were invoked by Y. Ne'eman, Phys. Rev. 134, B1355 (1964), to explain the breaking of SU(3) symmetry. Ne'eman's "fifth force" is rather strongly coupled to matter, and this leads to a number of contradictions with experiment, as discussed by D. Beder, R. Dashen, and S. Frautschi, to be published. Our main argument, and the astrophysical argument at the end of this Letter, can also both be applied to the fifth force, but our main argument has the advantage of applying even when the coupling constant is extremely small.

[3]J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964); A. Abashian et al., Phys. Rev. Letters 13, 243 (1964). My own interest in the possible existence of gauge particles coupled to strangeness was revived by an earlier experiment by R. Adair et al., Phys. Rev. 132, 2285 (1963), which seemed to show an anomalous K_1^0 regeneration in liquid hydrogen, but which appears to be contradicted by the data of Christenson et al.

[4]J. S. Bell and J. K. Perring, Phys. Rev. Letters 13, 348 (1964).

[5]J. Bernstein, N. Cabibbo, and T. D. Lee, Phys. Letters 12, 146 (1964). [Note added in proof. –The same suggestion is made in unpublished work by F. Gürsey and A. Pais, quoted by T. N. Truong, Phys. Rev. Letters 13, 358 (1964).]

[6]Coupling a massless spin-one particle to a nonconserved current would violate the Lorentz invariance of the S matrix (unless the coupling vanished in the low-frequency limit, in which case there could be no long-range interaction). See S. Weinberg, Phys. Rev. 135, B1049 (1964); Phys. Letters 9, 357 (1964); and to be published.

[7]The hyperphoton is emitted by the incoming K-meson line, this being the only term which becomes of order m^{-2} at low hyperphoton energy. Corresponding formulas are well known in electrodynamics; see, e.g., J. M. Jauch and F. Rohrlich, Theory of Photons and Electrons (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955), p. 392.

[8]This was suggested to me by J. Bell. Using the earth as source means that the hyperphoton field is very anisotropic, so detection of this field in an Eötvös-type experiment[1] becomes possible in principle. The hyperphoton potential energy of a nucleon is supposed to be the same as for a K meson, or about 0.5×10^{-8} eV, while its gravitational potential energy is 0.7 eV, so the ratio of the gravitational to hyperphoton force should be 1.4×10^8. However, the hypercharge of ordinary matter is closely proportional to its mass, the ratio varying by only 0.89 % from hydrogen to iron; hence it would be necessary to look for differences in the apparent ratio of gravitational to inertial mass at most of order 1.5×10^{10}.

[9]The thought unavoidably crosses one's mind that perhaps the events seen by Christenson et al. and Abashian et al. are actually due to the CP-conserving decay of K_2^0 into two pions and a very soft hyperphoton. However, although the ordinary inner bremsstrahlung spectrum is sharply peaked at low photon energy, Eq. (2) shows that this peak is absent in the hyperphoton emission spectrum. (There would be a low-energy peak for emission of very light spin-zero particles, but they would have to couple oppositely to K^0 and $\bar{K}^0$, and this is theoretically unappealing.)

[10]There is a difficulty in taking vector particles of zero bare mass coupled to nonconserved currents, pointed out be G. Feinberg (to be published). We are supposing here that the bare mass is smaller than the self-mass, though not necessarily zero.

BOOTSTRAP THEORY OF OCTET ENHANCEMENT*

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Recently it has become apparent that many violations of SU(3) follow an octet pattern. For example, (i) the Gell-Mann–Okubo sum rule[1,2] represents the main features of strong mass splittings among members of a supermultiplet, indicating that the splittings transform like the eighth component of an octet; (ii) the electromagnetic mass splittings within the baryon isospin multiplets seem to transform mostly like the third component of an octet[3]; (iii) nonleptonic weak decays also seem to be dominantly octet in view of the $|\Delta\vec{I}|=\frac{1}{2}$ rule.[4] If, as is generally assumed, both the weak and electric currents have octet transformation properties, then second-order electromagnetic and weak phenomena such as (ii) and (iii) will generally contain both $\underline{8}$ and $\underline{27}$ terms. The observed octet dominance thus requires some explanation. In case (i), we do not know what causes the violation–possibly the basic mechanism is pure octet; but, in any case, one wonders why higher order effects do not introduce nonoctet terms, particularly $\underline{27}$ terms.

Extrapolating from cases (i)-(iii), Coleman and Glashow[3] have suggested that octet dominance is a very general feature of SU(3) symmetry breaking. To explain this phenomenon, they proposed an, as yet undiscovered, octet of 0^+ bosons which could enhance the octet violations of SU(3) through the "tadpole" mechanism.

