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Sava Sakadžić, Lihong V. Wang, "Development of the correlation transfer equation of ultrasound-modulated multiply scattered light: a diagrammatic approach," Proc. SPIE 6086, Photons Plus Ultrasound: Imaging and Sensing 2006: The Seventh Conference on Biomedical Thermoacoustics, Optoacoustics, and Acousto-optics, 60861X (7 March 2006); doi: 10.1117/12.646264

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Event: SPIE BiOS, 2006, San Jose, California, United States

Development of the correlation transfer equation of ultrasound-modulated multiply scattered light: A diagrammatic approach

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ABSTRACT

In this paper, we develop a temporal correlation transfer equation (CTE) for ultrasound-modulated multiply scattered light. The equation can be used to obtain the temporal frequency spectrum of the optical intensity produced by a nonuniform ultrasound field in optically scattering media. Derivation of the CTE is based on the ladder diagram approximation of the Bethe-Salpeter equation. We expect the CTE to be applicable to a wide spectrum of conditions in the ultrasound-modulated optical tomography of soft biological tissues.

Keywords: ultrasound-modulated optical tomography, multiply scattered light, correlation transfer equation, Bethe-Salpeter equation, Dyson equation

1. INTRODUCTION

Development of soft biological tissue imaging systems based on ultrasound-modulated multiply scattered light has been a subject of intense research in recent years. The information obtained by exposing the biological tissues to visible, and near-infrared radiation, can be used for functional imaging and the detection of tissue abnormalities, which makes the optical imaging modalities very attractive for medical applications.

Ultrasound-modulated optical tomography (UOT) is a hybrid technique which combines the advantages of ultrasonic resolution and optical contrast.[1,2] In this technique, focused ultrasound and optical radiation of high temporal coherence are simultaneously applied to soft biological tissue, and the intensity of the ultrasound-modulated light is measured. This provides information about the optical properties of the tissue, spatially localized at the interaction region of the ultrasonic and electromagnetic waves.

Detection of ultrasound-modulated light is very challenging because of the uncorrelated optical phases among the speckles that are created by diffused light. Therefore, an intense research effort to develop more efficient detection systems for UOT experiments is taking place at present .[2–15]

Simultaneously, progress is being made on the theoretical understanding of the ultrasound modulation of light in optically strongly scattering media. Two mechanisms of modulation are included in the present models. The first mechanism is the dynamic light scattering of optical scatterers oscillating in an ultrasound field,[3,16] which is largely analogous to the dynamic light scattering of scatterers undergoing Brownian motion.[17] The second mechanism accounts for the ultrasound-induced changes in the optical index of refraction.[18,19] Both mechanisms are combined by Wang,[19] in a model based on the diffusing-wave spectroscopy (DWS) approach.[17,20] In this model,[19] the interaction of a plane ultrasound wave with diffused light is considered in an infinite and uniform optically scattering medium, assuming small values of the ultrasound-induced optical phase increments. Subsequently, equations are extended to account for anisotropic optical scattering,[21] Brownian motion,[21,22] pulsed ultrasound,[23] and strong correlations[23] between the ultrasound-induced optical phase increments. It is shown[23] that the correlations are weak only if $k_a l_{tr} \gg 1$, where l_{tr} is the optical transport mean free path; $k_a = 2\pi/\lambda_a$ is the magnitude of the ultrasound wave vector; and λ_a is the ultrasound wavelength. In a case of anisotropic optical scattering, equations derived for the isotropic case can be applied if l_{tr} is used instead of the optical mean free path.[21,23] In addition, a Monte Carlo algorithm is developed and used for comparison with the theoretical predictions,[21,24] as well as for modeling the scattering samples that have optically absorbing objects with cylindrical shape.[25]

Photons Plus Ultrasound: Imaging and Sensing 2006: The 7th Conference on Biomedical Thermoacoustics, Optoacoustics, and Acousto-optics, edited by A. A. Oraevsky, et. al
Proc. of SPIE Vol. 6086, 60861X (2006) • 1605-7422/06/$15 • doi: 10.1117/12.646264

