. The analyzed structure is shown in Figure 1.

in the mode volume) was obtained by introducing a novel type of photonic crystal lattice defect, consisting of the elongation of holes along the symmetry axes. We call this type of defect a *fractional edge dislocation*, by analogy with edge dislocations in solid state physics. Edge dislocations are formed by introducing extra atomic planes into the crystal lattice. On the other hand, we here insert only fractions of atomic planes along the symmetry axes of photonic crystal, as shown in Figure 1. Hole-to-hole distances are preserved under this deformation, and the half-spaces $y > p/2$ and $y < -p/2$ maintain the unperturbed photonic crystal geometry.

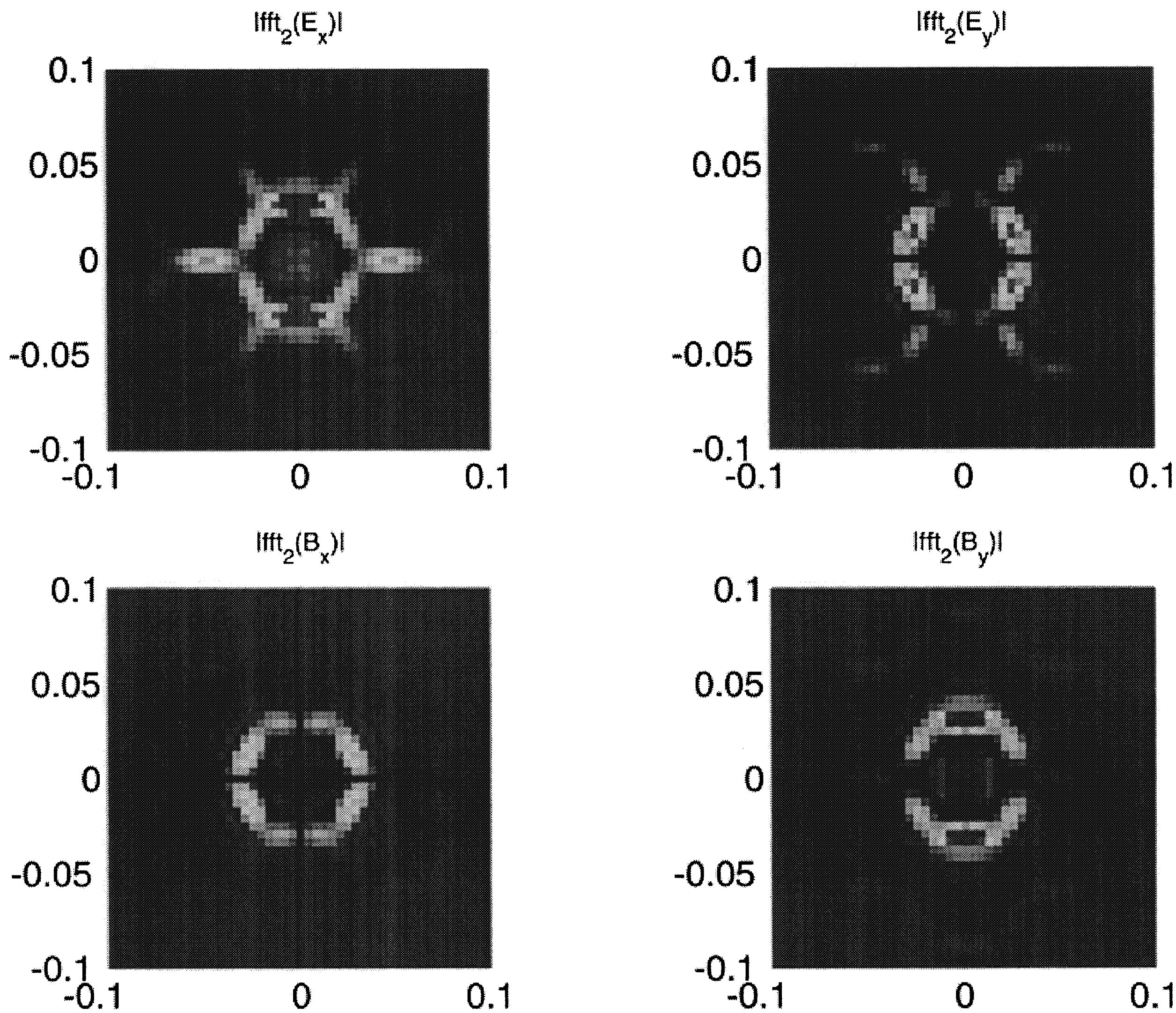

Figure 3. Fourier components of the x-dipole mode in the structure from Figure 1. A fractional edge dislocation is of the order $p = 0$ in this case. The light cone can be respresented as a disk with the radius approximately equal to 0.015. The horizontal and vertical axes correspond to $\frac{k_x}{2\pi}$ and $\frac{k_y}{2\pi}$, respectively.

