Design of Broad-Band PMD Compensation Filters

A. Eyal, *Member, IEEE* and A. Yariv, *Life Fellow, IEEE*

***Abstract*—We describe a new design approach for broad-band PMD compensation filters. An efficient algorithm for minimization of the maximum differential group delay within a given frequency band is described.**

***Index Terms*—Fiber-optic communication systems, optical filter design, PMD compensation, polarization-mode dispersion.**

IN RECENT YEARS, several approaches for polarization-mode dispersion (PMD) compensation have been studied. These approaches were designed to compensate for the effects of first-order PMD [1], [2] as well as high-order PMD [3]–[7], [12]. Typically, high-order PMD compensators (PMDCs) are multistage devices that have many degrees of freedom. A robust control algorithm is essential for using these broad-band PMDCs in practical applications. One major concern in controlling broad-band PMDCs is the possibility that they will converge to a local compensation optimum, which can be far from the absolute optimum. To overcome this problem some authors studied "feed forward" methods in which the Jones matrix (JM) of the PMD medium is determined online and the following compensation filter is adjusted so that its JM becomes as close as possible to the inverse JM of the PMD medium [8], [9]. The JM of the composite system, PMD medium and PMDC, becomes approximately frequency independent and PMD is compensated. Recently, a synthesis algorithm for such an inverse filter, in the absence of polarization-dependent loss or gain (PDL/G) was proposed [5]. The algorithm required knowledge of the medium JM and was based on expanding the elements of the desired JM as Fourier series and using the coefficients of the expansion to iteratively solve for the parameters of cascaded interferometric filters. The number of the terms in the Fourier expansion determined the number of basic cells in the filter. In general, however, the use of a finite number of terms in the Fourier expansion (typically smaller than 20) may result in large local deviations of the approximated matrix from the desired matrix. These deviations may cause the synthesis algorithm to fail or to produce large spikes in the PMD of the compensated system. In this letter, we demonstrate this and propose a new approach for designing a broad-band PMD compensation filters. The approach is general and is guaranteed to produce a solution. Instead of trying to synthesize an inverse medium the approach is based on minimization of the maximum differential group delay (DGD) in the band of interest. The method can be used as a basis for control algorithms of broad-band PMDCs and is applicable in the presence of PDL with only minor modifications.

Manuscript received December 19, 2001; revised April 4, 2002.

A. Eyal was with the Department of Applied Physics, California Institute of Technology, M.S. 128-95, Pasadena, CA 91125 USA. He is now with the Department of Interdisciplinary Studies, Faculty of Engineering, Tel-Aviv University, Israel.

A. Yariv is with the Department of Applied Physics, California Institute of Technology, Pasadena, CA 91125 USA.

Publisher Item Identifier S 1041-1135(02)06019-6.

Let the optical field at the input of an optical transmission medium be described by the Jones vector $\vec{\mathbf{E}}_{in}$. Assuming the absence of PDL the output field is then given by

$$\vec{\mathbf{E}}_{\mathrm{out}}(\omega) = e^{-\alpha + j\theta}\mathbf{U}(\omega)\vec{\mathbf{E}}_{\mathbf{in}}. \tag{1}$$

Here, α describes the medium attenuation/gain, θ is the overall phase, and $\mathbf{U}(\omega)$ is a frequency dependent unitary matrix. We assume that the common attenuation/gain and the common phase of the medium have been equalized by some other methods so that α and θ are frequency independent. Thus, without loss of generality, we choose $\exp(-\alpha + j\theta) = 1$. The output of the transmission medium is launched into a PMD compensation filter described by the matrix $\mathbf{H}(\omega)$. Ideally, complete cancellation of PMD can be achieved if the PMD filter satisfies $\mathbf{H}(\omega) = \mathbf{PU}^{-1}(\omega)$ where $\mathbf{P}$ is a frequency independent matrix. Recently, an algorithm for synthesis of $\mathbf{PU}^{-1}(\omega)$ using a cascade of N lossless interferometric filters has been proposed [5]. Each interferometric filter is characterized by two variables. An angle ϕ_m which determines its input coupling ratios, $\cos(\phi_m)$ and $\sin(\phi_m)$, and an angle ψ_m, which describes the differential phase between the arms of the interferometer. The transmission matrix of such a filter has the following form:

