Polarization ratios anomalies of 3D rough surface scattering as second order effects

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Abstract. In the present paper a detailed analysis of the behaviour of the electromagnetic scattering from various corrugated bi-dimensional surfaces is presented. We show that rigorous electromagnetic computations on two dimensional surfaces can in fact yield HH/VV polarization ratios greater than one, with values consistent with those observed experimentally. Furthermore we show that HH/VV ratios greater than one are ubiquitous in the case of surfaces of the form $f(x,y) = f_1(x) + f_2(y)$, known as crossed grating in optics. As demonstrated theoretically and numerically below these surfaces produce backscattered returns for which the first order Rice/Valenzuela term vanishes for off axis incidence. Further, the second order term becomes dominant and has the property that HH returns exceed VV returns for a significant range of incident angles. Our approach is based on the methods of [Bruno and Reitich, 1993] which yield accurate results for a large range of values of the surface height. In particular, these methods can be used well beyond the domain of applicability of the first order theory of [Rice, 1951]. The error in our calculations is guaranteed to be several orders of magnitude smaller than the computed values. The high order expansions provided by these methods are essential to determine the role played by the second order terms as they show that these terms indeed dominate most of the backscattering returns for the surfaces mentioned above. Classically, large HH/VV ratios were sought by means of first order approximations on one dimensional sinusoidal profiles. As we show below, in that case the first order terms do not vanish and the first order theories predict the behaviour of the backscattered returns, for small values of the height to period ratio. However, in the case of a two dimensional bisinusoidal surface, strong polarization dependent anomalies appear in the scattering returns as a *result* of the contributions of second order terms since, in that case, the first order contributions vanish.

1 Introduction

Recently, in the framework of remote sensing, experimental data [Trizna et al., 1991, Lee et al., 1997] has drawn attention to a peculiar feature of polarization effects of oceanic scattering. It was observed that radar cross sections for HH polarization (transmit H and receive H) can exceed radar cross sections for VV polarization (transmit V and receive V) in so called super-events. In the present paper we show that rigorous electromagnetic computations on two dimensional surfaces can yield HH/VV ratios greater than one, with values consistent with those observed experimentally. Furthermore we show that HH/VV ratios greater than one are ubiquitous in the case of surfaces of the form $f(x,y) = f_1(x) + f_2(y)$, (known as crossed grating in optics). As demonstrated below these surfaces produce

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backscattered returns for which the first order (Rice/Valenzuela) term vanishes for off axis incidence. Further, the second order term becomes dominant and has the property that HH returns exceed VV returns for a significant range of incident angles.

2 Perturbation expansions

We consider a time harmonic incident plane wave impinging on the doubly periodic surface $z = f(x,y)$, with x-axis period d_x and y-axis period d_y. We have: $f(x+d_x, y+d_y) = f(x,y)$. The incident electric field with wave vector $\mathbf{k}$ is :

$$\mathbf{E}^{inc} = \mathbf{A} \exp\left[i(\alpha x + \beta y - \gamma z)\right] \tag{1}$$

where the vector $\mathbf{A} = (A^1, A^2, A^3)$ specifies the state of polarization of the incident wave and $\mathbf{k}$ is determine from the incident angles ψ and θ as follows:

$$\mathbf{k} = \begin{pmatrix} \alpha = k\cos(\psi)\sin(\theta) \\ \beta = k\sin(\psi)\sin(\theta) \\ -\gamma = -k\cos(\theta) \end{pmatrix} \tag{2}$$

The angle θ is the angle between the vector $\mathbf{k}$ and the z-axis and the angle ψ is the angle between the projection of the vector $\mathbf{k}$ and the x-axis and $k = |\mathbf{k}| = 2\pi/\lambda$ where λ is wavelength of the incident radiation. The time harmonic Maxwell's equations for the scattered electric field $\mathbf{E}$ reduce to the following equations in the case of a perfect conductor:

$$\begin{cases} \Delta\mathbf{E}(x,y,z) + k^2\mathbf{E}(x,y,z) = 0 \qquad \nabla\cdot\mathbf{E}(x,y,z) = 0 \\ \mathbf{n}\times\mathbf{E}(x,y,f(x,y)) = -\mathbf{n}\times\mathbf{E}^{inc}(x,y,f(x,y)) \end{cases} \tag{3}$$

where $\mathbf{n}$ is the normal to the surface $z = f(x,y)$. To derive a perturbation series for the solution of this scattering problem we introduce the surface $f_\delta(x,y) = \delta f(x,y)$ where δ is a complex number, see [Bruno and Reitich, 1993]. The scattered field $\mathbf{E}(x,y,z;\delta)$ associated to the surface $f_\delta(x,y)$ can be written and computed as a Taylor series expansion in powers of δ, as follows:

$$\mathbf{E}(x,y,z;\delta) \;=\; \mathbf{E}(x,y,z;0) + \mathbf{E}_\delta(x,y,z;0)\delta + \mathbf{E}_{\delta\delta}(x,y,z;0)\frac{\delta^2}{2} + \dots \tag{4}$$

Solving the scattering problem for the surface $z = f(x,y)$ then amounts to evaluating the series (4) at $\delta = 1$.