Since the existing theoretical model is based on the DWS approach, applications are limited to simple geometries where it is possible to approximate the ultrasound field with a plane ultrasound wave, and where the probability density function of the optical path length between source and detector is analytically known. As a result, only transmission through,[19,21,23] and reflection from,[22,23] the infinite scattering slab filled with ultrasound, have been analytically studied. In most experiments, however, the optical parameters are heterogeneously distributed and a focused ultrasound beam is used. Therefore, a more general theoretical model, which can locally treat interactions between ultrasound and light in an optically scattering medium, is needed.

In this paper, based on the ladder diagram approximation of the Bethe-Salpeter equation,[26] we have derived a temporal correlation transfer equation (CTE) for ultrasound-modulated multiply scattered light. The work of the many authors in the last sixty years that established the link between multiple scattering theory and the radiative transfer equation is reviewed in several excellent articles.[27–30] Also, several authors have considered development of the CTE for scatterers moving with a given velocity distribution or undergoing Brownian motion.[31–34] In our case, both ultrasound-induced movement of the scatterers and ultrasound-induced change in the optical index of refraction have led to a new form of CTE.

Derivation of the CTE is presented in several steps. In Sec. 2, we first develop an expression for the Green's function in an ultrasound field, in a space free of optical scatterers. Next, we solve the Dyson equation[26] and obtain the value of a mean Green's function in the presence of optical scatterers, which can be used to obtain the ensemble averaged field for a given distribution of sources. Finally, in Sec. 3, based on the Bethe-Salpeter equation, we obtain an expression for a mutual coherence function and transform it into an integral form of the CTE.

2. DEVELOPMENT OF THE MEAN GREEN'S FUNCTION

We start by presenting an approximate expression for the Green's function of the electrical field component in a space free from optical scatterers in the presence of an ultrasound field. We assume that for moderate ultrasound pressure given by $P(\mathbf{r},t)$, the dielectric permeability of the media experiences small perturbation, and that it is well approximated with $\epsilon = \epsilon_0[1+2\eta P(\mathbf{r},t)/(\rho v_a^2)]$, where ϵ_0 is the dielectric permeability of unperturbed media; ρ is the mass density of the medium; v_a is the ultrasound speed; and η is the elasto-optical coefficient (in water at standard conditions $v_a \approx 1480$ ms^{-1}, and $\eta \approx 0.32$). Consequently, we locally approximate the optical index of refraction with $n(\mathbf{r},t) = n_0[1+\eta P(\mathbf{r},t)/(\rho v_a^2)]$, where $n_0 = \sqrt{\epsilon_0}$. We assume monochromatic light sources with angular frequency ω_0 and wave vector magnitude $k_0 = \omega_0/c_0$, where c_0 is the speed of light in a vacuum. The optical polarization effects are neglected for simplicity, and we consider only one component $\tilde{E}(\mathbf{r},t)$ of the electrical field. The time retardation is also neglected, since the time during which the light propagates through the sample is a small fraction of the ultrasound period. Due to the large ratio between the optical and ultrasound temporal frequencies, we approximate the quasi-monochromatic electrical field as $\tilde{E}(\mathbf{r},t) = E(\mathbf{r},t)\exp[-i\omega_0 t]$, where $E(\mathbf{r},t)$ is a slowly changing function of time.

For a point source at position $\mathbf{r}_0$, we approximate the Green's function $G_a(\mathbf{r},\mathbf{r}_0,t)$ of the slowly changing function $E(\mathbf{r},t)$ as

$$G_a(\mathbf{r},\mathbf{r}_0,t) \quad = \quad \frac{\exp(ik_0 n_0|\mathbf{r}-\mathbf{r}_0|[1+\xi(\mathbf{r},\mathbf{r}_0,t)])}{-4\pi|\mathbf{r}-\mathbf{r}_0|}. \tag{1}$$