We consider again some of the microcavities that we proposed in Reference 2. The unperturbed photonic crystal parameters are $r/a = 0.275$, $d/a = 0.75$ and $n = 3.4$, where r, a, d and n represent the hole radius, the periodicity of the triangular lattice, the thickness of the slab, and the refractive index of the semiconductor material, respectively. The choice of photonic crystal parameters is discussed in more details in our previous work.[2,9] In the FDTD method, we apply the discretization of 20 pixels per periodicity a. Therefore, a fractional edge dislocation of order $p = 1$ corresponds to the insertion of extra material whose thickness is equal to $1/20a$. In the microcavity of our interest, the central hole radius is decreased to $r_{def}/a = 0.2$ and a fractional edge dislocation of order p is applied along the x axis, as shown in Figure 1.

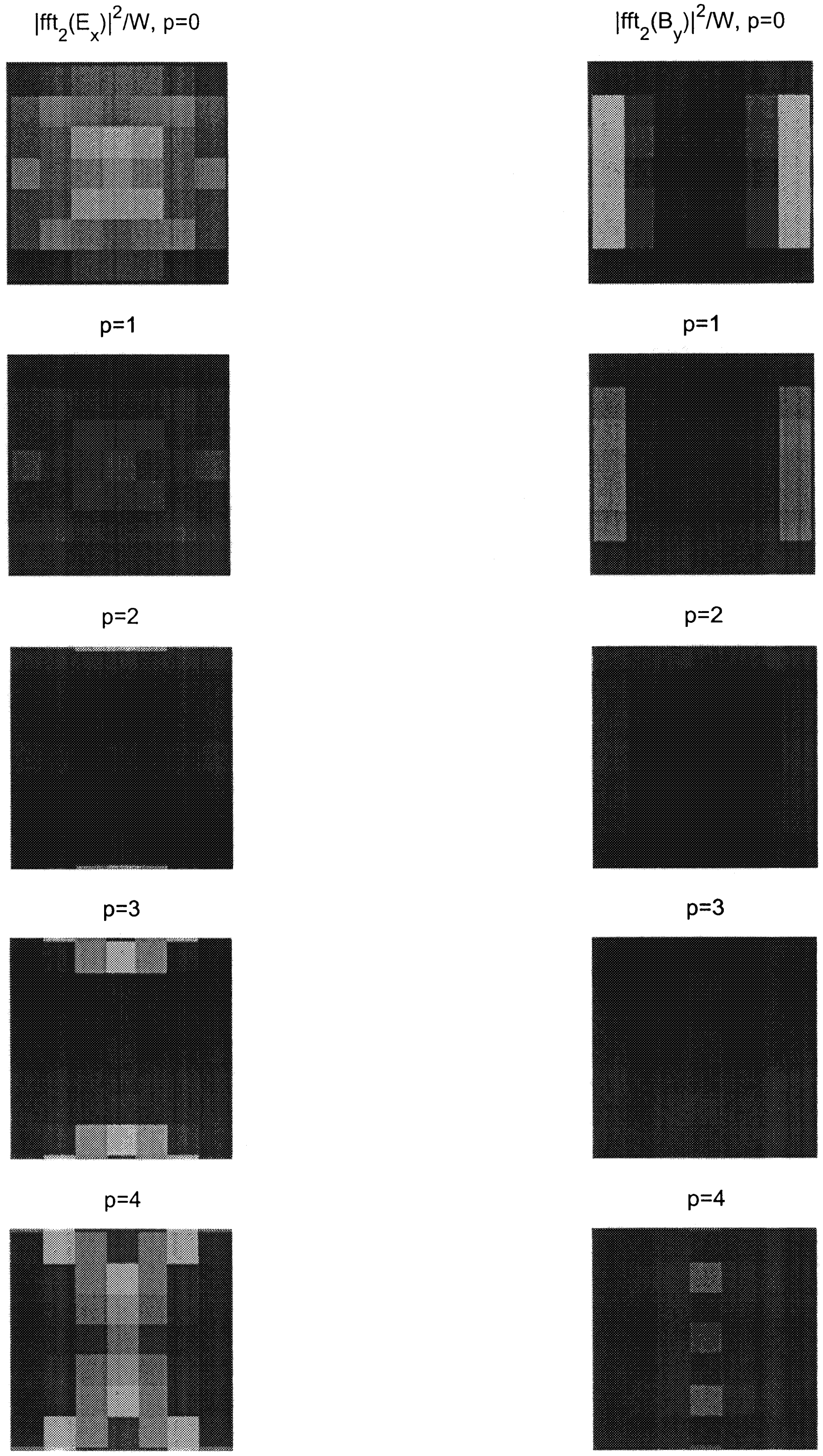

Figure 4. 2D Fourier transforms of the even field components of the x-dipole mode in the structure shown in Figure 1, as a function of the elongation parameter p. The light cone can be respresented as a disk inscribed into the square. Clearly, the intensities of Fourier transforms within the light cone are minimized for $p = 2$, where Q factor reaches its maximum.

Field components of the x-dipole mode in the analyzed structure are shown in Figure 2, as a function of the elongation parameter p. For the x-dipole mode, E_x and B_y components are even, while E_y and B_x components are odd with respect to both symmetry axes x and