$$\mathbf{H}(\omega) = \begin{bmatrix} h_{11} & -h_{12}^* \\ h_{12} & h_{11}^* \end{bmatrix}$$

$$\text{where } h_{11}(\omega) = e^{jNT\omega/2}\sum_{k=0}^{N-1} a_k e^{-jkT\omega}$$

$$h_{12}(\omega) = e^{jNT\omega/2}\sum_{k=1}^{N} b_k e^{-jkT\omega} \tag{2}$$

where T is the DGD of a single stage and a_k and b_k are complex coefficients. The series coefficients are related to the coupling coefficients, $\cos(\phi_m)$ and $\sin(\phi_m)$ and the differential phases, ψ_m in each of the N stages. The details of how to obtain the coupling coefficients $\cos(\phi_m)$ and $\sin(\phi_m)$ and the differential phases, ψ_m from a_k and b_k are described in [5]. Since $\mathbf{H}(\omega)$ is unitary, its elements satisfy: $|h_{11}(\omega)|^2 + |h_{12}(\omega)|^2 = 1$, and thus the filter does not affect the amplitude characteristics of the transmitted signal. The synthesis algorithm in [5] is based on expanding the elements of $\mathbf{U}^{-1}(\omega)$ as Fourier series and associating the expansion coefficients $\tilde{a}_k$ and $\tilde{b}_k$ with a_k and b_k. In general, however, even if $\mathbf{U}^{-1}(\omega)$ is taken to be exactly unitary the approximated matrix that results from truncating the Fourier expansion after N terms is in most cases nonunitary. The deviations from unitarity can be significant due to the ringing phenomenon that is associated with truncated Fourier series (also referred to as the Gibbs phenomenon). Though simple windowing can reduce the ringing the resulting matrix will, in general, remain nonunitary. When the unitarity condition is not satisfied, the synthesis algorithm will fail to obtain valid values for ϕ_m and ψ_m. To overcome this obstacle and to improve the performance of the filter we reformulate the problem of PMD compensation in the form of an optimization

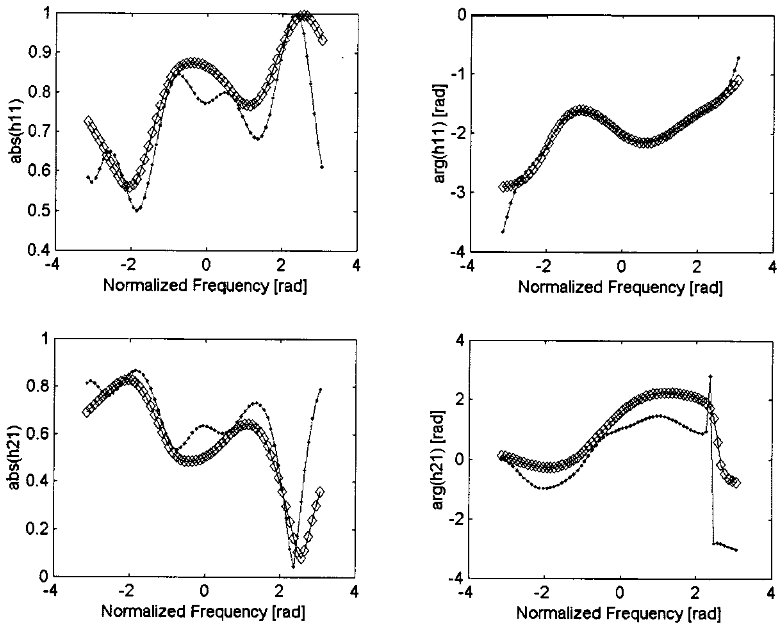

Fig. 1. The amplitudes and phases of the elements of $\mathbf{U}^{-1}(\omega)$ ($\diamond$) and of the synthesized filter $\mathbf{H}(\omega)$ ($\bullet$).

