Spin Independence, W Spin, Parity, and $SU(6)$ Symmetry*

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The assumption of strict spin independence of strong interactions between elementary particles forbids all three-meson and meson-baryon-baryon couplings, such as $\rho\pi\pi$, $NN\pi$, and $N^*N\pi$. It is shown that these catastrophies may be avoided, even in the nonrelativistic limit, by adopting a modified definition of spin independence, i.e., W-spin independence. A nonrelativistic definition of W spin is obtained which requires only a trivial change to yield the relativistic description. The consequences of the assumption of W-spin independence are explored.

INTRODUCTION

THE assumption of strict spin independence of the strong interactions between elementary particles[1] forbids the strongest known couplings such as $\rho\pi\pi$ and $N^*N\pi$. Some of the relativistic extensions of $SU(6)$ and spin independence[2] which have been proposed attribute this difficulty to the nonrelativistic nature of the static $SU(6)$ theory. Others require some form of symmetry breaking[3] to allow these decays. In this work we adopt a different point of view. We wish to find a higher symmetry in which the strongest couplings known in strong interactions are already present in the nonrelativistic version of the theory, and are not introduced by "relativistic corrections" or "symmetry breakers."

We first consider the implications of exact spin independence and show that not only the $\rho\pi\pi$ and $N^*N\pi$ couplings but, in fact, all Yukawa strong couplings are forbidden.[3] We then show that these catastrophies are avoided, *even in the nonrelativistic limit*, by a modified definition of spin independence, and that all the good $SU(6)$ results are retained. The assumption of this modified spin independence of collinear transition amplitudes leads to results in agreement with experiment and to interesting new predictions, and does not forbid any processes known experimentally to be strong. The nonrelativistic definition of W spin requires only a trivial modification to obtain a fully relativistic description.

The modified spin can be combined with $SU(3)$ to construct a nonrelativistic $SU(6)$ symmetry which is different from the static $SU(6)$ defined with ordinary spin.[1] Once this proper nonrelativistic $SU(6)$ symmetry is defined, no difficulty arises in the relativistic generalization. The relativistic result is just the W spin and $SU(6)_W$ subgroups of $U(6,6)$ and $U(6)\times U(6)$.[4–6]

In our treatment we shall make extensive use of the combined rotation–space-inversion transformation which has been used in various contexts[7] and which is described in detail by Bohr.[8] This transformation which we call R is just a reflection in a plane, i.e., has the effect of an ordinary mirror. However, since it is not immediately obvious what a mirror does to a spin, and to phases of wave functions, we specify the transformation as follows: R is a combination of a space inversion and a 180° rotation about an axis perpendicular to the plane. This transformation is particularly useful for states in which all momenta lie in the plane. For such states the R transformation leaves the momenta invariant and transforms *only the intrinsic spins and parities* of the particles. It is thus just a 180° rotation of all spins about an axis perpendicular to the plane combined with an "intrinsic" space inversion.

For convenience, let us choose the case where all momenta are in the yz plane and denote by R_x the combined transformation of space inversion and a 180°

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rotation about the x axis. Then

$$R_x \equiv P e^{i\pi J_x} \underset{yz}{\equiv} P_{\text{int}} e^{i\pi S_x}, \tag{1}$$

where $\mathbf{J}$ is the total-angular-momentum operator, P the space-inversion operator, $\mathbf{S}$ the total-spin operator and P_{int} the "intrinsic-space-inversion" operator. The notation $A \underset{yz}{\equiv} B$ means that the operators A and B are equal only when acting on states in which all momenta are in the yz plane.

THE DIFFICULTIES OF SPIN INDEPENDENCE

Consider the case where strong interactions are assumed to be "spin-independent", i.e., invariant under rotations of all the spins of all the particles in a given state without any change in the spatial degrees of freedom. Consider a state in which all particles have momenta in the yz plane. If we assume that strong interactions are invariant under both ordinary rotations and space inversion, then they are invariant under the R_x transformation. The interactions are thus invariant under the combination of the R_x transformation followed by a rotation of the spins about the x axis by $-180°$:

$$R_x e^{-i\pi S_x} = P e^{i\pi J_x} e^{-i\pi S_x} \underset{yz}{\equiv} P_{\text{int}} e^{i\pi S_x} e^{-i\pi S_x} = P_{\text{int}}. \tag{2}$$

The R_x transformation leaves the momenta invariant and the additional spin rotation $e^{-i\pi S_x}$ just cancels the effect of the spin rotation in the R_x transformation. Thus, the net effect of the combination of these transformations is just space inversion in the intrinsic space. The transformation (2) therefore simply defines the total intrinsic parity of the state, which is the product of the intrinsic parities of all the particles.

