* Professor L. V. King of McGill University has published a pamphlet on "Gyromagnetic Electrons and a Classical Theory of Atomic Structure and Radiation," Mercury Press, Montreal. In the present note the classical views of Professor King are not adhered to. However, it may be that there is a connection between the two treatments.

[1] *Nature*, February 20, 1926, p. 264.
[2] *Proc. Nat. Acad. Sci.*, **12**, 1926, pp. 80–85.
[3] *Zs. Physik*, **35**, 1926, p. 618.
[4] *Nature*, April 10, 1926.
[5] *Zs. Physik*, **36**, 1926, p. 259.
[6] Lorentz, *Theory of Electrons*, Teubner, **1916**, pp. 252–254.

THE QUANTUM THEORY AND THE BEHAVIOR OF SLOW ELECTRONS IN GASES

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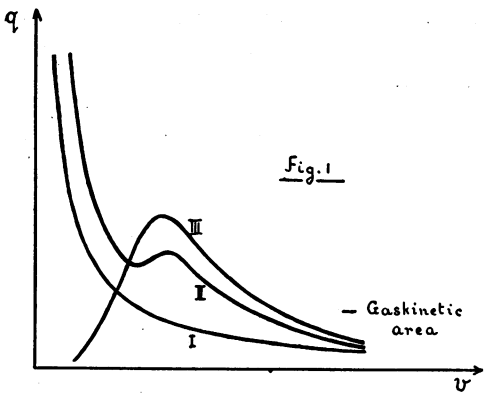
The new quantum theory[1] is based on the consideration that the only observable frequencies of the atoms and molecules are the so-called jump-frequencies given by the equation $\epsilon = h\nu$. In fact the experiments have shown that these are responsible for spectrum lines, anomalous dispersion and inelastic impacts. The revolution frequencies of the electrons in the stationary orbits, as given by the Bohr calculations, it is assumed can never be detected, so that Heisenberg[1] eliminated them from the theory intending to operate with observable data only. There are, however some experiments which have not yet been sufficiently discussed in relation to this problem: namely, the beautiful experiments on the behavior of slow electrons in gases. ("Slow" means that the electrons have less energy than corresponds to a resonance potential of the gas under investigation.) These experiments can be divided into two groups. One class deals with the deviations from the rectilinear motion which the electrons undergo in the field of force of the atoms, while the second class investigates the energy transfer. In this paper we give some considerations on the first type of experiments. The question of the energy transfer will be discussed in a following paper.

Deflections of the Electrons.—Experimentally the passage of electrons of definite velocities through gases under low pressures is observed. A quantitative measure of the deviation from rectilinear motion is obtained by introducing the concept of the effective area of an atom. This quantity $q(\varphi_k)$ is defined to be the area, surrounding the center of the atom, through

which the initial line of motion of the electron must pass, in order that it undergo a deflection greater than a certain critical angle φ_k determined by the construction of the apparatus. We should expect $q(\varphi_k)$ to be a function of the velocity (v) of the passing electrons, and experimentally[2] there are found to be three types of relation between q and v, as is shown in figure 1.



In practice φ_k is always chosen very small.[2] This is very convenient for the theoretical discussion because, under these circumstances, $q(\varphi_k)$ can be calculated by an application of the perturbation theory. One proceeds as follows: the undisturbed motion of the atomic system being assumed to be known, the perturbing force is taken equal, to a first approximation, to that exerted by an electron passing the atom in a straight line. The modified motion of the atom under this force is calculated and then the reaction of the disturbed atom on the electron is determined. This gives us a first approximation to the deflection angle φ. It can be shown, in this manner, that

$$\varphi \sim \frac{U(a)}{mv^2/2}\,\Phi\left(\frac{\tau_1}{\tau_2}\right) \qquad (1)$$

where $U(a)$ is the potential energy which the electron would have at a distance a from the atom and a is the shortest distance from the atom to the initial line of motion. The time τ_1 is the characteristic period of the unperturbed atom, and $\tau_2 = a/v$ gives a measure for the time during which the coupling between atom and electron is effective.

The mathematical theory leading to formula (1) will be published separately. Some justification for (1) on the basis of dimensional calculations can be given directly. As φ is without any dimension it must be a function of dimensionless quantities only. Now with the different parameters entering into the problem there can be found only three independent dimensionless quantities, namely, $p_1 = U(a)/(mv^2/2)$, $p_2 = \tau_1/\tau_2$, and $p_3 = m/M$, where m is the mass of the electron and M the mass of the atom. As p_3 is very small and practically of no influence we do not take it into account and get φ as a function of p_1 and p_2 alone. The physical significance of this result is simple: the deflection increases as the ratio of the potential energy $U(a)$ to the kinetic energy $mv^2/2$ of the electron, while the function $\Phi(\tau_1/\tau_2)$ evidently indicates a sort of resonance.