problem. We first define the frequency band of interest Ω. This band can be either simple or a union of disjoint sub-bands. Next we define a cost function that we wish to minimize. One possible choice of a cost function is the maximum DGD in Ω weighted by some positive function of frequency, $W(\omega)$. The role of $W(\omega)$ is to enable different amount of PMD cancellation in different frequency regions. This optimization criterion is simple to calculate and it does not require knowledge of the input state-of-polarization (SOP) or the modulation spectrum. This information is not always easily obtained, especially if Ω contains several channels from several different sources. Note, however, that this criterion does not necessarily provide optimal PMD cancellation. Other criteria that takes into account additional information about the system can, in some cases, provide better cancellation. The effect of first-order PMD, for example, can be canceled by transforming the system principal states of polarization such that one of them is aligned with the input SOP without canceling the DGD [6]. Generalizing this approach to broad-band PMD compensation may yield a different optimization criterion.

The optimization problem as we have defined it can be stated more concisely in the form

$$\min_{\phi_m,\,\psi_m}\left(\max_{\omega}\left(W(\omega)\cdot|DGD(\phi_m,\psi_m,\omega)|\right)\right)\quad \omega\in\Omega. \tag{3}$$

This is a nonconstrained nonlinear optimization problem for which efficient solution algorithms are available. The optimization variables were chosen to be ϕ_m and ψ_m, rather then the Fourier coefficients a_k and b_k, for the following reasons: this choice guarantees the unitarity of the synthesized system, the optimization variables are real and the domain of valid solutions is unbounded. In addition, in practical implementations of such a filter the controlled variables would usually be closely related to ϕ_m and ψ_m. To solve (3), we have used the "sequential quadratic programming" (SQP) method [8]. This is an iterative method, which is used to solve general nonlinear optimization problems. The first step in implementing this method is to choose the initial values for the optimization variables (ϕ_m and ψ_m). The initial guess is very important since it affects the number of iterations needed for convergence and can prevent convergence to local minima. In our case, a good initial guess can be obtained using a modification of the synthesis algorithm in [5]. First, we expand the elements of $\mathbf{U}^{-1}(\omega)$ as a Fourier series and obtain the expansion coefficients, $\tilde{a}_k$ and $\tilde{b}_k$. Then we obtain another set of coefficients $\tilde{\tilde{b}}_k$ by invoking unitarity

$$\left|\sum_{k=0}^{N-1}\tilde{\tilde{b}}_k e^{-jkT\omega}\right|^2 = 1-\left|\sum_{k=1}^{N}\tilde{a}_k e^{-jkT\omega}\right|^2. \tag{4}$$

The process of finding the coefficients $\tilde{\tilde{b}}_k$ from (4) is called spectral factorization and it gives rise to 2^N different sets of coefficients $(\tilde{\tilde{b}}_k)_i$ [11]. We define the column vectors $\tilde{\tilde{\mathbf{b}}}_i$ and $\tilde{\mathbf{b}}$ as vectors containing the coefficients $(\tilde{\tilde{b}}_k)_i$, and the "real" Fourier coefficients $\tilde{b}_k$, respectively. Next, we choose the set $(\tilde{\tilde{b}}_k)_{\mathrm{Im}in}$ which is the nearest to the real Fourier coefficients $\tilde{b}_k$ according to: $\min_i \|\tilde{\tilde{\mathbf{b}}}_i-\tilde{\mathbf{b}}\|$ where $\|\mathbf{b}\|\equiv\mathbf{b}^{+}\cdot\mathbf{b}$. Then the coefficients $\tilde{a}_k$ and $(\tilde{\tilde{b}}_k)_{\mathrm{Im}in}$ are associated with a_k and b_k and the synthesis algorithm is utilized to obtain the corresponding initial values for the optimization variables ϕ_m^0 and ψ_m^0. The optimization algorithm itself is an iterative process. In each iteration, the cost function is approximated as a second-order Taylor series in terms of the optimization variables. Corrections to the optimization variables are found by solving the second order sub-problem by means of Newton's like method [8].

