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Geometrical gauge theory of ghost and Goldstone fields and of ghost symmetries

(principal bundle/renormalization/supergroup/spontaneous symmetry breakdown/unified electro-weak interactions)

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ABSTRACT We provide a geometrical identification of the ghost fields, essential to the renormalization procedure in the non-Abelian (Yang–Mills) case. These are some of the local components of a connection on a principal bundle. They multiply the differentials of coordinates spanning directions orthogonal to those of a given section, whereas the Yang–Mills potential multiplies the coordinates in the section itself. In the case of a supergroup, the ghosts become commutative for the odd directions, and represent Nambu–Goldstone fields. We apply the results to chiral "flavor" $SU(3)_L \times SU(3)_R$ and to $SU(2/1)$. The latter reproduces a highly constrained Weinberg-Salam model.

It has been known since 1963 (1) that a principal fiber bundle provides a precise geometrical representation of Yang–Mills gauge theories. After 1975 (2), this correspondence has been extensively applied to the study of self-dual solutions of the Yang–Mills equation (monopoles, instantons) and of global properties of the bundle, etc.

We present here an entirely different domain of applications. First, we reproduce the recently suggested (3–5) identification of the Feynman–DeWitt–Faddeev–Popov ghost fields (6) essential to the renormalization procedure in the non-Abelian case, with local geometrical objects in the principal bundle. This will directly yield the Becchi-Rouet-Stora (BRS) equations (7) guaranteeing unitarity and Slavnov–Taylor invariance (8, 9) of the quantum effective Lagrangian. Except for the "antighost" variation, this quantum-motivated symmetry thus corresponds to "classical" (geometrical) notions, with its dependence on the gauge-fixing procedure (which determines the quantized Lagrangian) limited to section dependence, a mere choice of gauge.

We then consider the case of a supergroup (10) as an internal symmetry gauge, generalizing the recently suggested (11) role of $SU(2/1)$. We show how the ghosts geometrically associated to odd generators (12) may be identified with the Goldstone–Nambu (13) scalar fields of conventional models with spontaneous symmetry breakdown. As an example, we realize the chiral $SU(3)_L \times SU(3)_R$ "flavor" symmetry (14–16) by gauging the supergroup $Q(3)$ (see refs. 10 and 12).

Lastly, we recall some of the more relevant results concerning asthenodynamics (weak electromagnetic unification) as given (11) by the ghost-gauge $SU(2/1)$ supergroup.

Connections on a principal bundle: Gauge (potentials) and ghost fields

We start by reintroducing (17, 18) the concept of a connection on a principal fiber bundle $(P,M,\pi G,\cdot)$. Previous authors used definitions in which the connection (a one-form $\omega^a{}_{(YM)}$) was restricted to the base manifold M of dimension $m = 4$, so that writing

$$\omega^a{}_{(YM)} = \omega^a_\mu dx^\mu \quad (a = 1 \ldots n, \mu = 0, 1, \ldots 3) \qquad [1]$$

the ω^a_μ were identified with the Yang–Mills potentials. In our treatment, the connection ω has $m + n$ dimensions holonomically, ω^a_R $(R = \mu, i; \mu = 0, \ldots 3; i = 1, \ldots n)$, quite aside from the n components described by the a index and contracted with the abstract Lie algebra matrices λ_a.

We denote the (vertical) projection by $\pi:P \rightarrow M$, the structure group by G, and right-multiplication on P by the dot $(\cdot):P \times G \rightarrow P$, so that

$$\forall p \in P, \ \forall g,g' \in G, \begin{Bmatrix} \pi(p\cdot g) = \pi(p) \\ (p\cdot g)\cdot g' = p\cdot(gg') \end{Bmatrix} \qquad [2]$$

and for U_x, a neighborhood of $x \in M$, we get "local triviality" (a direct product) in P.