Invariance under the transformation (2) means that intrinsic parity is conserved. Thus the assumption that strong interactions are invariant under rotations of the spin variables alone, combined with the usual assumptions of invariance under rotation and space inversion, leads to a selection rule *forbidding a change in the internal parity of a state in any transition where all momenta are constrained to a plane.* All odd-parity particles, including pseudoscalar and vector mesons, can only be created in pairs in any process which is coplanar in some Lorentz frame. This selection rule forbids all baryon-baryon-meson and three-meson couplings. In particular it forbids the nucleon-nucleon-pion vertex as well as the $\rho\pi\pi$ and $N^*N\pi$ couplings.

The assumption of spin independence therefore does not lead to a good description of strong interactions in the nonrelativistic limit. Instead, it forbids all couplings which are generally assumed to dominate nonrelativistic strong-interaction processes. For example, nucleon-nucleon scattering by one-meson exchange with Yukawa vertices is forbidden by spin independence, even at low momenta where one should expect a nonrelativistic approximation to be valid. The conventional treatment of the nucleon electromagnetic form factors via intermediate vector mesons is also forbidden by spin independence for *arbitrarily small momentum transfers* where one would again expect a nonrelativistic approximation to be valid.

A MODIFIED SPIN INDEPENDENCE, W-SPIN INDEPENDENCE

Let us now attempt to modify the concept of spin independence in order to obtain a higher symmetry which gives the desirable $SU(6)$ results but which avoids the undesirable selection rule (2). We shall be guided by the following two interesting features of the previous analysis:

1. The law of conservation of intrinsic parity (2) does not appear as an absolute conservation law in a spin-independent theory. Intrinsic parity is conserved only in processes which take place in a plane. It is not conserved in processes involving large numbers of particles, e.g., a five-point function. The notion of a quantity which is conserved only in processes involving small numbers of particles is a fruitful one. The empirical discovery of such "partially conserved" quantities may lead to an understanding of more complicated dynamical properties of strong interactions which reduce to a simple conservation law only in certain specific cases.

The successful predictions of $SU(6)$ all involve processes involving small numbers of particles: properties of one-particle states, three-point vertex functions, and forward-scattering amplitudes. Each of these processes is "collinear"; it can be described in some Lorentz frame in which all momenta are in the z direction. We shall therefore restrict our attention to collinear processes and consider only the invariance properties of the amplitudes for such processes.

2. The undesirable selection rule (2) results from the combination of a new higher symmetry, spin independence, with the R_x symmetry already present. This combination of the $SU(2)$ spin group with the group of transformations already present leads to a larger group than $SU(2)$, under which the three-point vertex function is required to be invariant. If this group larger than the $SU(2)$ is combined with the conventional $SU(3)$, a *group larger than $SU(6)$* is obtained. It is this larger group which causes the trouble. There is "too much symmetry" which leads to undesired selection rules in addition to the desirable features of $SU(6)$ symmetry.

From the consideration of these points we reach the following conclusions. We wish to define a higher symmetry which need apply only to collinear processes. We want these processes to be invariant under some $SU(6)$ group in order to obtain the good $SU(6)$ results. However, we do not want too much symmetry. The minimum symmetry group which contains $SU(6)$ is of course $SU(6)$ itself. We can obtain this minimum symmetry only if the $SU(6)$ group is defined in such a way as *to include all other intrinsic symmetry operations such as the operation (1) which are already present* from other considerations. In group-theoretical language we can say that we wish to define an $SU(6)$ group which already contains the subgroup of the Lorentz group which leaves invariant all momenta in the z direction. Since $SU(3)$ is extraneous to all these considerations, the problem is one of finding an alternative $SU(2)$ group to replace the spin, where the new group should include transformations like (1).