Formula 1 is quite general, and it will be sufficient to apply it to two simple atomic models to show that it is capable of accounting for the curves of figure 1.

(*A*). *Polarizable Atoms.*—The atom, as a little planetary system, possesses the general property of deformability under the influence of external forces. Thus an electric field E will induce an electric dipole in the atom of the magnitude αE where α is a constant. The energy of this dipole in the field E is $-\alpha E^2/2$. Hence $U(a) = -\alpha e^2/2a^4$ and

$$\varphi \sim \frac{\alpha}{a^4 v^2} \cdot \frac{e^2}{m} \sim \frac{\alpha}{q^2 v^2}, \qquad \text{since } q = \pi a^2. \quad (2)$$

Keeping φ constant, as is done in the experimental arrangements, we have

$$qv = \text{const.} \sqrt{\alpha}.$$

Considering for a moment only the conditions in one and the same gas (α = const.) we obtain

$$qv = \text{const. (Hyperbola)}.$$

This relation is beautifully confirmed by some recent[2] experiments on H_2 and the vapors of Zn, Hg and Cd. The absolute values of q calculated on the basis of the rigorous theory, which was given in a previous paper,[3] check the experimental data within 10%.

A rough comparison of different gases can also be made because for electrons of the same velocity, q^2/α should be constant, or since α is proportional to the molar refractivity P, q^2/P should be constant. Taking the values of P given by optical experiments we obtain the following results.

TABLE 1

	P IN CM.^{3}	Q IN CM.^{2}/CM.^{3}	Q^2/P
Cd	20	90	405
Hg	14	80	460
Zn	16	80	400
H_2	2	32	512

Q is defined as $Q = 3.56 \cdot 10^{16} q$, where $3.56 \cdot 10^{16}$ is the number of molecules per cm.^{3} at 1 mm. pressure. The values of Q given in the table are those found for 2-volt electrons.

The agreement is sufficiently good. More accurate computations on the basis of the complete theory will be given in another place.

(*B*). *Atoms Having a Permanent Asymmetry.*—In general it will not be sufficient to take into account merely the induced dipole. The atom may possess some permanent asymmetry, so that it must be considered as an electric dipole or quadrupole. This fact is familiar from the theories of dielectric constant, equation of state, etc. However, the atom may display one type of asymmetry in its reaction to a slowly changing field, and

another when reacting to a field varying as rapidly as that of a passing electron. Thus a single charged nucleus with an electron revolving in a circle around it would behave as an electric quadrupole in dielectric constant measurements, but would act as a dipole (revolving, it is true) when acting on an electron passing it with a velocity comparable to that of the orbital electron. It follows that in our case the atom (even of a noble gas) will act to the first approximation as an electric dipole (of moment μ), rotating or oscillating with the frequency ω, even though it shows higher symmetry in other phenomena.

Since for a dipole $U(a) \sim \mu e/a^2$, we will have

$$\varphi \sim \frac{\mu e}{a^2 m v^2} \Phi\left(\frac{\omega a}{v}\right). \qquad (3)$$

For φ = const. and not considering the factor Φ, there would result

$$qv^2 = \text{const.}$$

The effective area must increase with decreasing v much more rapidly than in case A. This is the case for all gases which do not follow the simple curve 1.

Furthermore we would expect μ, and with it q, to increase with the atomic volume for atoms in the same column of the periodic system. This effect is found also and is especially conclusive when the data for He, Ne, Ar, Kr and Xe are compared.

It is also seen that q must be greater than one would expect it to be, supposing the polarizability alone to cause the deflections of the electrons. Experimentally this is always the case. As an example we may cite Argon (P = 4.17 cm.^{3}) which has an area Q = 70 cm.^{2}/cm.^{3}, whereas Zinc (P = 15 cm.^{3}) in spite of its much higher polarizability has only Q = 30 cm.^{2}/cm.^{3} (Q is here the real effective area of an atom for 4-volt electrons multiplied by the number of atoms present in one cm.^{3} at 1 mm. pressure and 0°C.).

Finally for this type of atoms there must occur a decrease of q at the velocity for which v/a becomes comparable with ω. This will give a maximum in the q-v curve. This is found in every gas possessing the other characteristics just discussed. Unfortunately, a quantitative comparison is impossible, as we do not know the revolution frequencies in complicated atoms. (The investigation of atomic hydrogen which would be of extreme interest and importance, has not yet been carried out experimentally.) However, this difficulty is removed when we accept the principles of the new quantum theory. According to them we must replace the revolution frequencies by those of the absorption lines belonging to the normal state. Table 2 is based on this hypothesis. If the velocity v for which q = max.

is given in volts (V_1) and V_2 is the first resonance potential of the atom, $V_2/\sqrt{V_1}$ should be approximately constant for atoms having the same external shells.