To check our new design approach and to demonstrate the tradeoff between the filter complexity and the compensation efficiency, we have performed a numerical simulation. The PMD medium was emulated by a cascade of 500 randomly oriented birefringent waveplates. The DGD of each waveplate was 0.1 ps. The band of interest was chosen to be -500 GHz $<\ \Omega\ <$ 500 GHz so the period of the filter was $P = 1000$ GHz. The compensation filter comprised $N = 5, 6, 7, \text{and } 8$ stages. Each stage had a DGD of $1/P = 1$ ps. The magnitude and the phase of the elements of the desired Jones matrix for one example of a PMD medium are plotted in Fig. 1. Also plotted in Fig. 1 are the corresponding values of the compensation filter (for $N = 7$) in the initial stage of the optimization algorithm. The

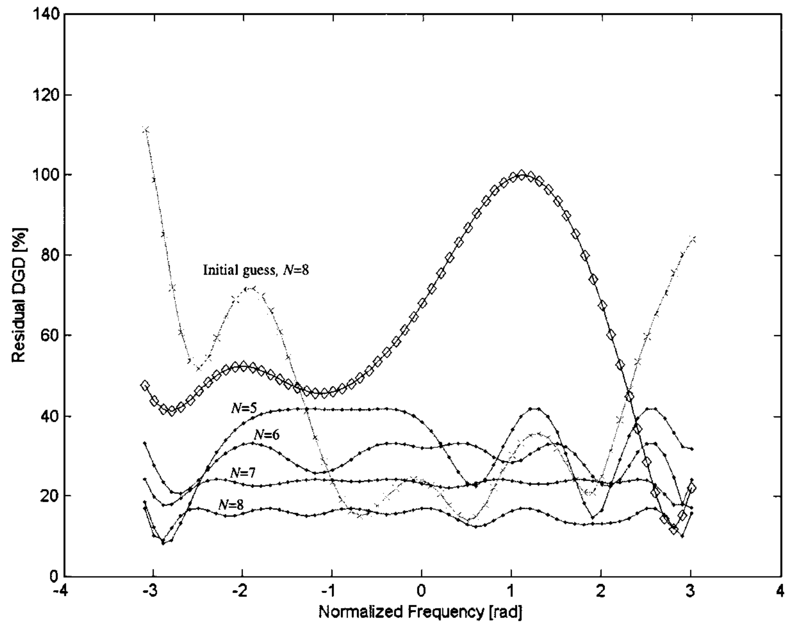

Fig. 2. The DGD of the uncompensated system (◇), the residual DGD after compensation by a Fourier based synthesized filter with 8-stages (designed according to [5]) (×) and the residual DGD after compensation by optimized filters with 5, 6, 7, and 8 stages (●) relative to the maximum DGD in the uncompensated medium.

errors in magnitudes between the desired h_{11} and h_{21} and the corresponding initial values of the filter elements are similar in both elements. In contrast, the phase error in h_{11} is clearly much smaller than in h_{21}. This is a result of the procedure for obtaining the initial conditions for the filter. In this procedure, the value of the synthesized h_{21} is chosen from a finite set of valid options, so that the coefficients $(\tilde{\tilde{b}}_k)_{\mathrm{Imin}}$ can be significantly different from $\tilde{b}_k$. The DGD of the PMD medium before compensation together with the residual DGD after compensation are plotted in Fig. 2. It can be seen that for the Ns chosen the residual DGD reduces as N increases by roughly 8%–9% for each additional stage. Also plotted in Fig. 2 is the residual DGD for an 8-stages filter that was designed according to [5] with the essential modifications needed to satisfy the unitarity condition that were described above. The angles found by this approach were used as an initial guess for the optimization algorithm. It can be seen that this filter provides only partial compensation of PMD and for some frequencies it even increases the DGD. In contrast, the optimized PMD filters provide significant PMD cancellation over the entire transmission band with uniform ripple. Similar results were obtained in all the examples that were tested. The number of iterations required for the optimization algorithm to converge depended strongly on the initial guess and on the termination criterion. It was found that for a randomly chosen initial guess the number of iterations was typically bigger than 100. By using the modified synthesis procedure of [5] to obtain an initial guess the number of iterations reduced to 10–60 depending on the termination criterion. Significant improvement of PMD compensation compared to the initial guess was obtained already after ten iterations. In applications where the algorithm will be used to dynamically correct for small perturbations in the JM of a PMD medium, it is expected that the number of iterations will be even smaller.