Let us now specify precisely those invariance properties of a collinear transition amplitude which follow only from invariance under rotations and reflections. We consider only transformations in the intrinsic variables, spin and parity, and choose one Lorentz frame so that all momenta are in the z direction. Since there is no orbital angular momentum in the z direction, $J_z=S_z$, and angular-momentum conservation requires invariance under the group of continuous spin rotations about the z axis. The transition amplitude is also invariant under the transformation $P_{\text{int}}e^{i\pi S_x}$, by Eq. (1). For a collinear process in the z direction, all momenta are also in the xz plane and the transformation (1) can also be defined about the y axis. This gives $P_{\text{int}}e^{i\pi S_y}$ as a symmetry operation. We thus find that the collinear transition amplitude must be invariant under a group which includes rotations about the z axis and reflections about any plane containing the z axis.

We now wish to find an $SU(2)$ group, or a set of "spin" transformations which include this rotation-reflection group. Clearly, S_z must be a generator of this group. We now need to find two more generators which must satisfy angular-momentum commutation rules with S_z and which must generate the finite transformations $P_{\text{int}}e^{i\pi S_x}$ and $P_{\text{int}}e^{i\pi S_y}$.

For simplicity, let us restrict ourselves to systems of spin-$\frac{1}{2}$ particles. This describes all known particles, if the quark model is assumed for higher spin particles. However, once we define an $SU(6)$ group and obtain a consistent set of transformation properties for all particles with the use of the quark model, we do not need to assume the existence of quarks. They are simply a useful mathematical device for finding the right transformation properties of the observed particle states.

For a spin-$\frac{1}{2}$ particle, we can write $\mathbf{S}=\boldsymbol{\sigma}/2$, where σ_x, σ_y, and σ_z are the usual Pauli matrices. Then R_x can be written in a more convenient form with the aid of the following identities:

$$R_x \underset{\overline{yz}}{=} P_{\text{int}}e^{i\pi S_x}=P_{\text{int}}e^{i\pi\sigma_x/2}$$
$$=P_{\text{int}}[\cos(\pi/2)+i\sigma_x\sin(\pi/2)]=iP_{\text{int}}\sigma_x. \quad (3a)$$

$$e^{i\pi P_{\text{int}}S_x}=e^{i\pi P_{\text{int}}\sigma_x/2}$$
$$=\cos(\pi/2)+iP_{\text{int}}\sigma_x\sin(\pi/2)=iP_{\text{int}}\sigma_x. \quad (3b)$$

Thus

$$R_x \underset{\overline{yz}}{=} P_{\text{int}}e^{i\pi S_x}=e^{i\pi P_{\text{int}}S_x}. \quad (3c)$$

From the exponential form (3*c*) of R_x, we see that it can be generated by the operator $P_{\text{int}}S_x$. We can now define three operators which have the desired properties:

$$W_z=S_z, \quad (4a)$$

$$W_x=P_{\text{int}}S_x,$$
$$W_y=P_{\text{int}}S_y. \quad (4b)$$

These operators satisfy angular-momentum commutation rules. Furthermore, W-spin transformations include the transformations R_x and R_y as 180° W-spin rotations about the x and y axes:

$$e^{i\pi W_x}=R_x, \quad (5a)$$

$$e^{i\pi W_y}=R_y. \quad (5b)$$

We see immediately that the assumption of W-spin invariance does not lead to the bad selection rule (2). If S_x is replaced by W_x on the left-hand side of (2), the right-hand side is multiplied by an additional factor P_{int} and the resulting transformation is just the identity and gives no new selection rule.

$$R_xe^{-i\pi W_x}\underset{\overline{yz}}{=} 1. \quad (6)$$

From a group-theoretical point of view we can say that the rotation-reflection subgroup of the Lorentz group is isomorphic to a group in which the reflections are replaced by 180° rotations perpendicular to the z-axis. It is clearly possible to embed such a group which contains only some of the rotations in a three-dimensional space in a group isomorphic to the full three-dimensional rotation group. The operators (4) represent a specific construction of such a group.