As is known, see [Petit, 1980], the field scattered from a bi-periodic surface can be represented outside the groove region (that is for $z > \max f(x,y)$) as a sum of outgoing plane waves with certain amplitudes $\mathbf{B}_{p,q}$ as follows $\mathbf{E}(x,y,z) = \sum_{p,q} \mathbf{B}_{p,q}\, e^{i\alpha_p x + i\beta_q y + \gamma_{p,q} z}$ where

$$\alpha_\ell = \alpha + \ell K_x \qquad \beta_m = \beta + mK_y \qquad \gamma_{\ell,m} = \sqrt{k^2 - \alpha_\ell - \beta_m} \tag{5}$$

and where the surface $z = f(x,y)$ is given by its Fourier series:

$$f(x,y) = \sum_{\ell,m=-F}^{F} f_{\ell,m}\, e^{i\ell K_x x + imK_y y} \qquad K_x = \frac{2\pi}{d_x} \qquad K_y = \frac{2\pi}{d_y}$$

The outgoing (p,q) plane wave $e^{i\alpha_p x + i\beta_q y + \gamma_{p,q} z}$ will contribute backscatering returns if the following 3D Bragg conditions are satisfied:

$$\alpha_p = -\alpha \qquad \beta_q = -\beta \qquad \gamma_{p,q} = \gamma \tag{6}$$

From the definitions (2) and (5), the conditions (6) are equivalent to the following:

$$\frac{\lambda}{d_x} = \frac{2\sin(\theta)}{\sqrt{p^2 + \left(\frac{d_x}{d_z}\right)^2 q^2}} \qquad \tan(\psi) = \frac{q}{p}\frac{d_x}{d_y} \tag{7}$$

For a given incidence (θ, ψ), for which p and q satisfy (7), it can be shown that the first order backscattered field is given by:

$$\mathbf{E}_\delta(x,y,z,;0) = \begin{pmatrix} 2i(\gamma A^1 + 2\alpha A^3) \\ 2i(\gamma A^2 + 2\beta A^3) \\ -2i(\alpha A^1 + \beta A^2) + 4i\frac{A^3}{\gamma}(\alpha^2 + \beta^2) \end{pmatrix} f_{p,q}\, e^{i\alpha_p x + i\beta_q y + \gamma_{p,q} z} \tag{8}$$

3 Numerical Results - Perfectly conducting surfaces

We consider the simplest two dimensional surface namely a bisinusoid surface with periods $d_x = d_y = 1$ and height h defined by:

$$f(x,y) = \frac{h}{4}\left(\cos(2\pi x) + \cos(2\pi y)\right) \tag{9}$$

The fact that the surface $f(x,y)$ is of the form $f(x,y) = f_1(x) + f_2(y)$ implies that the Fourier coefficients $f_{\ell,m}$ of $f(x,y)$ are such that $f_{p,q} = 0$ for $p \cdot q \neq 0$. In this case, formula (8) shows that unless $p = 0$ or $q = 0$, that is unless $\psi = 0$ or $\psi = \frac{\pi}{2}$, the first order backscattered field vanishes. When the incident field is aligned with the x or y axis, that is when $\psi = 0$ or $\psi = \frac{\pi}{2}$, then the ratio of HH to VV backscattered returns turns out to be:

$$\frac{\sigma_{HH}(\theta,0)}{\sigma_{VV}(\theta,0)} = \frac{\sigma_{HH}(\theta,\pi/2)}{\sigma_{VV}(\theta,\pi/2)} = \left(\frac{\cos(\theta)^2}{1+\sin(\theta)^2}\right)^2 \tag{10}$$

which is the classical first order result found in [Valenzuela, 1978] for a perfect conductor (take the limit $\varepsilon \to +\infty$ in formulas 4.10 and 4.11 on page 211). When $\psi \neq 0$ or $\psi \neq \frac{\pi}{2}$, that is for $p \cdot q \neq 0$, the first order term vanishes (since $f_{p,q} = 0$ for $p \cdot q \neq 0$) and the second order term becomes dominant. This is illustrated in Figure 1 where the second order calculation is compared to a high order (21) converged reference solution (see [Bruno and Reitich, 1993, Sei et al., 1999] for details on the accuracy of the numerical algorithm used). Most interestingly, the second order term has the striking feature that HH returns exceed VV returns for a large range of incidence angles as illustrated in Figure 2, in sharp contrast to the first order returns. As expected from formula (8) the fact that the second order term is dominant is not a special feature of the bisinusoid surface (9); similar results were obtained for arbitrary surfaces of the form $f(x,y) = f_1(x) + f_2(y)$.

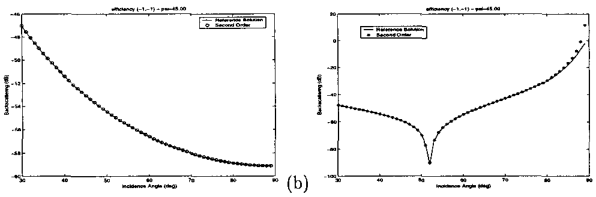

Figure 1: Comparison of the second order calculation to the exact high order calculation for the surface (9) with $h = 0.03$, $\psi = 45°$ and $30° \leq \theta \leq 89°$. (a) HH polarization. (b) VV Polarization.

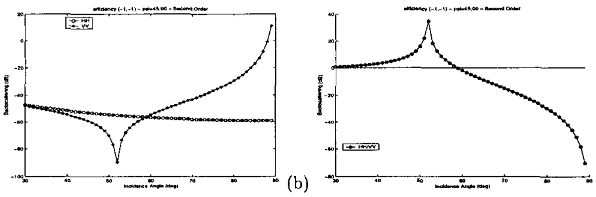

Figure 2: HH and VV backscattered returns for the surface (9) with $h = 0.03$, $\psi = 45°$ and $30° \leq \theta \leq 89°$ (a). Corresponding HH to VV ratios (b).

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