In summary, we have presented a new design method for a PMD compensating filter. This method uses an optimization algorithm to minimize the maximum DGD in a given transmission band. The approach was tested on several different examples of PMD media and performed significantly better than synthesis of the inverse of the transmission matrix using its Fourier expansion. The optimization algorithm can be used as a basis for control algorithms of broad-band multistage PMDCs.

ACKNOWLEDGMENT

The authors would like to thank Y. Li and G. T. Paloczi for helpful discussions about this letter.

REFERENCES

[1] B. W. Hakki, "Polarization mode dispersion compensation by phase diversity detection," *IEEE Photon. Technol. Lett.*, vol. 9, pp. 121–123, Jan. 1997.

[2] D. Sobiski, D. Pikula, J. Smith, C. Henning, D. Chowdhury, E. Murphy, E. Kolltveit, and F. Annunziata, "Fast first-order PMD compensation with low insertion loss for 10Gbit/s system," *Electron. Lett.*, vol. 37, pp. 46–48, Jan. 2001.

[3] R. Noe, D. Sandel, M. Yoshida-Dierolf, S. Hinz, V. Mirvoda, A. Schopflin, C. Glingener, E. Gottwald, C. Scheerer, G. Fischer, T. Weyrauch, and W. Haase, "Polarization mode dispersion compensation at 10, 20, and 40 Gb/s with various optical equalizers," *J. Lightwave Technol.*, vol. 17, pp. 1602–1616, Sept. 1999.

[4] Y. Li, A. Eyal, and A. Yariv, "Higher order error of discrete fiber model and asymptotic bound on multistaged PMD compensation," *J. Lightwave Technol.*, vol. 18, pp. 1205–1213, Sept. 2000.

[5] L. Moller, "Filter synthesis for broad-band PMD compensation," *IEEE Photon. Technol. Lett.*, vol. 12, pp. 1258–1260, Sept. 2000.

[6] C. K. Madsen, "Optical all-pass filters for polarization mode dispersion compensation," *Opt. Lett.*, vol. 25, pp. 878–880, June 2000.

[7] M. Shtaif, A. Mecozzi, M. Tur, and J. A. Nagel, "A compensator for the effects of high-order polarization mode dispersion in optical fibers," *IEEE Photon. Technol. Lett.*, vol. 12, pp. 434–436, Apr. 2000.

[8] L. Moller and L. Buhl, "Method for PMD vector monitoring in picosecond pulse transmission systems," *J. Lightwave Technol.*, vol. 19, pp. 1125–1129, Aug. 2001.

[9] P. C. Chou, J. M. Fini, and H. A. Haus, "Demonstration of feed-forward PMD compensation technique," *IEEE Photon. Technol. Lett.*, vol. 14, pp. 161–163, Feb. 2002.

[10] R. Fletcher, *Practical Methods of Optimization*, 2nd ed. New York: Wiley, 1987.

[11] S. P. Wu, S. Boyd, and L. Vandenberghe, "FIR filter design via spectral factorization and convex optimization," in *Applied and Computational Control, Signals and Circuits*, B. Datta, Ed. Cambridge, MA: Birkhauser, 1998, vol. 1, ch. 5, pp. 215–245.

[12] F. Roy, "A simple dynamic polarization mode dispersion compensator," in *OFC' 99*, 1999, Paper TuS4.