THE W-SPIN CLASSIFICATION OF PARTICLES

The W-spin $SU(2)$ group, Eq. (4), is defined for systems of spin-$\frac{1}{2}$ particles, and has been constructed so that W-spin invariance of collinear processes does not lead to the undesirable selection rule (2). We now wish to examine the consequences of W-spin invariance to see whether this is a better nonrelativistic approximation for strong interactions than ordinary spin independence. The first step is to define the W-spin transformation properties of mesons and baryons in such a way that the W-spin group still contains the whole rotation-reflection group required by ordinary rotation-reflection invariance. One way to define these transformation properties is to use a mathematical model in which mesons and baryons are composed of spin-$\frac{1}{2}$ quarks and antiquarks in relative S states with zero relative momentum. This is just a mathematical device and does not assume the existence of quarks. From the definition (4) we see that *W spin differs from ordinary spin in having the component normal to the momentum multiplied by the intrinsic parity of the particle.* Thus, the W spin defined for the antiparticle has a negative sign in the definition of W_x and W_y because of the odd intrinsic parity. Quarks, which are defined to have positive intrinsic parity, have their W spin equal to their ordinary spin at rest. The antiquarks which are defined to have negative intrinsic parity have their x and y components of W spin opposite to those of their ordinary spin at rest. The directions of a W-spin rotation about the x or y axis are thus opposite for quark and antiquark. The W spin of a system containing quarks and antiquarks is quite different from the total ordinary spin of the system. The z component of the W spin is just the z component of the total spin; however, the x and y components of the W spin are the differences between the total quark spin and the total antiquark spin.

Let us now examine the classification of the system of a quark and an antiquark at rest, according to W spin and ordinary spin. In both cases we have two spins of $\frac{1}{2}$ which give rise to four states, a triplet and a singlet. For convenience, we will denote the quark and antiquark states with spin up and spin down relative to the positive z-direction by

$$Q\uparrow,\ \bar{Q}\uparrow,\ Q\downarrow, \text{ and } \bar{Q}\downarrow.$$

Since $W_z=S_z$ for the system, the states having the eigenvalues ± 1 for both S_z and W_z must correspond to the triplet spin state in both cases.

$$|Q\uparrow\bar{Q}\uparrow\rangle,\quad S_z=W_z=+1;\quad S=W=1 \tag{7a}$$

$$|Q\downarrow\bar{Q}\downarrow\rangle,\quad S_z=W_z=-1;\quad S=W=1. \tag{7b}$$

The states having the zero eigenvalue of $S_z=W_z$ can be classified as triplet or singlet according to S or W spin by noting that the triplet state is obtained from the state (7a) by the application of the corresponding lowering operator. We see immediately that the S- and W-spin classifications are different because the lowering operators for S and W spin have a different phase when acting on the antiquark. In particular, the W-spin lowering operator when acting on a system of quarks and antiquarks is the difference between the S-spin quark lowering operator and the S-spin antiquark lowering operator.

$$W_-=W_-^Q+W_-^Q=S_-^Q-S_-^{\bar{Q}} \tag{8}$$

where the superscripts Q and $\bar{Q}$ denote the part of the particular operator acting only on quarks or on antiquarks. The S-spin triplet state is obtained by operating on the state (7a) with the S-spin lowering operator S_-,

$$S_-|Q\uparrow\bar{Q}\uparrow\rangle=|(S_-Q\uparrow)\bar{Q}\uparrow\rangle+|Q\uparrow(S_-\bar{Q}\uparrow)\rangle;\quad S_z=W_z=0,\quad S=1. \tag{9a}$$

We have explicitly exhibited the two terms resulting from the operation of S_- on the quark and on the antiquark without actually carrying through the operation in order to avoid the necessity of choosing a phase convention which is irrelevant for our purposes. The analogous W-spin triplet state is obtained by operating with the W-spin lowering operator (8). Because of the negative sign in the antiquark term we obtain the opposite phase from that of Eq. (9a)

$$W_-|Q\uparrow\bar{Q}\uparrow\rangle=|(S_-Q\uparrow)\bar{Q}\uparrow\rangle-|Q\uparrow(S_-\bar{Q}\uparrow)\rangle;\quad S_z=W_z=0,\quad W=1. \tag{9b}$$

The two states (9a) and (9b) are obviously orthogonal to one another. Thus, the W-spin triplet state, (9b) is the S-spin singlet state and vice versa. Let us denote the three states of the S-spin triplet by (V_{+1}, V_0 and V_{-1}) and the S-spin singlet state by P. The subscripts denote the eigenvalue of S_z and the letters V and P denote vector and pseudoscalar for the case where the vector and pseudoscalar mesons are considered to be made of a quark and an antiquark in a relative S state. The W-spin triplet then consists of the state (V_{+1},P,V_{-1}) and the W-spin singlet is the state V_0. The W-spin classification of the states P and V_0 are just interchanged from the corresponding S-spin classification. This has been called "W-S flip."

