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# Application of time reversal to thermoacoustic tomography

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## ABSTRACT

Reconstruction for thermoacoustic tomography in an arbitrary detection geometry is proposed by time-reversing the measured field back to the time when the thermoacoustic sources are excited. Time reversal of the field can be implemented efficiently by applying the delay-and-sum algorithm. The theoretical conclusions are supported by a numerical simulation of three-dimensional thermoacoustic tomography.

**Keywords:** time reversal, thermoacoustic tomography

## 1. INTRODUCTION

Developing reconstruction methods in thermoacoustic tomography (TAT) has been a craft work. The derivation of an algorithm depends strongly on the specific detection configuration, and exact reconstruction algorithms are only available in a few special geometries. Although it has been noted that the energy deposition, the to-be-reconstructed value in TAT, is equivalent to the initial pressure distribution, no method for deriving the initial pressure distribution from the signals measured across the detection surface ( or to time-reverse the measured signals) is available. In our recent work,[1] we find that when time reversal is considered in the time domain, exact time-reversal methods that use only the field on an arbitrary closed surface can be derived for a wide variety of applications such as tomography with diffracting sources, inverse diffraction, and ultrasound therapy. In this paper, we apply time-reversal methods to deriving exact and approximate reconstruction algorithms for TAT in an arbitrary configuration for the first time.

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In the METHOD section, we first show that the energy deposition in TAT is equivalent to the initial pressure distribution. Then, we introduce some basic concepts of time reversal and an exact time-reversal formula as well as an approximate one in the form of delay-and-sum algorithm. In the RESULTS section, we present a reconstruction formula in TAT by time-reversing the acoustic field to the time when the to-be-reconstructed point is excited. The reconstruction algorithm is verified by a numerical simulation of 3-D TAT. Conclusions are presented in the final section.

## 2. METHOD

### 2.1. Reconstruction in TAT

In this subsection, we will show that the energy deposition is equivalent to the initial pressure distribution in TAT. The thermoacoustic signal $p_0(\mathbf{r},t)$ is related to the energy deposition $H(\mathbf{r},t) = I_0\eta(t)\varphi(\mathbf{r})$ by the following wave equation

$$\nabla^2 p_0(\mathbf{r},t) - \frac{1}{v_s^2}\frac{\partial p_0(\mathbf{r},t)}{\partial t^2} = -\frac{\beta}{C}\frac{\partial H(\mathbf{r},t)}{\partial t}\,, \tag{1}$$

where $I_0$ is a scaling factor proportional to the incident radiation intensity; $\varphi(\mathbf{r})$ describes the microwave absorption properties of the medium at $\mathbf{r}$; $\eta(t)$ describes the shape of the irradiating pulse and is a nonnegative function whose integration over time equals the pulse energy; $C$ is the specific heat; $\beta$ is the coefficient of volume thermal expansion; and $v_s$ is the acoustic speed in the background. We will consider a delta pulse (the effect of finite pulse length has been discussed elsewhere[2]). Substituting a delta pulse into (1), and applying the integration of $t$ over $[0-, 0+]$ twice, where 0+ (0-) is an infinite small, positive (negative) real number, we have

$$p_0(\mathbf{r}, 0+) = \varepsilon\varphi(\mathbf{r}), \tag{2}$$

where $\varepsilon = \beta v_s^2 I_0/C$. It is assumed that the thermoacoustic pressure is finite. Therefore, the reconstruction in TAT is equivalent to reverting the pressure field to time zero. This concept can also be adopted to derive the TAT reconstruction algorithm in a known acoustically heterogeneous medium.

### 2.2. Time reversal

The time reversal of an acoustic or electromagnetic wave is based on the invariance of the wave equation in a lossless medium under the transform $t \to -t$ ($t$ represents the time). The time reversal of a wave can be understood as the generation of the back-propagation field from the measured forward-propagation field and/or its normal derivative after removing the initial sources. The concept of time reversal has been implemented experimentally and applied to a wide range of studies such as inverse scattering,[3–5] wavefront distortion correction,[6,7] and multiple scattering phenomena.[8] Here we will introduce some basic concepts of time reversal and an exact time-reversal formula as well as an approximate one in the form of delay-and-sum. We start from the wave equation for pressure $p_0(\mathbf{r},t)$ in a non-absorbing and non-dispersive medium[9]

$$\nabla^2 p_0(\mathbf{r},t) - \frac{1}{v_s^2}\frac{\partial^2 p_0(\mathbf{r},t)}{\partial t^2} = -q(\mathbf{r},t) \ , \tag{3}$$

where $q(\mathbf{r},t)$ is the source term, which is nonzero only in $R$ [the space enclosed by the detection surface $\Sigma$] and within the time period $[0, T_s]$. We have

$$p_0(\mathbf{r},t) = \int_0^{T_s} dt_0 \int_R d\mathbf{r}_0 q(\mathbf{r}_0,t_0) g_+(\mathbf{r},t\,|\mathbf{r}_0,t_0) \ , \tag{4}$$

where $g_\pm(\mathbf{r},t\,|\mathbf{r}_0,t_0) = \delta(t - t_0 \mp |\mathbf{r}-\mathbf{r}_0|/v_s)\,/(4\pi\,|\mathbf{r}-\mathbf{r}_0|)$ is a diverging ($g_+$) or converging ($g_-$) Green's function.