The dot $(\cdot)$ induces a map t from the Lie algebra A of G into P_*, the tangent manifold to P. Thus, for $\forall \lambda_a, \lambda_b, \lambda_e \in A$ $(a,b,e = 1 \ldots n)$ with

$$[\lambda_a,\lambda_b] = C^e{}_{ab}\lambda_e \qquad [3]$$

we have

$$t:A \rightarrow P_*, \quad \lambda \rightarrow \tilde{\lambda} \in P_*. \qquad [4]$$

One proves that t is a homomorphism of A, with the Lie bracket (LB) operation realized on P_* as a Poisson bracket (PB)

$$\widetilde{[\lambda,\lambda']}_{LB} = [\tilde{\lambda},\tilde{\lambda}']_{PB}. \qquad [5]$$

However, this map t has no inverse because the image of A (of dimension n) does not span P_*, of dimension $(n + m)$.

A linear mapping from P_* to A, *the connection* ω, is now chosen to provide the missing inverse

$$\omega:P_* \rightarrow A, \quad \forall\lambda \in A, \quad \omega(\tilde{\lambda}) = \lambda. \qquad [6]$$

ω is Lie-algebra valued, and belongs to the cotangent manifold $^*P$. It is thus a one-form. If z^R are local coordinates over P, one may explicitly write

$$\left.\begin{aligned} &\forall v \in P_*, \quad v = v^R(z)\frac{\partial}{\partial z^R} \\ &\qquad (R,S = 1, 2, \ldots n + m) \\ &\omega = \omega^a{}_S(z)\, dz^S\lambda_a \\ &\omega(v) = v \rfloor \omega = \omega^a_R v^R \lambda_a \equiv \omega^a(v)\lambda_a \end{aligned}\right\} \qquad [7]$$

($\rfloor$ denotes a contraction, $\partial/\partial z^R \rfloor dz^S = \delta^S_R$). For $\tilde{\lambda}_b$ as v, we have $\omega^a(\tilde{\lambda}_b) = \delta^a_b$.

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Abbreviations: LB, Lie bracket; PB, Poisson bracket; BRS, Becchi-Rouet-Stora.

Because P_* is larger than A, there is a nontrivial kernel H of ω. In other words, to each point $p \in P$, ω associates a subspace $H_p \subset P_{*p}$. This is known as the "horizontal" tangent vector space at p, and defines an exact splitting of P_*:

$$h \in H_p \Leftrightarrow \omega_p(h) = 0$$

$$P_{*p} = V_p + H_p, \quad H_p = \mathrm{Ker}\,(\omega_p) \qquad [8a]$$

$$V_p = \mathrm{Im}_t(A), \quad (\tilde{\lambda})_p \in V_p.$$

One also assumes an equivariance condition

$$H_{p \cdot g} = H_p \cdot g. \qquad [9a]$$

We now introduce the Lie derivative ∇_v or convective derivative along a vector field v in P_* [7]. Its action on functions, vector fields, and one-forms reads:

$$\nabla_v f(z) = v^R \frac{\partial}{\partial z^R} f \qquad [10a]$$

$$\nabla_v v' = [v,v']_{\mathrm{PB}} \qquad [10b]$$

$$\nabla_v \omega = d(v \rfloor \omega) + v \rfloor d\omega. \qquad [10c]$$

The equivariance condition [9a] can be written infinitesimally as

$$\nabla_{\tilde{\lambda}} h \rfloor \omega = 0. \qquad [9b]$$

Taking the Lie derivative of [8a], we have

$$\nabla_{\tilde{\lambda}}(h \rfloor \omega) = \nabla_{\tilde{\lambda}} h \rfloor \omega + h \rfloor \nabla_{\tilde{\lambda}} \omega = 0,$$

yielding by [9b]

$$h \rfloor \nabla_{\tilde{\lambda}} \omega = 0$$

(i.e., $\nabla_{\tilde{\lambda}}\omega$ is vertical). It can thus be rewritten with a linear representation and a factorized z dependence,

$$\nabla_{\tilde{\lambda}} \omega = \dot{f}(z)[\lambda,\omega]_{\mathrm{LB}}. \qquad [8b]$$