The classification of the baryons is trivial, since these are composed only of quarks and no antiquarks in the quark model. The W spin of a baryon is therefore the same as its ordinary spin.

THE CONSEQUENCES OF W-SPIN INVARIANCE

Let us now examine the assumption that all collinear vertex functions are invariant under W spin, with the mesons and baryons classified as indicated by the quark model. This assumption has been shown to allow the ρ and N^* decays,[4] and to lead to no selection rules which forbid processes known experimentally to be strong.[4,9] We shall summarize these results here.

The ρ and N^* decays are allowed by W spin, as is evident from the following relations:

$$\begin{array}{ccccc} \rho & \to & \pi & + & \pi \\ W=0 & & W=1 & & W=1 \\ W_z=0 & & W_z=0 & & W_z=0, \end{array} \tag{10a}$$

$$\begin{array}{ccccc} N^* & \to & N & + & \pi \\ W=\frac{3}{2} & & W=\frac{1}{2} & & W=1 \\ W_z=\frac{1}{2} & & W_z=\frac{1}{2} & & W_z=0. \end{array} \tag{10b}$$

The following meson couplings have been shown to be forbidden by W spin[4,9]:

$$\begin{array}{cccccc} P & \to & P & + & P & \text{(forbidden)} \\ W=1 & & W=1 & & W=1 & \\ W_z=0 & & W_z=0 & & W_z=0, & \end{array} \tag{11a}$$

$$\begin{array}{cccccc} P & \to & V_0 & & V_0 & \text{(forbidden)} \\ W=1 & & W=0 & & W=0 & \\ W_z=0 & & W=0 & & W=0, & \end{array} \tag{11b}$$

$$\begin{array}{cccccc} T_0 & \to & P & + & V_0 & \text{(forbidden)} \\ W=0,2 & & W=1 & & W=0 & \\ W_z=0 & & W_z=0 & & W_z=0, & \end{array} \tag{11c}$$

[9] H. Harari and H. J. Lipkin, Phys. Rev. 140, B1617 (1965).

where T denotes a 2+ meson constructed from two quarks and two antiquarks. The W spin of the T_0 state is determined by the same method used above for P and V.

These selection rules lead to no new predictions, because they are also obtained independently from angular momentum and parity considerations. The connection between the two derivations becomes evident when one considers the R transformation. All the mesons in the relations (11) are eigenstates of R_x; P is odd, V_0 and T_0 are even. Thus, all the couplings (11) are forbidden by R. We now see that these couplings are forbidden by the assumption either of W-spin invariance or by ordinary rotation-reflection invariance, since either assumption includes invariance under R.

Interesting nontrivial predictions have been obtained for the B^*BV vertex,[4,9] where B^* and B denote spin-$\frac{3}{2}$ decuplet and spin-$\frac{1}{2}$ octet baryons, respectively. If onyl ordinary rotation-reflection invariance is assumed, there are three different independent amplitudes for this vertex. They can be labeled either as helicity amplitudes or, using the analogy between the vector meson and the photon, as $E2$, $M1$, and $L2$. Here $L2$ denotes the longitudinally polarized vector-meson state.

$$\begin{array}{ccccc} B^* & \rightarrow & B & + & V_{\pm 1} \\ W=\frac{3}{2} & & W=\frac{1}{2} & & W=1\,, \end{array} \tag{12a}$$

$$\begin{array}{ccccc} B^* & \rightarrow & B & + & V_0 \\ W=\frac{3}{2} & & W=\frac{1}{2} & & W=0\,. \end{array} \tag{12b}$$

We see immediately that the coupling of the longitudinally polarized V_0 is forbidden (12b). All the other couplings are given by (12a). These all involve members of the same W-spin multiplets $B^*(W=\frac{3}{2})$, $B(W=\frac{1}{2})$ and $V_{\pm 1}(W=1)$, and are therefore all proportional to the same reduced matrix element with proportionality factors which are just Clebsch-Gordan coefficients. Thus, there is only *one* coupling for the B^*BV vertex allowed by W-spin invariance. This turns out to be the $M1$ coupling. This prediction, which also applies to the $(B^*B\gamma)$ vertex is in good agreement with experiment.