For moderate ultrasound pressures and distances not far from the source the small fractional phase perturbation $\xi(\mathbf{r},\mathbf{r}_0,t)$ is obtained as an integral of the optical path increments along the line between $\mathbf{r}_0$ and $\mathbf{r}$ as

$$\xi(\mathbf{r},\mathbf{r}_0,t) = \frac{\eta}{\rho v_a^2|\mathbf{r}-\mathbf{r}_0|}\int_{\mathbf{r}_0}^{\mathbf{r}} P(\mathbf{r}',t)dr'. \tag{2}$$

We consider now a medium with discrete and uncorrelated point-like optical scatterers. We assume independent optical scattering and an optical wavelength λ_0 that is much smaller than the optical mean free path l_s between the consecutive scattering events (weak scattering approximation). We also assume that a monochromatic ultrasound field in an optically strongly scattering media similar to soft biological tissue is uniform on scales that are comparable with the optical transport mean free path (l_{tr}), and locally approximate ultrasound pressure with $P(\mathbf{r},t) = P_0 \cos(\omega_a t - \mathbf{k}_a \cdot \mathbf{r} + \phi)$, where $\mathbf{k}_a = k_a \hat{\mathbf{\Omega}}_a$ is the ultrasound wave vector, and P_0, ω_a, ϕ, and $\hat{\mathbf{\Omega}}_a$ are pressure amplitude, angular frequency, local initial phase, and propagation direction of the ultrasound, respectively ($|\hat{\mathbf{\Omega}}_a| = 1$). This allows us to write the explicit expression for the phase perturbation $\xi(\mathbf{r}, \mathbf{r}_0, t)$ as

$$\xi(\mathbf{r}, \mathbf{r}_0, t) = \frac{1}{2} M \cos\left(\omega_a t - \mathbf{k}_a \cdot \frac{\mathbf{r} + \mathbf{r}_0}{2} + \phi\right) \mathrm{sinc}\left(\mathbf{k}_a \cdot \frac{\mathbf{r} - \mathbf{r}_0}{2}\right), \tag{3}$$

where $\mathrm{sinc}(x) = \sin(x)/x$, and $M = 2\eta P_0/(\rho v_a^2)$.

The precision of Eq. (1), where the small fractional phase perturbation $\xi(\mathbf{r}, \mathbf{r}_0, t)$ is given by Eq. (3), is worse for large values of $|\mathbf{r} - \mathbf{r}_0|$. However, for further derivations, it is sufficient that it is approximately valid for $|\mathbf{r} - \mathbf{r}_0|$ on the order of a few l_{tr}. This should be satisfied in soft biological tissues at visible and near-infrared optical wavelengths ($l_{tr} \approx 1$ mm), when the ultrasound pressure is not greater than 10^5 Pa, and in the medical ultrasound frequency range.[23]

The scattering cross section σ_s is related to the optical scattering amplitude $f(\hat{\mathbf{\Omega}}_{sc}, \hat{\mathbf{\Omega}}_{inc})$ as

$$\sigma_s = \int_{4\pi} \left| f(\hat{\mathbf{\Omega}}_{sc}, \hat{\mathbf{\Omega}}_{inc}) \right|^2 d\Omega_{sc}, \tag{4}$$