To fix f(z) we take the Lie derivative of [6]:

$$\nabla_{\tilde{\lambda}} \omega(\tilde{\lambda}') = [\tilde{\lambda},\tilde{\lambda}']_{\mathrm{PB}} \rfloor \omega + \tilde{\lambda}' \rfloor \nabla_{\tilde{\lambda}} \omega = 0,$$

which vanishes because $\omega(\tilde{\lambda}') = \lambda'$, a constant. Replacing the last term by [8b] and using [5], we find that $f(z) = -1$. The equivariance condition can thus be stated as

$$\nabla_{\tilde{\lambda}} \omega = -[\lambda,\omega]_{\mathrm{LB}}. \qquad [9c]$$

We now define the curvature two-form (17, 18)

$$\Omega = d\omega + \frac{1}{2}[\omega,\omega] \qquad [11]$$

and contract it with a vertical vector field $\tilde{\lambda}$

$$\tilde{\lambda} \rfloor \Omega = \tilde{\lambda} \rfloor d\omega + \frac{1}{2}[\tilde{\lambda} \rfloor \omega,\omega] - \frac{1}{2}[\omega,\tilde{\lambda} \rfloor \omega].$$

The first term is given by [10c], the last two are given by [6]

$$= \nabla_{\tilde{\lambda}} \omega + \frac{1}{2}[\lambda,\omega] - \frac{1}{2}[\omega,\lambda],$$

and, when [9c] replaces the first term, the expression vanishes. The *curvature two-form is thus purely horizontal,*

$$\tilde{\lambda} \rfloor \Omega = 0. \qquad [12]$$

This equation is the *Cartan-Maurer structural equation of a principal fiber bundle.* Up to this point, we have just used textbook geometry. We can now identify the ghost fields.

Because we are in P_*, a gauge choice corresponds locally to defining a section—i.e., a surface Σ in P—locally diffeomorphic to the base manifold M. We fit the z^R coordinates to Σ by lifting local x^μ coordinates from the base M, and α^i (group parameters) coordinates from G, by using the maps (π^{-1}, τ^{-1}) with τ a projection onto the fiber to get the equation for Σ:

$$\Sigma{:}\alpha^i(x) = 0, \quad i = 1, \ldots n. \qquad [13]$$

We now express the vertical connection form ω in this basis

$$\frac{\partial}{\partial x^\mu} \rfloor \omega = \Phi_\mu, \quad \frac{\partial}{\partial \alpha^i} \rfloor \omega = X_i$$

$$\omega = X_i d\alpha^i + \Phi_\mu dx^\mu. \qquad [14]$$

We identify the ghost (6) fields C^a as (ℓ is a constant length)

$$\ell C^a = X_i^a d\alpha^i \qquad [15]$$

while φ_μ^a is the Yang—Mills potential.

According to [7], had we taken a topologically trivial P and a global flat section, $C^a{}_{(0)}$ would have coincided explicitly with the Cartan left-invariant one-forms of the rigid group. It would then carry no x^μ dependence and would not be a true field. However, under a gauge transformation,

$$\delta\omega^a(x,\alpha) = D\epsilon^a(x,\alpha) \qquad [16]$$

so that $C^a{}_{(0)} = 1/\ell(\alpha^{-1}\,d\alpha)^a$ receives x^μ-dependent contributions,

$$\delta C^a = \frac{1}{\ell} d\alpha^i \left[\frac{\partial}{\partial \alpha^i} \epsilon^a(x,\alpha)\right] - \frac{1}{\ell} C^a{}_{be} C^b \epsilon^e(x,\alpha). \qquad [17]$$

We now rewrite Ω of [11] in component form. Defining "partial" exterior derivatives

$$df = sf + \bar{d}f; \quad sf = d\alpha^i \frac{\partial}{\partial \alpha^i} f; \quad \bar{d}f = dx^\mu \frac{\partial}{\partial x^\mu} f. \qquad [18]$$