y. Therefore, it is expected that E_y and B_x (i.e., L_x and N_y) do not contribute significantly to the radiated power in this case, since their Fourier transforms are equal to zero along both k_x and k_y axes. This is also illustrated in Figure 3. Therefore, in the case of the analyzed x-dipole mode we can approximate the expression 9 even further:

$$P_4 = 2\frac{\eta}{8\lambda^2 k^2} \iint\limits_{|\vec{k}_{||}| \le k} d_{k_x} d_{k_y} \left(\left| FT_2(H_y) \right|^2 + \frac{1}{\eta^2} \left| FT_2(E_x) \right|^2 \right) \tag{12}$$

In order to minimize the radiated power, it is necessary to minimize (within the light cone) the Fourier transforms of the even field components E_x and B_y. In general case, these Fourier transforms are non-zero at small values of $|\vec{k}_{||}|$ (i.e., in the light cone). However, they can be minimized by balancing the intensities of positive and negative field lobes. Indeed, we can observe in Figure 2 that by varying the elongation parameter p, we also tune the sizes of the central (negative) lobes in E_x and B_y, as well as the intensity distribution between the positive and negative lobes. Therefore, the tuning in p is expected to lead to tuning in Fourier transforms of the even field components, and subsequently to tuning in radiated powers.

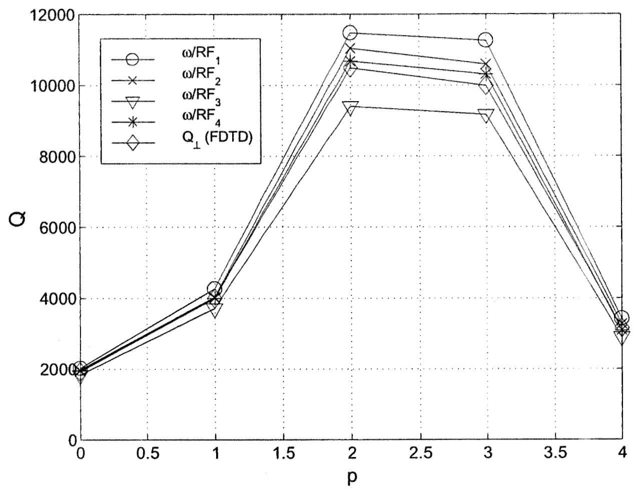

Figure 5. Q factors estimated from the FDTD, or from the Fourier transforms of the tangential field components. The plane S is positioned at a distance equal to $\lambda/2$ from the surface of the membrane.