where $\hat{\mathbf{\Omega}}_{inc}$ and $\hat{\mathbf{\Omega}}_{sc}$ are directions of the incident and scattered waves, respectively, and we assume that a scattering potential is spherically symmetric, such that $f(\hat{\mathbf{\Omega}}_{sc}, \hat{\mathbf{\Omega}}_{inc})$ is the function of $\hat{\mathbf{\Omega}}_{sc} \cdot \hat{\mathbf{\Omega}}_{inc}$ only. The scattering phase function $p(\hat{\mathbf{\Omega}}_{sc}, \hat{\mathbf{\Omega}}_{inc})$ is defined as $p(\hat{\mathbf{\Omega}}_{sc}, \hat{\mathbf{\Omega}}_{inc}) = \sigma_s^{-1} |f(\hat{\mathbf{\Omega}}_{sc}, \hat{\mathbf{\Omega}}_{inc})|^2$, and it satisfies $\int_{4\pi} p(\hat{\mathbf{\Omega}}_{sc}, \hat{\mathbf{\Omega}}_{inc}) d\Omega_{sc} = 1$. In addition, from the optical theorem, we have $\sigma_s + \sigma_a = 4\pi(k_0 n_0)^{-1} \mathrm{Im}[f(\hat{\mathbf{\Omega}}_{inc}, \hat{\mathbf{\Omega}}_{inc})]$, where σ_a is the optical absorption cross section, and Im[] is an imaginary part. If ρ_s is the density of the optical scatterers, then the optical extinction, scattering and absorption coefficients are defined as $\mu_t = \mu_s + \mu_a$, $\mu_s = \sigma_s \rho_s$, and $\mu_a = \sigma_a \rho_s$, respectively.

We write the expression for field $E_s(\mathbf{r}, t)$ in the far field approximation produced by the scattering of the plane wave $\exp[ik_0 n_0 \hat{\mathbf{\Omega}}_{inc} \cdot \mathbf{r}]$ at $\mathbf{r}_s$ as

$$E_s(\mathbf{r}, t) = -4\pi G_a(\mathbf{r}, \mathbf{r}_s, t) \exp\left[ik_0 n_0 \mathbf{e}_s(t) \cdot (\hat{\mathbf{\Omega}}_{inc} - \hat{\mathbf{\Omega}}_{sc})\right] f(\hat{\mathbf{\Omega}}_{sc}, \hat{\mathbf{\Omega}}_{inc}) \exp\left[ik_0 n_0 \hat{\mathbf{\Omega}}_{inc} \cdot \mathbf{r}_s\right]. \tag{5}$$

In Eq. (5), $\hat{\mathbf{\Omega}}_{sc} = (\mathbf{r} - \mathbf{r}_s)/|\mathbf{r} - \mathbf{r}_s|$, and the first exponent on the right hand side accounts for the Doppler shift caused by the ultrasound-induced movement of the scatterer. The position of the scatterer having resting position $\mathbf{r}_s$ is given by $\mathbf{r}_s + \mathbf{e}_s(t)$, where $\mathbf{e}_s(t)$ is the small ultrasound-induced scatterer displacement given by $\mathbf{e}_s(t) = \hat{\mathbf{\Omega}}_a P_0 S_a (k_a \rho v_a^2)^{-1} \sin(\omega_a t - \mathbf{k}_a \cdot \mathbf{r}_s - \phi_a + \phi)$. In general, S_a and ϕ_a are the coefficients of the deviation of the scatterer amplitude and the phase of the displacement from the movement of the surrounding fluid.[23, 35] However, we expect that an endogenous optical scatterer in soft biological tissue closely follows in both phase and amplitude the ultrasound-induced tissue vibrations, in which case $S_a \approx 1$ and $\phi_a \approx 0$.

The mean Green's function $G_s(\mathbf{r}_b, \mathbf{r}_a, t)$ provides the ensemble averaged value of the electrical field (referred to also as a mean or coherent field), and it can be obtained by solving the Dyson equation.[26, 31, 32] The Dyson equation in the Bourret approximation is given by

$$G_s(\mathbf{r}_b, \mathbf{r}_a, t) = G_a(\mathbf{r}_b, \mathbf{r}_a, t) - 4\pi \int G_a(\mathbf{r}_b, \mathbf{r}_s, t) e^{ik_0 n_0 \mathbf{e}_s(t) \cdot (\hat{\mathbf{\Omega}}_{as} - \hat{\mathbf{\Omega}}_{sb})} f(\hat{\mathbf{\Omega}}_{sb}, \hat{\mathbf{\Omega}}_{as}) G_s(\mathbf{r}_s, \mathbf{r}_a, t) \rho_s \, d\mathbf{r}_s, \tag{6}$$

where $\hat{\mathbf{\Omega}}_{as}$ and $\hat{\mathbf{\Omega}}_{sb}$ are unity vectors in directions $\mathbf{r}_s - \mathbf{r}_a$ and $\mathbf{r}_b - \mathbf{r}_s$, respectively.