Cohomology implies

$$\bar{d}^2 = sd + \bar{d}s = s^2 = 0. \qquad [19]$$

$\bar{d}$ is our "ordinary" horizontal d which depends on the section Σ and s is the exterior differential normal to the section. Ω can be broken into three pieces—i.e., terms in $d\alpha^i \wedge d\alpha^j$, in $d\alpha^i \wedge dx^\mu$ and in $dx^\mu \wedge dx^\nu$. Applying [12] implies the vanishing of the first two pieces. By [14, 15, 18] we have

$$sC^a = -\frac{\ell}{2}[C,C]^a \qquad [20]$$

$$s\Phi^a = \ell \bar{D}_\mu C^a. \qquad [21]$$

These are the BRS equations (7) for Φ_μ^a and C^a. $\ell^{-1}s$ is thus the BRS operator.

One of us (J.T.-M.) has shown (14) how the covariant quantization path integral, used in summing over all configurations of the potential satisfying BRS, can be given a geometrical form. In this representation, Feynman diagrams involve nonintegrated exterior forms (the ghosts) together with anticommuting Lagrange multipliers (the antighosts). One can then check that the minus sign required by ghost loops, which led to the assignment of Fermi statistics to spin-zero fields $C^a(x)$, *is indeed just the sign due to self anticommutation of one-forms.*

Nambu–Goldstone fields

When the Lie group G is replaced by a Lie supergroup (10) and the Lie algebra A by a graded Lie algebra (12), some connection one-forms commute instead of anticommuting.

For an internal graded Lie algebra, the one-forms obey a $Z(2) \times Z(2)$ gradation (18, 19)

$$\eta^{pa} \wedge \xi^{qb} = (-1)^{pq+AB} \xi^{qb} \wedge \eta^{pa}, \qquad [22]$$

where η^{pa} and ξ^{qb} are, respectively, a p-form and a q-form, the indices a and b represent a basis of a graded Lie algebra, and A and B are their respective gradings. The connections

$$\omega^i = G^i_\mu dx^\mu + \phi^i \quad [23]$$

thus commute when i represents an odd-grading in the graded Lie algebra ($A = 1$). *ϕ^i is thus a Lorentz-scalar physical Bose field.* We have recently conjectured (11) that these fields be identified with Nambu–Goldstone (Higgs–Kibble) fields when the Weinberg–Salam (20, 21) model's $SU(2) \times U(1)$ gauge group is embedded in the supergroup $SU(2/1)$. The internal supergroup represents a ghost symmetry (i.e., a symmetry between physical and ghost fields) because it changes the statistics without changing the spins. The Goldstone–Nambu (or, after further spontaneous breakdown, the Higgs fields) thus become in this approach the appropriate gauge fields for the odd part of the ghost symmetry.

In the study of Goldstone-type realizations of *global* symmetries, the Goldstone field corresponded to that part of the invariance group that was not a symmetry of the vacuum and could thus not be realized linearly on single-particle-state multiplets. It is indeed instructive (15, 16) to choose as an example the one case of that type we understood between 1960 and 1967: the pion's (and 0^- octet) role as the zero-mass Goldstone particle in chiral $W(3)_{\mathrm{ch}} = SU(3)_{\mathrm{L}} \otimes SU(3)_{\mathrm{R}}$.

In the nonlinear picture (22, 23), the vacuum is invariant under the positive parity $SU(3) \subset W(3)$ charges X^+. The remaining *8* of generators [under that $SU(3)$] corresponding to the axial-vector charges X^- is realized nonlinearly. The *8* of 0^- mesons η acts as realizer,

$$\exp(-i\eta \cdot X^-)(0,\psi) = (\eta,\psi). \quad [24]$$

The η are in fact parameters of the axial generators. We denote the more common parameter of the (linear) vector subgroup by α.