An $SU(6)$ group can be constructed by combining $SU(3)$ with W spin. The assumption of invariance of collinear three point vertex functions[2] under $SU(6)_W$ leads to a number of predictions for various processes which are in generally good agreement with experiment.

THE RELATIVISTIC FORMULATION OF W SPIN

Now that we have defined a spin whose conservation in the nonrelativistic limit leads to no serious disagreement with experiment, we can consider its relativistic extension. However, it is immediately evident that the definition (4) is already relativistic and needs no extension. If the spin-$\frac{1}{2}$ particles are described by Dirac spinors, the operator β is just the intrinsic parity in the rest frame. We can therefore express the W-spin operators in terms of the Dirac algebra

$$W_z=\sigma_z/2\,, \tag{13a}$$

$$W_x=\beta\sigma_x/2\,, \tag{13b}$$

$$W_y=\beta\sigma_y/2\,. \tag{13c}$$

These operators are invariant under Lorentz transformations in the z direction; therefore, the W-spin classification for a particle with arbitrary momentum in the z direction is the same as the classification at rest. This is all that is needed for a relativistic description of collinear processes.

The operators (13) have been well known in the treatment of electron-polarization phenomena.[10] For studies of scattering of relativistic electrons and positrons the relative phase of the W-spin operators defined for electrons and positrons has no physical significance, and W-S flip does not appear. One might consider the use of W spin for bound states consisting of both positrons and electrons; however, this does not seem to have been done.

The operators (13) have also been combined with $SU(3)$, without the W-S flip, for the consideration of the nucleon electromagnetic form factors.[6] Here the W-S flip is irrelevant as the antibaryons and zero helicity meson states do not appear. The W-S flip appears naturally in derivations of W spin as a subgroup in the $U(6,6)$ theory[4] and the positive-parity nonchiral $U(6)\times U(6)$ symmetry.[5]

W SPIN AND THE CLASSIFICATION OF PARTICLES AT REST

The assumption that collinear vertex functions are invariant under W spin or under the group $SU(6)_W$ implies the existence of a larger group which is used for classifying the states of particles at rest. This can be seen from the simple example of the W-spin triplet containing the states (V_{+1},P,V_{-1}). Except for the singular case of zero-mass particles, the existence of the vector-meson states V_{+1} and V_{-1} requires that the third polarization state V_0 must also exist. Thus, the existence of this particular W-spin triplet requires an additional W-spin singlet to form a "supermultiplet" containing two W-spin multiplets, a triplet and singlet.

The structure of the larger supermultiplets is easily obtained as follows for an arbitrary W-spin multiplet: Let us consider the set of particle states corresponding to a given W-spin multiplet in the rest frame of the particle. From ordinary rotational invariance it follows that an ordinary spatial rotation must transform any particle state into another particle state having the same mass. A rotation of a one-particle state in its rest frame is simply a spin rotation. If we combine ordinary spin rotations together with the W-spin rota-

[10] M. E. Rose, *Relativistic Electron Theory* (John Wiley & Sons, Inc., New York, 1961), p. 72.

tions (4) we obtain a larger Lie group than W spin for the classification of particles at rest. This is the group having the six generators $(S_x,S_y,S_z,P_{\rm int}S_x,P_{\rm int}S_y,P_{\rm int}S_z)$. This is just the group $SU(2)\times SU(2)$, the direct product of spin rotations for all positive parity particles and spin rotations from all negative-parity particles. In a quark model, this is the direct product of the spin rotations for all quarks and the spin rotations for all antiquarks.