An alternative mechanism for octet enhancement in terms of "bootstrap" ideas has been proposed by Cutkosky and Tarjanne.[5,6] The bootstrap mechanism does not specifically require octets of spin-zero particles, and employs S-matrix concepts which appear to be quite distinct from the field-theoretic ideas of the tadpole mechanism. The purposes of the present paper are to emphasize that the bootstrap mechanism can provide a simple, unified explanation of octet enhancement in strong, electromagnetic, and weak symmetry breaking, and to present a model of how octet enhancement works for mass splittings of the $J=\frac{1}{2}^+$ B octet and the $J=\frac{3}{2}^+$ Δ decuplet.

We begin by assuming the bootstrap theory of strong interactions, describing each strongly interacting particle as a bound or resonant state of strongly interacting particles. Unstable particles and stable ones are treated on the same footing. If SU(3) symmetry held exactly, each supermultiplet would appear as a set of degenerate poles in the various scattering amplitudes with appropriate quantum numbers.

Now for purposes of describing the octet enhancement mechanism, we consider the specific case of electromagnetic mass corrections of order e^2, ignoring the effect of strong violations. The electromagnetic interaction causes shifts in the positions of the poles in each supermultiplet from their original common value. To understand the behavior of the shifts, first consider the more familiar case of electromagnetic corrections to an isotopic-spin multiplet. The e^2 corrections have parts proportional to 1, T_3, and T_3^2. In terms of irreducible representations one has a singlet mass correction $dM_{1,1}$ transforming like $I=0$, a triplet correction $dM_{3,n}$ transforming like the nth component of the $I=1$ state (n runs from 1 to 3; in practice the T_3 component is involved), and a correction $dM_{5,n}$ transforming like the nth component of $I=2$. Similarly the e^2 corrections to an SU(3) supermultiplet include a shift $dM_{1,1}$ that transforms like a singlet, a shift $dM_{8,n}$ transforming like the nth component of an octet ($n=1,\cdots,8$), and a 27-plet term $dM_{27,n}$ ($n=1,\cdots,27$).

The shifts in mass of the bound or resonant supermultiplet states are caused in part by direct electromagnetic effects such as photon exchange. We call these effects the driving terms D. In our isotopic-spin example a driving term transforming according to a definite representation, e.g., $D_{3,n}$, drives only the mass shift transforming in the same way, e.g., $dM_{3,n}$. Similarly in SU(3), $dM_{8,n}$ is driven only by $D_{8,n}$, and so forth.

In the bootstrap theory, the shifts in mass of the bound states also occur in response to electromagnetic mass shifts of external particles and of exchanged particles. For example, if we think of the Δ decuplet as a resonance occurring in the $B+\Pi\to B+\Pi$ and other scattering amplitudes (Π is the 0^- octet), dM^Δ is affected by shifts dM^B in the external- and exchanged-baryon masses, by shifts dM^Π, by shifts dM^Δ in Δ exchange, and so forth. Just as with the direct driving terms D, a shift $dM_{8,n}{}^B$ affects only (in our lowest order considerations) the shift $dM_{8,n}{}^\Delta$ which transforms in the same way.[7] Thus we obtain relations for the octet symmetry violations of the form

$$dM_{8,n}{}^{B_s}=A_8{}^{B_sB_s}dM_{8,n}{}^{B_s}+A_8{}^{B_sB_a}dM_{8,n}{}^{B_a}+A^{B_s\Delta}dM_{8,n}{}^{\Delta}+\cdots+D_{8,n}{}^{B_s},$$

$$dM_{8,n}{}^{B_a}=A_8{}^{B_aB_s}dM_{8,n}{}^{B_s}+A_8{}^{B_aB_a}dM_{8,n}{}^{B_a}+A^{B_a\Delta}dM_{8,n}{}^{\Delta}+\cdots+D_{8,n}{}^{B_a},$$

$$dM_{8,n}{}^{\Delta}=A_8{}^{\Delta B_s}dM_{8,n}{}^{B_s}+A_8{}^{\Delta B_a}dM_{8,n}{}^{B_a}+A_8{}^{\Delta\Delta}dM_{8,n}{}^{\Delta}+\cdots+D_{8,n}{}^{\Delta}; \tag{1}$$

where the two independent octet terms in the baryon mass, the D type (labeled B_s) and F type (labeled B_a), must be kept separate. Note that the A matrix contains no SU(3) violation; that is why it connects $\underline{8}$ only to $\underline{8}$, n only to n, and is independent of n. The A matrix does connect B_s and B_a since D and F terms do mix even when SU(3) symmetry is preserved. Similar relations apply to the $\underline{1}$ and $\underline{27}$ shifts, except that each of these cases contains only one baryon term instead of two.