For a generic element g of $W(3)$ we get

$$g^{-1}\exp(-i\eta \cdot X^-) = \exp(-i\eta' \cdot X^-)\exp(-i\alpha \cdot X^+), \quad [25]$$

where $\eta \to \eta'$ is caused by the positive parity part of g^{-1}, whereas α is produced by the negative parity element acting on η, which is itself such an element. The resulting group action is given by

$$g^{-1}(\eta,\psi) = [\eta', D(\exp -i\alpha \cdot X^+)\psi]. \quad [26]$$

This action clearly exhibits a $Z(2)$ grading provided by parity. Can we represent it linearly by a supergroup? In ref. 12 we had indeed constructed the relevant f, d coefficient superalgebra explicitly. It now appears as $Q(3)$ in ref. 10. For $Q(3)_{\mathrm{ch}}$, take a set of sixteen (6×6) matrices,

$$X^+: \begin{vmatrix} \lambda_m & \\ & \lambda_m \end{vmatrix} \quad X^-: \begin{vmatrix} & \lambda_n \\ \lambda_n & \end{vmatrix}_{\gamma_5} \quad [27]$$

[λ_m, λ_n are $SU(3)$ matrices (14)], and define the brackets,

$$[X^+_m, X^+_n] = if_{mn\ell} X^+_\ell$$

$$[X^+_m, X^-_n] = if_{mn\ell} X^-_\ell$$

$$[X^-_m, X^-_n]_D := X^-_m X^-_n + X^-_n X^-_m - \tfrac{2}{3}(\mathrm{Tr}\, X^-_m X_n) I = 2\, d_{mn\ell} X^+, \quad [28]$$

where the $d_{mn\ell}$ are $SU(3)$ totally symmetric Clebsch–Gordan coefficients for $\underset{\sim}{8} \times \underset{\sim}{8} \to \underset{\sim}{8}_{\mathrm{sym}}$ (14).

The symmetric bracket between two odd elements thus differs from an anticommutator (in this defining representation) by a trace. In the adjoint representation, it will again be an anticommutator.

We now take this $G = Q(3)$ in P and study the connections. Under the X^+ generated $SU(3)$ subgroup, we have two octets,

$$\omega^a = \ell C^a(x) + dx^\mu \phi^a_\mu(x)$$

$$\omega^i = \ell\eta^i(x) + dx^\mu G^i_\mu(x). \quad [29]$$

In the even subgroup, $\phi^a_\mu(x)$ is a $J = 1$ octet and $C^a(x)$ is the corresponding ghost $\underset{\sim}{8}$. In the odd piece, the ω^i also form an $\underset{\sim}{8}$ under X^+, but they are commutative, as we have seen in [22] and [23]. Thus, $\eta^i(x)$ is a 0^- octet of bosons—i.e., physical fields! In fact, *we can identify these "exorcized ghosts" as the Goldstone–Higgs multiplet of the theory!* They are accompanied, howevever, by a new type of ghost, the $J = 1$ Fermi statistics $G^i_\mu(x)$. The role of the latter is perhaps not entirely understood at this point, but one can already see them in action in one-loop renormalization group equations: they provide a relatively heavier weighted contribution of ghost type [e.g., in conserving θ_w *in* $SU(2/1)$]. Notice that the entire counting system for such internal supergauges has to be reordered, because the Higgs fields η^i will be coupled universally, thus providing new diagrams of order g^3, etc. . . .