Time reversal of $p_0(\mathbf{r},t)$ at time $T_0$, is defined as

$$p_r(\mathbf{r},T_0) = p_0(\mathbf{r},T_0), p_r'(\mathbf{r},T_0) = -p_0'(\mathbf{r},T_0) \ , \tag{5}$$

where prime represents the temporal derivative in this paper; $p_r(\mathbf{r},t)$ is the time-reversed field of $p_0(\mathbf{r},t)$; and $T_0$ is chosen to be large enough so that $p_0(\mathbf{r},t) = 0$ for $\mathbf{r} \in R, t > T_0$. Then, $p_r(\mathbf{r},t)$ can be uniquely determined by the initial conditions at $T_0$. The above definition of time reversal is analogous to the fact that a particle will move back along its trajectory if its velocity is reversed and position unchanged. According to this definition, we mean $p_r(\mathbf{r},\hat{t})$ when we say time-reversing a field back to time $t$, where a hat over a time variable $t$ represents $2T_0 - t$.

In the case of a point source $\delta(\mathbf{r}-\mathbf{r}_0)\delta(t-t_0)$ in $R$, (5) becomes

$$g_r(\mathbf{r},T_0\,|\mathbf{r}_0,t_0) = g_+(\mathbf{r},T_0\,|\mathbf{r}_0,t_0), g_r'(\mathbf{r},T_0\,|\mathbf{r}_0,t_0) = -g_+'(\mathbf{r},T_0\,|\mathbf{r}_0,t_0). \tag{6}$$

It can be easily verified that the time-reversed field

$$g_r(\mathbf{r},t\,|\mathbf{r}_0,t_0) = g_-(\mathbf{r},t\,|\mathbf{r}_0,\hat{t_0}) - g_+(\mathbf{r},t\,|\mathbf{r}_0,\hat{t_0}), t > T_0 \ , \tag{7}$$

because it satisfies both the homogeneous wave equation and the initial-value conditions shown above. This result is also obtained by Cassereau[7] and Porter.[10] (7) means that if we time-reverse, at $T_0$, the field of a point source which is located at $\mathbf{r}_0$ and excited at $t_0$, the time-reversed field converges to $\mathbf{r}_0$ at time $\hat{t_0}$ and then diverges with an opposite amplitude. The diverging wave $g_+(\mathbf{r}, t\left|\mathbf{r}_0, \hat{t_0}\right)$, however, does not have a counterpart in the forward propagation. It exists because, unlike the forward propagation, there is no source inside $\Sigma$ for $g_r(\mathbf{r}, t)$. The diverging wave with an opposite amplitude exactly cancels the source term related to the converging one.

In the case of an arbitrary source, similar results can be obtained after considering the linearity of the wave equation with respect to the source

$$p_r(\mathbf{r}, t) = \int_0^{T_s} dt_0 \int_R d\mathbf{r}_0 q(\mathbf{r}_0, t_0) g_r(\mathbf{r}, t\left|\mathbf{r}_0, t_0\right) \ . \tag{8}$$

After substituting (7) into (8), using $g_-(\mathbf{r}, t\left|\mathbf{r}_0, t_0\right) = g_+(\mathbf{r}, -t\left|\mathbf{r}_0, -t_0\right)$, and a variable transform, we obtain for $\mathbf{r}_d \in \Sigma$ ( although it is valid for any $\mathbf{r}$)

$$p_r(\mathbf{r}_d, t) = p_0(\mathbf{r}_d, 2T_0 - t) + p_{div}(\mathbf{r}_d, t) \ , \tag{9}$$

$$p_{div}(\mathbf{r}_d, t) = -\int_{2T_0 - T_s}^{2T_0} dt_0 \int_R d\mathbf{r}_0 q(\mathbf{r}_0, \hat{t_0}) g_+(\mathbf{r}_d, t\left|\mathbf{r}_0, t_0\right) \ . \tag{10}$$