By applying the method of stationary phase to Eq. (6), we obtain the following solution

$$G_s(\mathbf{r}_b, \mathbf{r}_a, t) = -\frac{\exp\left[iK(\mathbf{r}_b, \mathbf{r}_a, t)|\mathbf{r}_b - \mathbf{r}_a|\right]}{4\pi|\mathbf{r}_b - \mathbf{r}_a|}, \tag{7}$$

where the term in exponent is $K(\mathbf{r}_b, \mathbf{r}_a, t) = k_0 n_0 [1 + \xi(\mathbf{r}_b, \mathbf{r}_a, t)] + 2\pi \rho_s f(\hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}})/(k_0 n_0)$.

3. DEVELOPMENT OF CTE

The mutual coherence function of the electrical field component is given by $\Gamma(\mathbf{r}_a, \mathbf{r}_b, t, \tau) = \langle E(\mathbf{r}_a, t)E^*(\mathbf{r}_b, t+\tau)\rangle$, where $\mathbf{r}_a$ and $\mathbf{r}_b$ are two closely spaced points, and $\langle\,\rangle$ represents the ensemble average. Under the weak scattering approximation, $\Gamma(\mathbf{r}_a, \mathbf{r}_b, t, \tau)$ satisfies the ladder approximation of the Bethe-Salpeter equation[26, 31, 32, 34]

$$\Gamma(\mathbf{r}_a, \mathbf{r}_b, t, \tau) = \Gamma_0(\mathbf{r}_a, \mathbf{r}_b, t, \tau) + \int\int v^a_{s'}(t) v^{b\,*}_{s''}(t+\tau)\Gamma(\mathbf{r}_{s'}, \mathbf{r}_{s''}, t, \tau)\rho(\mathbf{r}_{s'}, t; \mathbf{r}_{s''}, t+\tau) d\mathbf{r}_{s'} d\mathbf{r}_{s''}. \tag{8}$$

In Eq. (8), $\Gamma_0(\mathbf{r}_a, \mathbf{r}_b, t, \tau) = \langle E(\mathbf{r}_a, t)\rangle\langle E^*(\mathbf{r}_b, t+\tau)\rangle$ is the mutual coherence function of the coherent (unscattered) field, and $\mathbf{r}_{s'}$ and $\mathbf{r}_{s''}$ are the positions of the same scatterer at two different time moments ($\mathbf{r}_{s'} = \mathbf{r}_s + \mathbf{e}_s(t)$, $\mathbf{r}_{s''} = \mathbf{r}_s + \mathbf{e}_s(t+\tau)$). The function $\rho(\mathbf{r}_{s'}, t; \mathbf{r}_{s''}, t+\tau)$ is the probability density of finding the same scatterer s at position $\mathbf{r}_{s'}$ and time t, and at position $\mathbf{r}_{s''}$ and time $t+\tau$. The expression for the operator $v^p_q(t)$ is $v^p_q(t) = -4\pi G_s(\mathbf{r}_p, \mathbf{r}_q, t) f(\hat{\mathbf{\Omega}}_{qp}, \hat{\mathbf{\Omega}}_{inc})$, and the product from the integral in Eq. (8) is

$$v^a_{s'}(t) v^{b\,*}_{s''}(t+\tau) = \frac{f(\hat{\mathbf{\Omega}}_{s'a}, \hat{\mathbf{\Omega}}') f^*(\hat{\mathbf{\Omega}}_{s''b}, \hat{\mathbf{\Omega}}')}{|\mathbf{r}_a - \mathbf{r}_{s'}||\mathbf{r}_b - \mathbf{r}_{s''}|} \exp\left(iK(\mathbf{r}_a, \mathbf{r}_{s'}, t)|\mathbf{r}_a - \mathbf{r}_{s'}|\right) \exp\left(-iK^*(\mathbf{r}_b, \mathbf{r}_{s''}, t+\tau)|\mathbf{r}_b - \mathbf{r}_{s''}|\right). \tag{9}$$