We may solve [21] and write

$$G^i_\mu = -\ell s^{-1}\bar{D}_\mu\eta^i := \epsilon_\mu\eta^i$$

$$\phi^a_\mu = -\ell s^{-1}\bar{D}_\mu C^a := \epsilon_\mu C^a. \quad [30]$$

For a matter field ψ, an $SU(3)$ triplet in representation [27], the BRS equation is

$$s\psi^n = \ell[C,\psi]^n := r^n, \quad [31]$$

where we have defined an *effective* ghost field r^n. If ψ^n is a Lorentz-spinor *fermion* (the β-positive components in fact), r^n will be a Lorentz-spinor *boson*—i.e., a ghost. However, because G is a supergroup, the ψ^n fill up only half of a representation like [27]. The other half consists of a triplet t^u of Lorentz-spinor opposite parity *bosons* (i.e., ghosts). Thus the BRS equation becomes

$$st^u = \ell[C,t]^u := \psi^u, \quad [32]$$

thus relating them to Lorentz-spinor *fermions* ψ^u that complement the r^n in making a six-dimensional (and Dirac β diagonalized) representation of $W(3)$. To have all new ghosts (r^n, t^u) appear as composite and the ψ^u appear as additional (inverse parity) matter fields, we may write

$$r^n = s\psi^n, \quad t^u = s^{-1}\psi^u. \quad [33]$$

Summing up, we have seen that gauging a supergroup G produces as gauge fields both the vector mesons ϕ^a_μ coupled to the even subgroup G^+ and a Goldstone–Higgs multiplet η^i behaving as $A(G^-)$ under $A(G^+)$ itself. At the same time, the theory contains the renormalization ghosts C^a and a new set of vector ghosts G^i_μ. Matter fields ψ^n and ψ^u are split between two analogous representations of G, even though when taken together they fit exactly the quantum numbers of one such representation. In their split assignment, they are accompanied by composite ghost fields that complete the two representations. All of this will be true of $SU(2/1)$ as well.

Notice that the resulting gauge Lagrangian in the case of $Q(3)$ (in its physical part) is exactly that of the "flavor" $SU(3)$ of the sixties with phenomenological constituent quark fields and with the 1^- mesons ρ, K^*, ϕ^o/ω^o as gauge fields, *plus a universally coupled 0^- meson multiplet* π, K, η. This is just the Lagrangian postulated by Gürsey and Radicati, *which gave rise to* SU(6) as its static symmetry (24). There is no $U(1)$ problem!

Table 1. Kinematics of $SU(2/1)$

Representation	Particles	Ghosts
$\underset{\sim}{8}$, $J = 1$	$\phi^1_\mu, \phi^2_\mu, \phi^3_\mu, \phi^8_\mu$	$G^4_\mu, G^5_\mu, G^6_\mu, G^7_\mu$
$\underset{\sim}{8}'$, $J = 0$	$\eta^4, \eta^5, \eta^6, \eta^7$	C^1, C^2, C^3, C^8
$\underset{\sim}{3}$, $J = 1/2$	ν^0_L, e^-_L	r^-_L (composite)
$\underset{\sim}{3}'$, $J = 1/2$	e^-_R	t^0_R, t^-_R (composite)
$\underset{\sim}{4}$, $J = 1/2$	$u^{2/3}_L, d^{-1/3}_L$	$r^{-1/3}_L, r^{2/3}_L$ (composite)
$\underset{\sim}{4}'$, $J = 1/2$	$d^{-1/3}_R, u^{2/3}_R$	$t^{2/3}_R, t^{-1/3}_R$ (composite)

$SU(2/1)$ as the ghost theory of asthenodynamics (the weak electromagnetic interactions)

The idea of a supergroup as an internal gauge group involving the ghosts of renormalization was first suggested (11) in the context of a basic theory of the unified weak electromagnetic interaction. It reproduces the Weinberg–Salam model (20, 21) in an extremely constrained form, imposed by $SU(2/1) \supset SU(2)_L \times U(1)$. The kinematics of $SU(2/1)$ are astonishingly precise in fitting just the observed particle representations of $SU(2)_L \times U(1)_U$ (Table 1). A "family" is thus $(\underset{\sim}{3} + \underset{\sim}{3}') + 3 \times (\underset{\sim}{4} + \underset{\sim}{4}')$. Note that $SU(2/1)$ *predicts* that the $I_L = 1/2$ multiplet is in a $\underset{\sim}{4}$ representation when the charges are fractional and in a $\underset{\sim}{3}$ when they take on integer values! Similarly, it predicts that the Higgs–Goldstone multiplet is an isodoublet $I_L = 1/2$, $U = \pm 1$. Also, $\theta_W = 30°$, and $m_\eta \approx 245$ GeV. The $\lambda\phi^4$ self-coupling of the Higgs–Goldstone multiplet is $\lambda \approx 4/3\, g^2$. We refer the reader to the original article (11) and to a recent discussion at the classical level (25).