The addition of the $SU(3)$ degrees of freedom to the above analysis is trivial and leads to the group $SU(6) \times SU(6)$. This is the direct product of an $SU(6)$ group for all positive-parity particles and one for all negative-parity particles. This group contains $SU(6)_W$ as a subgroup. One can clearly add the operators $P_{\rm int}$ and the identity to the generators to obtain the groups $U(2)\times U(2)$ and $U(6)\times U(6)$.

Note that for $SU(6)_W$ multiplets which contain only quarks or only antiquarks, like the baryons and the antibaryons, a given $U(6)\times U(6)$ supermultiplet contains only a single $SU(6)_W$ multiplet. The representation of $U(6)\times U(6)$ is always a singlet in one of the $U(6)$ groups, either the quark or the antiquark. On the other hand, for $SU(6)_W$ multiplets obtained from states containing both quarks and antiquarks, there is more than one $SU(6)_W$ multiplet in the corresponding multiplet of $U(6)\times U(6)$. The obvious example is that of the mesons which must be classified in the $(\mathbf{6},\bar{\mathbf{6}})$ representation of $U(6)\times U(6)$ which contains 36 states, corresponding to a **35** and a singlet of $SU(6)_W$.

The larger group $U(6)\times U(6)$ is needed for classifying states of particles at rest, but cannot be used as an invariance group for vertex functions. On the one hand, it contains the group of ordinary spin rotations and therefore automatically leads to all the contradictions with experiment found in any theory which assumes spin independence. On the other hand, the relativistic generalization [analogous to (13)], of the group $U(6)\times U(6)$ is not invariant under Lorentz transformations in any direction. The subgroup of $U(6)\times U(6)$ invariant under Lorentz transformations only in the z direction is just the group $SU(6)_W$.

So far we have considered only the transformation properties of states of physical particles. It is also of interest to examine the classification of field operators and virtual particles off the mass shell. For applications to electromagnetic and weak interactions it is of interest to examine the classification of the currents which are coupled in these interactions. These properties are not automatically determined by the transformation properties at rest of the physical particles and depend on the details of the particular theory. The relevant properties are the field equations in the case of the field theory, and the commutation relations of the currents in a theory which begins with the algebra of currents.

In a Dirac quark field theory the $U(6)\times U(6)$ and $SU(6)_W$ transformation properties of the field operators are obtained as follows: The field operators which create particles or antiparticles at rest are classified into multiplets which are exactly the same as the corresponding multiplets used to classify the state of these particles at rest. The field operators which annihilate these particles at rest are then classified in the corresponding conjugate representation. The classification for field operators which create or annihilate a particle having a finite momentum in the z direction is the same in $SU(6)_W$ as for the operator of the corresponding state at rest. This is not true for $U(6)\times U(6)$. This procedure specifies completely the $SU(6)_W$ classification for quark field operators which create or annihilate particles or antiparticles moving in the z direction. The transformation properties for operators creating and annihilating other particles are obtained by suitably combining the quark operators. The Fourier components of current operators with momentum transfer in the z direction are bilinear products of quark creation and annihilation operators and can be classified accordingly, in $SU(6)_W$.

W SPIN WITHOUT A QUARK MODEL

The W spin and $SU(6)_W$ classification of particles has been constructed with the aid of a quark model. Let us now consider other possible classifications. We assume that the classifications of particles at rest is given by the group $U(6)\times U(6)$. Since W spin is a subgroup of $U(6)\times U(6)$, the W-spin classification is uniquely determined. The only remaining degree of freedom not specified by these considerations is the intrinsic parity of the particles. In a quark model the parity of any $U(6)\times U(6)$ multiplet is uniquely determined. We can now consider the possible existence of $U(6)\times U(6)$ multiplets having the opposite parity to that defined by the quark model. For states in such "anomalous parity multiplets," Eq. (6) is replaced by

$$R_x e^{-i\pi W_x} \underset{\overline{\overline{yz}}}{=} -1. \qquad (14)$$

The quantity $R_x e^{-i\pi W_x}$ thus has the eigenvalue $+1$ for normal parity multiplets and -1 for anomalous parity multiplets. If W spin, angular momentum and parity are conserved, then $R_x e^{-i\pi W_x}$ is also conserved. It then follows that particles of anomalous parity can only be produced in pairs and cannot decay into particles of normal parity.

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