We introduce new vectors in the center-of-gravity coordinate system as $\mathbf{r} = (\mathbf{r}_a + \mathbf{r}_b)/2$, $\mathbf{r}_d = \mathbf{r}_a - \mathbf{r}_b$, $\mathbf{r}_{s,av} = (\mathbf{r}_{s''} + \mathbf{r}_{s'})/2$, $\mathbf{r}_{ds} = \mathbf{r}_{s'} - \mathbf{r}_{s''}$, and also $\hat{\mathbf{\Omega}} = (\mathbf{r} - \mathbf{r}_{s,av})/|\mathbf{r} - \mathbf{r}_{s,av}|$. Since $|\mathbf{r}_d| \ll |\mathbf{r} - \mathbf{r}_{s,av}|$ and $|\mathbf{r}_{ds}| \ll |\mathbf{r} - \mathbf{r}_{s,av}|$, we assume $f(\hat{\mathbf{\Omega}}_{s'a}, \hat{\mathbf{\Omega}}') \approx f(\hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}')$, $f(\hat{\mathbf{\Omega}}_{s''b}, \hat{\mathbf{\Omega}}') \approx f(\hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}')$, and

$$\begin{aligned}
|\mathbf{r}_a - \mathbf{r}_{s'}| &\approx |\mathbf{r} - \mathbf{r}_{s,av}| + (\mathbf{r}_d - \mathbf{r}_{ds}) \cdot \hat{\mathbf{\Omega}}/2, \\
|\mathbf{r}_b - \mathbf{r}_{s''}| &\approx |\mathbf{r} - \mathbf{r}_{s,av}| - (\mathbf{r}_d - \mathbf{r}_{ds}) \cdot \hat{\mathbf{\Omega}}/2, \\
(|\mathbf{r}_a - \mathbf{r}_{s'}||\mathbf{r}_b - \mathbf{r}_{s''}|)^{-1} &\approx |\mathbf{r} - \mathbf{r}_{s,av}|^{-2}.
\end{aligned} \tag{10}$$

Eq. (9) can be now presented as

$$v^a_{s'}(t) v^{b\,*}_{s''}(t+\tau) = \sigma_s \frac{p(\hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}')}{|\mathbf{r} - \mathbf{r}_{s,av}|^2} e^{iK_r(\mathbf{r}_d - \mathbf{r}_{ds})\cdot\hat{\mathbf{\Omega}}} e^{-\mu_t|\mathbf{r} - \mathbf{r}_{s,av}|} e^{i\Psi_n(\mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_{s'}, \mathbf{r}_{s''}, t, \tau)}, \tag{11}$$

where $K_r = n_0 k_0 + 4\pi \mathrm{Re}[f(\hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}})]\rho_s/(2k_0 n_0)$, and Re[ ] is the real part. $\Psi_n(\mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_{s'}, \mathbf{r}_{s''}, t, \tau)$ is the difference of the ultrasound-induced phase increments given by

$$\Psi_n(\mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_{s'}, \mathbf{r}_{s''}, t, \tau) = k_0 n_0 |\mathbf{r}_a - \mathbf{r}_{s'}|\xi(\mathbf{r}_a, \mathbf{r}_{s'}, t) - k_0 n_0 |\mathbf{r}_b - \mathbf{r}_{s''}|\xi(\mathbf{r}_b, \mathbf{r}_{s''}, t+\tau). \tag{12}$$