TABLE 2

	V_2	$\sqrt{V_1}$	$V_2/\sqrt{V_1}$
Ne	16.6	4.5	3.7
Ar	11.5	3.5	3.3
Kr	9.9	3.1	3.2
Xe	8.3	2.3	3.6

Helium is not given, since its external shell is different from that of the other inert gases it cannot be compared directly with them. In fact it deviates widely, as its ratio $V_2/\sqrt{V_1}$ is equal to 13.5.

Comparing the stray fields of different atoms in the periodic system, we can deduce from our discussion the following very remarkable fact. The atoms with complete external shells as Ne, Ar, Xe and Kr show very small stray fields as far as the average over long times is concerned. The momentary constellation, however, has the character of an electric dipole. This means that the phases of the four (Stoner[4]) or eight revolving electrons (Bohr) are such as to put their instantaneous center of gravity always outside of the nucleus. Its average position, however, coincides with the nucleus. The same consideration is valid for CH_4 which has a complete shell also. Atoms or molecules with two revolving electrons such as Zn, Cd, Hg, He and H_2 show very small stray fields even in their momentary aspects, as follows from our results, that no effect attributable to an appreciable asymmetry is to be found in the action of these atoms on passing electrons. This means that the two electrons revolve in such phase relation, that their center of gravity is nearly at rest.

The results connected with formula 3 and table 2, require us to assume that Bohr's set of "virtual" oscillators has a real existence, and acts mechanically on passing electrons. It seems further, on account of the resonance confirmed by the experiments, as if the optical frequencies were really the revolution frequencies. This suggests that the old discrepancy between their values, as given by classical calculations, and the observed values from spectral lines is due to some misinterpretation of the observed facts. As indeed the observer is not situated in the same high electric field as the electron, his unit of time may differ from that of the electron. We shall not develop this idea further in the present paper. Neither will we discuss the very important question of the Ramsauer effect of great transparency of some atoms for slow electrons as shown by curve 3, as the author cannot contribute anything new to the suggestions made by previous writers on the subject.[3,5]

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[1] W. Heisenberg, *Zs. Physik*, **33**, p. 879, 1925; M. Born und P. Jordan, *Ibid.*, **34**, p. 859, 1925.

[2] For a detailed account of the methods reference may be made to C. Ramsauer, *Ann. Physik.*, **72**, p. 345, 1923, and *Jahrb. Radioaktivität*, **19**, p. 345, 1923; R. B. Brode, *Physic. Rev.*, **25**, p. 636, 1925, and *Proc. of the Royal Soc.*, **109**, p. 397, 1925.

[3] F. Zwicky, *Physik. Zeitschr.*, **24**, p. 171, 1923.

[4] E. C. Stoner, *Phil. Mag.*, **48**, p. 719, 1924.

[5] F. Hund, *Zs. Physik*, **13**, p. 241, 1923; G. Wentzel, *Ibid.*, **15**, p. 172, 1923.

TRANSFER OF ENERGY FROM ELECTRONS TO ATOMS

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In the preceding paper[1] we have assumed that the electric dipole induced in an atom by a passing electron has the value corresponding to a static field which is equal to the instantaneous field of the electron. This is, however, not quite exact. Due to the rapid variation of the field there will be some sort of a time lag. The deflections will therefore be smaller than calculated in section A of preceding paper. But the most important result of the more rigorous calculation is that it yields an approximation to the transfer of energy which escaped our attention while we were using the approximation method A in the mentioned publication.[1] It is possible to solve also this problem on the basis of the perturbation theory. As the general treatment involves very complicated mathematics we will demonstrate our ideas with a simple model which gives all the principal effects. We choose for this purpose a linear oscillator with the characteristic frequency $\nu_0 = \omega/2\pi$. We then calculate the perturbation caused by an electron passing it with the velocity v. The axis of the oscillator will be taken normal to the unperturbed straight line orbit of the electron, and the distance from the oscillator to the intersection of both lines we call a. The elastic constant of the oscillator is f and m_0 its movable mass (with the coördinate x) which bears the charge ϵ. We have then, assuming the inverse square law

$$m_0 \frac{d^2x}{dt^2} + fx = \frac{e\epsilon a}{(a^2 + v^2t^2)^{3/2}} = F(t); \qquad \omega^2 = \frac{f}{m_0}. \tag{1}$$

The particular integral $\bar{x}$ of differential equations of this type can be found in a general way in putting

$$\bar{x} = A(t) \sin \omega t + B(t) \cos \omega t$$

where $A(t)$ and $B(t)$ have to be determined as functions of t, so as to satisfy