As in the case of a point source, the diverging component $p_{div}(\mathbf{r}_d, t)$ has no counterpart in the forward propagation and is, in general, not available from the experimental measurements of $p_0$ except in special cases. Nevertheless, it is shown that $p_r(\mathbf{r}, t)$ before a specified time can be derived using only $p_0(\mathbf{r}_d, t)$[1]

$$p_r(\mathbf{r}, t) = -\int_{T_0}^{t^+} dt_0 \oint_\Sigma dS_d p_0(\mathbf{r}_d, \hat{t_0}) \frac{\partial g_1(\mathbf{r}, t\left|\mathbf{r}_d\right., t_0)}{\partial n} \ , \tag{11}$$

where $\partial/\partial n$ is the derivative along the normal of $\Sigma$ at $\mathbf{r}_d$ pointing away from the volume $R$; $g_1(\mathbf{r}, t\left|\mathbf{r}_d, t_0\right)$ with $\mathbf{r}, \mathbf{r}_d \in (R \cup \Sigma)$ is Green's function subject to the homogeneous Dirichlet boundary condition on $\Sigma$; and $t^+$ is infinitesimally greater than $t$. An explicit expression of $g_1$, in the form of a series of eigenfunctions, can be found for the boundaries that fit with separable coordinates.[11] In TAT, after assuming that the object is not close to the detection surface and following the method shown in our recent work,[1] we arrive at the following approximate algorithm in the form of delay-and-sum

$$p_r(\mathbf{r},t) \approx \frac{1}{2\pi}\oint_{\Sigma} dS_d \frac{\mathbf{n}\cdot(\mathbf{r}_d-\mathbf{r})}{|\mathbf{r}-\mathbf{r}_d|^2}[\frac{p_0(\mathbf{r}_d,t_{rd})}{|\mathbf{r}-\mathbf{r}_d|} - p_0'(\mathbf{r}_d,t_{rd})/v_s] \;, \tag{12}$$

where $t_{rd} = 2T_0 - t + |\mathbf{r}-\mathbf{r}_d|/v_s$.

## 3. RESULTS

Combining (2) and (11) or (12), we have the following reconstruction algorithm in TAT

$$\varphi(\mathbf{r}) = \frac{1}{\varepsilon}p_0(\mathbf{r},0) = \frac{1}{\varepsilon}p_r(\mathbf{r},2T_0). \tag{13}$$

A three-dimensional TAT is numerically simulated (Fig. 1). The objects are two spheres with a radius of 8 mm. $\varphi(\mathbf{r})$ is set to be 1 in the sphere and zero otherwise. 2048 detection positions are randomly distributed over a sphere with a radius of 80 mm and the center at the origin. The imaging space is a cube with a side length of 112 mm, centered at the origin, and mapped with a 64 by 64 by 64 mesh. In the forward problem, $p(\mathbf{r},t)$ is computed by the integration of the object value along a series of spheres. The signals are within [0 1] MHz. The acoustic speed is 1500 m/s. In the inverse problem, we use (12) to time-reverse fields to save computation time. Figures 1(a)-(b) show that the shape and position of the object is reconstructed correctly. The line graphs in Fig. 1(c) show that the object is reconstructed quantitatively.

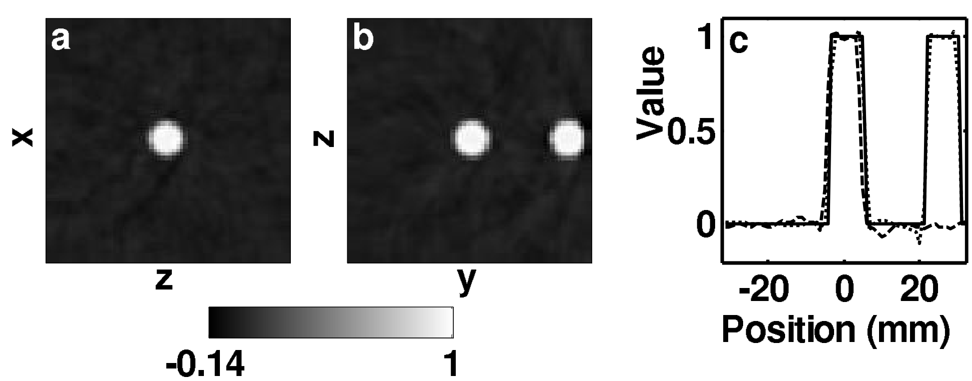


**Figure 1.** (a) Reconstructed image section in DT along the $y = 0$ and (b) $x = 0$ planes. (c) The line graphs along the $z = 0$ (dotted), $y = 0$ (dashed) lines in (b) and the corresponding real value (solid).

## 4. CONCLUSIONS

Reconstruction for thermoacoustic tomography in an arbitrary detection geometry is proposed by time-reversing the measured field back to the time when the thermoacoustic sources are excited. Time reversal of the field can be implemented efficiently by applying the delay-and-sum algorithm. The theoretical conclusions are supported by the numerical simulation of three-dimensional thermoacoustic tomography.

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