Let us relabel the mass relations, such as (1), in the form

$$(1-A_8)^{\alpha\beta}dM_{8,n}{}^{\beta}=D_{8,n}{}^{\alpha}, \tag{2}$$

where α and β run over the independent octets of mass matrices such as the Δ term, the D and F baryon terms, and so forth. We can invert (2), obtaining

$$dM_{8,n}^{\ \alpha} = \left(\frac{1}{1-A_8}\right)^{\alpha\beta} D_{8,n}^{\ \beta}. \tag{3}$$

Now in terms of Eqs. (1)-(3), the problem set forth at the beginning of this paper was that D_8 and D_{27} are of comparable magnitude but dM_8 emerges as the dominant mass splitting. It is easy to see how this can occur in our formalism. If one or more of the eigenvalues of A_8 is near unity, then $dM_{8,n}$ will contain a large term multiplying the associated eigenvector(s). If the matrix A_{27} lacks eigenvalues near unity, the octet is preferentially enhanced.

The mechanism just described for electromagnetic octet enhancement applies generally to any violations of SU(3) which are linear in the masses, since the eigenvalues and eigenvectors of A_8 and A_{27} are independent of the driving term D and independent of the axis n along which the violation lies in SU(3) space.[8] Thus, barring the unlikely case that the driving term has no component along the enhanced eigenvector, isospin-conserving strong mass splittings ($n=8$) and isospin-violating electromagnetic mass splittings ($n=3$) (as well as the hard-to-observe weak mass splittings) should all exhibit octet enhancement and all lie along the same eigenvector (i.e., all have the same ratios among the independent octet mass matrices such as the Δ term and the D and F baryon terms).

Nonleptonic weak interactions also appear to be octet dominated, but here it is couplings, not mass shifts, that one normally observes. The formalism we have discussed for linear mass shifts can evidently be generalized to include linear changes in coupling constants together with mass differences, so that our equations would be replaced by linear equations involving dg's and dM's. Again octet enhancement can result from finding octet eigenvalues near unity, with 27-plet eigenvalues far away, and a universal pattern of octet enhancement may emerge for the strong, electromagnetic, and (parity-conserving) weak SU(3) symmetry violations in coupling constants.[9]

To test the correctness of the bootstrap theory of octet enhancement, an extensive program of calculations involving the various supermultiplets will be required. We report here on a simple estimate of a small piece of the general problem —the mass shifts of the B octet and Δ decuplet. We specialize to the two-particle amplitude $B+\Pi \to B+\Pi$ for both supermultiplets, and use the static model with a linear denominator function. The left-hand cut is simplified to two poles representing B and Δ exchange, and the right-hand cut to two poles representing the appearance of B and Δ in the direct channel. We consider only mass shifts dM^B and dM^Δ in this approximate calculation, leaving the changes in coupling and pion mass shifts in the driving term.

To calculate the A matrix we first separate the effects of exchanged and external masses; i.e., we write $A = A_{\text{exch}} + A_{\text{ext}}$. Group theory can be used[5] to find ratios like $A_{1\ \text{exch}}{}^{\Delta B} : A_{27\ \text{exch}}{}^{\Delta B}$, $A_{1\ \text{ext}}{}^{BB} : A_{8\ \text{ext}}{}^{B_s B_s}, \cdots$; thus all the dynamics is contained in the A_1's. $A_{1\ \text{exch}}$ can be determined from a simple SU(3)-symmetric calculation using the S-matrix method of Dashen and Frautschi.[10] Then $A_{1\ \text{ext}}$ is determined by using the fact that the static model is invariant under a uniform shift of B and Δ masses.

The only free parameter in A is the F/D ratio λ for the $BB\Pi$ couplings. Several experimental and dynamical arguments[11] indicate that λ lies between 0.3 and 0.5. With λ in this range we find that the eigenvalues of A_{27} are far from unity, whereas A_8 always has an eigenvalue near one. The eigenvectors and eigenvalues of A_8 vary smoothly with λ and at $\lambda \approx 0.46$, A_8 has an eigenvalue of exactly one, whose associated eigenvector[12] is

$$dM_8^{\ B_s}/dM_8^{\ B_a} \approx -0.24, \quad dM_8^{\ \Delta}/dM_8^{\ B_a} \approx 1.15. \tag{4}$$

The appearance of an eigenvalue exactly equal to one is quite likely due to the present, oversimplified model. However, a study of some possible improvements of the model indicates that the qualitative features of this result, i.e., a large octet enhancement along a direction similar to that of Eq. (4), are a general property of the B-Δ system in SU(3) and would survive a more detailed calculation.