The Fourier components of the x-dipole mode in the structure with $p = 0$ are shown in Figure 3. When the elongation parameter p changes in the analyzed range from 0 to 4, peaks in the Fourier space preserve their position, but their intensities are tuned. This can be observed in Figure 4. Clearly, Fourier components within the light cone are minimized for $p = 2$, where Q factor reaches its maximum. *Therefore, the optimization of Q factor of the dipole mode (after the application of fractional edge dislocations) is a result of suppression of the wave-vector components within the light cone. This suppression is a product of balancing between the energies of the positive and negative field lobes of the even field components. The Q factor optimization is achieved in this case without a significant mode delocalization.*

In our FDTD calculations,[2] the total Q factor is separated into the lateral ($Q_{||}$) and vertical ($Q_\perp$) quality factors. $Q_\perp$ is a measure of radiation loss, while $Q_{||}$ corresponds to the loss through mirrors in lateral directions, which can be reduced by adding more layers of photonic crystal. The boundary for separation of vertical from lateral loss (i.e., $Q_\perp$ from $Q_{||}$) is positioned approximately at $\lambda/2$ from the surface of the membrane, as suggested in our early work.[10] We have discussed in our recent publication[2] that this choice of boundary excludes some small portion of radiation loss from $Q_\perp$, and the total Q factor of the analyzed dipole mode achievable by increasing the number of the PC layers around the defect (also referred to as the limit of total Q factor) is somewhat smaller than $Q_\perp$.

The radiation Q factors are evaluated using the method presented in this article, and results are shown in Figure 5. The plane S (above which we integrate the radiated power) is positioned at $\lambda/2$ above the surface of the membrane, same as the boundary for separation of $Q_\perp$ from $Q_{||}$. A very good agreement with $Q_\perp$ from the FDTD simulations is observed. However, $Q_\perp$ is somewhat smaller, due to numerical inaccuracy. Radiation factors RF_2, RF_3 and RF_4 are estimated under the assumption that most of the radiation is collected at vertical incidence. This is not really true in the case of the x-dipole, for which reason there is an offset between the Q factors evaluated from $RF_i, i = 2,3,4$ and Q estimated from RF_1, which does not make any assumptions on the direction of radiation.

4. CONCLUSION

We have presented a method for estimating the Q factor of a mode and its radiation loss from the known Fourier transform of the near field distribution. By applying this approach to high Q structures that we have proposed recently,[2] we have proven that the optimization of the Q factor of the dipole defect mode (after the application of fractional edge dislocations) results from the suppression of the wave-vector components within the light cone. This suppression is a result of balancing between the positive and negative lobes in the even field components. Although our analysis focuses on the dipole mode only, similar approach can be applied to any type of microcavity formed in planar photonic crystals.

5. ACKNOWLEDGMENTS

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REFERENCES

1. J. Vučković, M. Lončar, and A. Scherer, "Design of photonic crystal optical microcavities," *Proceedings of the SPIE meeting Photonics West 2001, San Jose* , Jan. 2001.
2. J. Vučković, M. Lončar, H. Mabuchi, and A. Scherer, "Design of photonic crystal microcavities for cavity QED," *Physical Review E* **65**, p. 016608, Jan. 2001.
3. S. Johnson, S. Fan, A. Mekis, and J. Joannopoulos, "Multipole-cancellation mechanism for high-Q cavities in the absence of a complete photonic bandgap," *Applied Physics Letters* **78**, pp. 3388–3390, May 2001.
4. E. Miyai and K. Sakoda, "Quality factor for localized defect modes in a photonic crystal slab upon a low-index dielectric substrate," *Optics Letters* **26**, pp. 740–742, May 2001.
5. J. Vučković, M. Lončar, H. Mabuchi, and A. Scherer, "Photonic crystal microcavities for strong coupling between an atom and the cavity field," *Proceedings of the LEOS 2000, Rio Grande, Puerto Rico* , Nov. 2000.
6. T. Yoshie, J. Vučković, A. Scherer, H. Chen, and D. Deppe, "High quality two-dimensional photonic crystal slab cavities," *Applied Physics Letters* **79**, pp. 4289–4291, Dec. 2001.
7. J. Vučković, M. Lončar, H. Mabuchi, and A. Scherer, "Optimization of Q-factor in microcavities based on free-standing membranes," *To appear in IEEE Journal of Quantum Electronics* , 2002.
8. S. Ramo, J. Whinnery, and T. V. Duzer, *Fields and waves in communication electronics*, John Wiley and Sons, Inc., New York, 1994.
9. J. Vučković, *Photonic crystal structures for efficient localization or extraction of light*, Ph.D. Thesis, California Institute of Technology, 2002.
10. O. Painter, J. Vučković, and A. Scherer, "Defect Modes of a Two-Dimensional Photonic Crystal in an Optically Thin Dielectric Slab," *Journal of the Optical Society of America B* **16**, pp. 275–285, Feb. 1999.