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1. Lubkin, E. (1963) *Ann. Phys. N.Y.* **23**, 233–283.
2. Wu, T. T. & Yang, C. N. (1975) *Phys. Rev. D* **12**, 3845–3857.
3. Thierry-Mieg, J. (1978) These de Doctorat d'Etat (Université de Paris-Sud, Orsay, France).
4. Thierry-Mieg, J., *J. Math. Phys.*, in press.
5. Ne'eman, Y. (1979) in *Proceedings of the 19th International Conference on High Energy Physics, Tokyo, 1978*, eds. Homma, S., Kawaguchi, M. & Miyazawa, H. (Phys. Soc. of Japan, Tokyo), pp. 552–554.
6. Faddeev, L. D. & Popov, V. N. (1967) *Phys. Lett. B* **25**, 29–32.
7. Becchi, C., Rouet, A. & Stora, R. (1975) *Commun. Math. Phys.* **42**, 127–162.
8. Slavnov, A. A. (1972) *Teor. Mat. Fiz.* **10**, 153–161.
9. Taylor, J. C. (1971) *Nucl. Phys. B* **33**, 436–444.
10. Rittenberg, V. (1978) in *Group Theoretical Methods in Physics*, Proceedings of the 6th International Conference Tübingen, 1977, eds. Kramer, P. & Rieckers, A., Lecture Notes in Physics 1979 (Springer, Berlin), pp. 3–21.
11. Ne'eman, Y. (1979) *Phys. Lett. B* **81**, 190–194.
12. Corwin, L., Ne'eman, Y. & Sternberg, S. (1975) *Rev. Mod. Phys.* **47**, 573–604.
13. Nambu, Y. & Jona-Lasinio, F. (1961) *Phys. Rev.* **122**, 345–358.
14. Gell-Mann, M. & Ne'eman, Y. (1967) *The Eightfold Way* (Benjamin, New York).
15. Weinberg, S. (1968) *Phys. Rev.* **166**, 1568–1577.
16. Gell-Mann, M., Oakes, R. J. & Renner, B. (1968) *Phys. Rev.* **175**, 2195–2199.
17. Kobayashi, S. & Nomizu, K. (1963) *Foundations of Differential Geometry* (Interscience, New York), Vol. 1, pp. 63–66.
18. Ne'eman, Y. (1979) *Symétries, Jauges et Variétés de Groupe* (Presses de l'Université de Montréal, Montréal, Canada).
19. Ne'eman, Y. & Regge, T. (1978) *Riv. Nuovo Cimenta Ser. III*, **5**, 1–43.
20. Weinberg, S. (1967) *Phys. Rev. Lett.* **19**, 1264–1266.
21. Salam, A. (1968) in *Elementary Particle Theory, Proceedings of the Eighth Nobel Symposium*, ed. Svartholm, N. (Almquist & Wiksell, Stockholm), pp. 367–377.
22. Coleman, S., Weiss, J. & Zumino, B. (1969) *Phys. Rev.* **177**, 2239–2247.
23. Joseph, A. & Solomon, A. (1970) *J. Math. Phys.* **11**, 748–761.
24. Dyson, F. J. (1965) *Symmetry Groups* (Benjamin, New York).
25. Ne'eman, Y. & Thierry-Mieg, J. (1980) in *Proceedings of the Salamanca (1979) International Conference on Differential Geometry Methods in Physics*, ed. Pérez-Rendon, A. (Lecture Notes in Mathematics, Springer, New York), in press.