Keeping in mind the crude nature of the approximations made, we turn to the comparison with experiment. For the strong mass shifts

$$dM_8^{\ B_s}/dM_8^{\ B_a} \approx -0.25$$

and

$$dM_8^{\ \Delta}/dM_8^{\ B_a} \approx 1.25.$$

Neglecting strong violations of SU(3), the observed electromagnetic mass shifts in the B octet satisfy

$$dM_8^{\ B_s}/dM_8^{\ B_a} \approx -0.4 \pm 0.1.$$

Thus experiment supports the prediction that both strong and electromagnetic mass shifts lie along an eigenvector of A_8 whose eigenvalue is close to one.[13]

A series of more detailed papers treating the calculation of the A matrix, computation of some driving terms which may allow an estimate of the absolute magnitude of the baryon electromagnetic mass shifts, some effects of higher order terms, and other related topics is in preparation.

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[1]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[2]S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

[3]S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964).

[4]N. Cabibbo, Phys. Rev. Letters 12, 62 (1964).

[5]R. E. Cutkosky and P. Tarjanne, Phys. Rev. 132, 1355 (1963). The general nature of the bootstrap mechanism was stressed by R. E. Cutkosky, Bull. Am. Phys. Soc. 8, 591 (1963).

[6]Extensive work along related lines has been done by R. H. Capps; e.g., Phys. Rev. 134, B1396 (1964).

[7]S. L. Glashow, Phys. Rev. 130, 2132 (1963).

[8]If strong symmetry violation is spontaneous, $D_{\mathrm{strong}} = 0$ and the enhanced eigenvalue must equal unity in the linear approximation. When higher orders are included, however, the eigenvalue is no longer required to be exactly one. In any case the strong mass shifts are expected to lie along the eigenvector associated with eigenvalue $A_8 \sim 1$ whether or not $D_{\mathrm{strong}} = 0$.

[9]The parity-violating weak interaction can be studied with the same techniques, but in this case a different A matrix with generally different eigenvalues and eigenvectors is involved.

[10]R. F. Dashen and S. C. Frautschi, Phys. Rev. 135, B1190 (1964).

[11]A. Martin and K. Wali, Phys. Rev. 130, 2455 (1963); R. Dalitz, Phys. Letters 5, 53 (1963).

[12]Here the numbers $dM_8^{\Delta}, \cdots,$ are the coefficients of normalized octet mass matrices.

[13]The Coleman-Glashow model[3] likewise predicts that electromagnetic mass splittings will follow the same pattern as strong mass splittings, but does not predict ratios among the strong splittings since they are needed to fix the parameters of the model.

BROKEN SYMMETRIES AND WEAK INTERACTIONS. II.†

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(Received 14 September 1964)

In an earlier Letter with this title,[1] parity-preserving nonleptonic decays were related to the violation of SU(3) coupling-constant equalities. It was assumed, for the discussion of pionic hyperon decays, that SU(3) symmetry is strictly maintained within the π-baryon couplings and within the K-baryon couplings, but that the SU(3) equality of π and K coupling constants is broken. It is somewhat discomforting for this point of view to conclude that the fractional violation of the coupling-constant equality is significantly different for the two SU(3) coupling constants $f^{(1)}$ and $f^{(2)}$. We wish to point out that the situation is improved by including another contribution, the importance of which was not appreciated in I.

It was there remarked that weak mixing of the baryon octuplet and weak meson mixing cancel completely if the breakdown of SU(3) symmetry is limited to the mass displacements produced by the vacuon $\langle S_{33}\rangle$. There are other effects, however, which enable this parity-preserving decay mechanism to operate. One is the coupling-constant inequality, $f_K \neq f_\pi$. A second one is the weak mixing of the baryon octuplet with the singlet of the broken W_3 nonuplet. A third one is the failure of the Gell-Mann–Okubo octuplet mass formula, which we have attributed to strong octuplet-singlet mixing. The last effect was not considered in I, owing perhaps to the psychological influence of the oft repeated statement that the GMO formula is accurate to within one half of one percent. The more relevant number is the mass ratio

$$\rho = [\Lambda - \tfrac{1}{3}(2N + 2\Xi - \Sigma)]/(\Lambda - N) = 0.045,$$

and its consequences are not negligible.

The p-wave coupling constants implied by the three contributing factors are

$$f_{\Sigma^+} = -\theta_+[\tfrac{1}{2}\rho(f^{(1)} + f^{(2)}) + \tfrac{1}{3}\gamma f_{\Phi Y}];$$

$$f_{\Sigma^-} = \theta_+[\Delta f^{(2)} - \tfrac{1}{2}\rho(f^{(1)} + f^{(2)}) - \tfrac{1}{3}\gamma f_{\Phi Y}];$$

$$f_\Lambda = \theta_+ 6^{-1/2}[2\Delta f^{(1)} - \Delta f^{(2)} - 3\rho f^{(1)}];$$

$$f_{\Xi^-} = \theta_+ 6^{-1/2}\left[2\Delta f^{(2)} - \Delta f^{(1)} + 3\rho\frac{\Lambda - N}{\Xi - \Lambda} f^{(2)}\right].$$