By using the expressions from Eq. (10), the expression in Eq. (12) is approximated as $\Psi_n(\mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_{s'}, \mathbf{r}_{s''}, t, \tau) \approx \Psi_n(\mathbf{r}, \mathbf{r}_{s,av}, \hat{\mathbf{\Omega}}, t, \tau)$ where

$$\Psi_n(\mathbf{r}, \mathbf{r}_{s,av}, \hat{\mathbf{\Omega}}, t, \tau) = \frac{2\Lambda_n k_a}{\mathbf{k}_a \cdot \hat{\mathbf{\Omega}}} \sin\left(\frac{1}{2}\omega_a \tau\right) \sin\left[\omega_a\left(t + \frac{1}{2}\tau\right) - \mathbf{k}_a \cdot \frac{\mathbf{r} + \mathbf{r}_{s,av}}{2} + \phi\right] \sin\left(\mathbf{k}_a \cdot \frac{\mathbf{r} - \mathbf{r}_{s,av}}{2}\right). \tag{13}$$

In Eq. (13), $\Lambda_n = 2k_0 n_0 \eta P_0/(k_a \rho_s v_a^2)$.

The probability density function $\rho(\mathbf{r}_{s'}, t; \mathbf{r}_{s''}, t+\tau)$ in Eq. (8) is given by

$$\rho(\mathbf{r}_{s'}, t; \mathbf{r}_{s''}, t+\tau) = \rho_s \delta\left(\mathbf{r}_{sd} - \Delta\mathbf{e}(\mathbf{r}_s, t, \tau)\right), \tag{14}$$

where $\Delta\mathbf{e}(\mathbf{r}_s, t, \tau) = \mathbf{e}_s(t+\tau) - \mathbf{e}_s(t)$, and $\mathbf{r}_{sd} = -\mathbf{r}_{ds}$. By replacing the integration over positions $\mathbf{r}_{s'}$ and $\mathbf{r}_{s''}$ with an integration over $\mathbf{r}_{sd}$ and $\mathbf{r}_{s,av}$, Eq. (8) becomes

$$\begin{aligned}
\Gamma(\mathbf{r}_a, \mathbf{r}_b, t, \tau) = \Gamma_0(\mathbf{r}_a, \mathbf{r}_b, t, \tau) + \int \mu_s p(\hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}') \exp[iK_r(\mathbf{r}_d - \mathbf{r}_{ds}) \cdot \hat{\mathbf{\Omega}}] \exp[i\Psi_n(\mathbf{r}, \mathbf{r}_{s,av}, t, \tau)] \\
\times \exp[-\mu_t|\mathbf{r} - \mathbf{r}_{s,av}|]\Gamma(\mathbf{r}_{s'}, \mathbf{r}_{s''}, t, \tau)\delta(\mathbf{r}_{sd} - \Delta\mathbf{e}(\mathbf{r}_s, t, \tau)) d\mathbf{r}_{sd} d|\mathbf{r} - \mathbf{r}_{s,av}| d\Omega,
\end{aligned} \tag{15}$$

where we used $d\mathbf{r}_{s,av} = |\mathbf{r} - \mathbf{r}_{s,av}|^2 d|\mathbf{r} - \mathbf{r}_{s,av}| d\Omega$.

We assume quasi-uniformity ($|\partial_{\mathbf{r}}\Gamma| \ll |\partial_{\mathbf{r}_d}\Gamma|$) of the mutual coherence function $\Gamma(\mathbf{r}, \mathbf{r}_d, t, \tau)$, written in respect to the center-of-gravity coordinates, and relate the time varying specific intensity of the quasi-monochromatic light $I(\mathbf{r}, \hat{\mathbf{\Omega}}_q, t, \tau)$ to the mutual coherence function as[27, 28, 34, 36]

$$\Gamma(\mathbf{r}_a, \mathbf{r}_b, t, \tau) \; = \; \int I(\mathbf{r}, \hat{\mathbf{\Omega}}, t, \tau) \exp[iK_r \hat{\mathbf{\Omega}} \cdot \mathbf{r}_d] d\Omega. \tag{16}$$

The integral form of the CTE is then obtained by substituting Eq. (16) into Eq. (15), performing the integration which involves Dirac's delta function, and subsequently removing the integrals over $\hat{\mathbf{\Omega}}$, together with exponents $\exp[iK_r \hat{\mathbf{\Omega}} \cdot \mathbf{r}_d]$ which are common for all terms. We write the final result as

$$I(\mathbf{r}, \hat{\mathbf{\Omega}}, t, \tau) = I_0(\mathbf{r}, \hat{\mathbf{\Omega}}, t, \tau) + \int \mu_s p(\hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}') e^{-\mu_t |\mathbf{r} - \mathbf{r}_s|} I(\mathbf{r}_s, \hat{\mathbf{\Omega}}', t, \tau) \Phi(\mathbf{r}, \mathbf{r}_s, \hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}', t, \tau) d|\mathbf{r} - \mathbf{r}_s| d\Omega'. \tag{17}$$

In Eq. (17), the term $\Phi(\mathbf{r}, \mathbf{r}_s, \hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}', t, \tau)$ is equal to $\exp[i\Psi_d(\mathbf{r}_s, \hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}', t, \tau)] \exp[i\Psi_n(\mathbf{r}, \mathbf{r}_s, \hat{\mathbf{\Omega}}, t, \tau)]$, and it accounts for the ultrasound-induced phase increments due to both mechanisms of modulation. The displacement term $\Psi_d(\mathbf{r}_s, \hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}', t, \tau) = K_r(\hat{\mathbf{\Omega}} - \hat{\mathbf{\Omega}}') \cdot \Delta\mathbf{e}(\mathbf{r}_s, t, \tau)$ is given by

$$\Psi_d(\mathbf{r}_s, \hat{\mathbf{\Omega}}, \hat{\mathbf{\Omega}}', t, \tau) = \Lambda_d \left[(\hat{\mathbf{\Omega}} - \hat{\mathbf{\Omega}}') \cdot \hat{\mathbf{\Omega}}_a \right] \sin\left(\frac{1}{2}\omega_a \tau\right) \cos\left(\omega_a t + \frac{1}{2}\omega_a \tau - \mathbf{k}_a \cdot \mathbf{r}_s - \phi_a + \phi\right), \tag{18}$$

where $\Lambda_d = 2K_r S_a P_0 / (k_a \rho_s v_a^2)$.

Compared to the CTE equation where the optical scatterers are undergoing Brownian motion,[28, 34] in Eq. (17) we have a similar term $\Phi()$, but this time it is due to two mechanisms of the ultrasound-induced optical phase increments. The time varying specific intensity $I(\mathbf{r}, \hat{\mathbf{\Omega}}, t, \tau)$ in a case of ultrasound-modulation depends on both time t and time increment τ. Of most practical interest are the temporal harmonics of $I(\mathbf{r}, \hat{\mathbf{\Omega}}, \tau) = \omega_a (2\pi)^{-1} \int_0^{2\pi/\omega_a} I(\mathbf{r}, \hat{\mathbf{\Omega}}, t, \tau) dt$, which can be obtained by the temporal Fourier transform of $I(\mathbf{r}, \hat{\mathbf{\Omega}}, \tau)$. An analytical solution for $I(\mathbf{r}, \hat{\mathbf{\Omega}}, \tau)$ is difficult to obtain due to correlations among the ultrasound-induced optical phase increments. However, it should be possible to adapt the numerical codes developed for the Boltzman's equation to calculate the temporal harmonics of the ultrasound-modulated light based on Eq. (17). Also, in the diffusion regime, and for ultrasound wavelengths which satisfy $k_a l_{tr} \gg 1$, it is possible to significantly simplify the expression for $I(\mathbf{r}, \hat{\mathbf{\Omega}}, \tau)$ by pre-averaging[34] the ultrasound-induced optical phase increments in Eq. (17).

4. CONCLUSION

In conclusion, based on the ladder diagram approximation of the Bethe-Salpeter equation, we have developed an integral form of the CTE for ultrasound-modulated light. The derivations are valid within the weak scattering approximation, the medical ultrasound frequency range and moderate ultrasound pressures. We expect that the CTE can help to better model UOT experiments for estimations of sensitivity, resolution, and signal-to-noise ratios. Further development of the theory is necessary to address tightly focused ultrasound fields with very high ultrasound pressures.

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