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Statistics of Remnant Speckles in an Adaptively Corrected Imaging System

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ABSTRACT

Understanding the statistics of remnant speckles in the halo of an adaptively-corrected point-spread function is critically important to using adaptive optics in high-dynamic-range searches for faint companions. It has been clear for some time that photon (Poisson) statistics alone do not adequately account for noise in the halo, as the coherent nature of speckles gives them a temporal persistence that leads to a much larger noise contribution, termed "speckle noise". I consider in this paper the physical mechanism for speckle formation, and show that residual speckles, in the case of highly corrected adaptive optics systems, tend to be "pinned" to secondary maxima ("Airy rings") in the underlying diffraction-limited point-spread function, affecting their spatial distribution in an important way. Further, in current practical adaptive optics systems, the structure of the Airy rings will shift over relatively short time scales in response to flexure-induced non-common-path errors, modifying the temporal evolution of the statistics of the speckle distribution as well.

Keywords: adaptive optics, speckles

1. INTRODUCTION

Adaptive optics systems are becoming operational on a number of large telescopes. In conditions of reasonably good initial (uncorrected) seeing, they are able to deliver diffraction-limited performance at near-infrared wavelengths for apertures in the 5- to 10-meter class. By their very nature, adaptive optics systems continually alter the point-spread function of the telescope: roughly, they concentrate a fraction S, the Strehl ratio, of the light from the spatially-extended, temporally-varying halo, which is subject to atmospheric seeing fluctuations, into a more compact distribution corresponding to the diffraction pattern of the telescope's aperture. This diffraction-limited "core" is generally reasonably stable, though characterized by slow drifts over minutes of time, and subject to more rapid variations as well if the strength or nature of the underlying seeing varies. The distribution of remnant light in the halo, which has not been concentrated into the core by the adaptive correction, remains more highly variable, and is the subject of investigation of this paper.

In practice, even relatively high-quality adaptive corrections are currently characterized by Strehl ratios of only perhaps $\sim 60\%$, meaning that 40% of the total light from the source is still spatially dispersed in a fast-changing halo. Very ambitious adaptive optics system contemplated for the future, featuring deformable mirrors with thousands or tens of thousands of actuators, might produce Strehl ratios well above 90%. There will always be a halo of remnant speckles, even with such an "extreme" adaptive optics system, and understanding the complex statistical behavior of these halos, temporally and spatially, will be important to success with the ambitious observational tasks envisioned for these systems, particularly searching for faint companions (stars, substellar objects, or exosolar planets) in the vicinity of relatively bright stars[1,2].

In this paper, I discuss the spatial statistical behavior of remnant speckles in a highly corrected adaptive optic imaging system. It has commonly been assumed that the post-correction speckles may be found randomly distributed anywhere within a smooth spatial envelope describable by a Gaussian or similarly smooth functional form. This distribution follows from experience gained in classical speckle interferometry, where no hardware correction of the image is involved. I show instead that speckles, when the adaptive correction is high, are found only at the positions of diffraction rings of the diffraction-limited point-spread function. This behavior is noticeable in experimental data at Strehl ratios of $S \sim 60\%$, is even more apparent in simulations at higher Strehl, and may be derived algebraically from the Fourier-optical expression for a diffraction-limited image.

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Besides clarifying the fundamental physics of the adaptive imaging process, the insight into speckle behavior presented in this paper may have important practical applications to the search for faint companions. In particular, engineering a diffraction-limited point-spread function so that is has a region substantially free of secondary maxima will tend to sweep that same region free of speckles and hence of speckle noise, which is the dominant noise source affecting searches for faint companions. The tendency of speckles to localize only on "Airy" rings will also affect their motions in the focal plane and hence have an impact on their temporal statistics as well: speckles may be thought of as "pinned" to secondary maxima of the diffraction-limited point-spread function, causing an apparent lengthening of the time spent at a ring position.

2. QUALITATIVE EXPLANATION OF THE ORIGIN OF SPECKLES

Focal-plane speckles have been familiar to optical astronomy since long before the advent of adaptive optics[3]. Seeing degrades the images of ground-based telescopes when the diameter of the telescope, D, substantially exceeds the transverse scale of atmopheric turbulence, r_0, which is a parameter describing the transverse scale over which atmospherically-degraded wavefronts remain roughly coherent. Then exposures longer than the timescale of atmospheric fluctuations exhibit a blur circle of size FWHM $\sim \lambda/r_0$, much larger than the diffraction-limited point-spread diameter of FWHM $\sim \lambda/D$. Multiple coherent patches of size r_0 over the telescope pupil combine incoherently to produce the seeing blur circle.

If short exposures are made, the focal-plane blur spot is resolved into a number of bright and dark interference maxima and minima that move rapidly within the envelope of the blur spot. These structures can be as small as of order the diffraction limit of the telescope, λ/D, and they arise from chance phase coherence among a few r_0-size seeing cells on the telescope aperture that may be separated by distances as large as D. Since they carry diffraction-limited imaging information, they have been exploited in a number of "speckle interferometry" schemes to achieve spatial resolution higher than normally allowed by the atmosphere.

3. NOISE IMPLICATIONS OF SPECKLES FOR ADAPTIVE OPTICS

Any adaptive optics system with Strehl ratio S less than unity will leave a fraction (1-S) of image power in a spatially-distributed speckle halo. The speckles remaining in the halo of the adaptively-corrected image are unusually problematic for companion searches. They move about as coherent entities, and hence cause brightness variations equal to their full intensity as they traverse a search region of interest; this is far worse than the photon noise (Poisson noise) from a single speckle, which is proportional to the square-root of the intensity of a speckle. For this reason, speckle noise is expected to be the limiting noise source in future searches that might be made using extremely high-correction adaptive optics systems. These systems may employ deformable mirrors with thousands or tens of thousands of actuators to achieve Strehl ratios in excess of 99%; there are some compelling reasons, such as spectroscopic capability, that observing exosolar planets with extreme adaptive optics may be attractive and may yield information not obtainable with current radial-velocity detection methods.

4. EXPERIMENTAL EVIDENCE FOR INTERACTION BETWEEN SPECKLES AND THE DIFFRACTION-LIMITED PSF

Our experience with the Palomar adaptive optics system[4–6] has suggested that past assumptions about speckle behavior are incorrect, at least when the degree of adaptive correction is reasonably high. Figure 1 shows short-exposure remnant speckle patterns for moderately high (Strehl $\sim$ 30%) and high (Strehl $\sim$ 60%) adaptive correction. A computed diffraction-limited point-spread function is shown at upper left for comparison.

The high-Strehl data set, in particular, exhibits a tendency for individual speckles to lie on rings, presumably the secondary maxima of the underlying diffraction-limited point-spread function. This conclusion is supported by the occurrence of the brightest speckles along rings that are brightest in the diffraction-limited point-spread function (where the fractional obscuration by the secondary mirror causes radial beating in the point-spread function). These speckles are similar to the bright knots observed along the first Airy ring of the corrected point-spread function[6], and in a general sense must also arise from residual aberrations, though there may be an instrumental component (e.g. flexure-induced non-common-path errors) to those aberrations.

The impression that speckles are occurring along rings of the diffraction-limited point-spread function, particularly at high Strehl, is further strengthened by the morphologies of many of the speckles, which appear to be arcs tracing out segments of Airy rings.

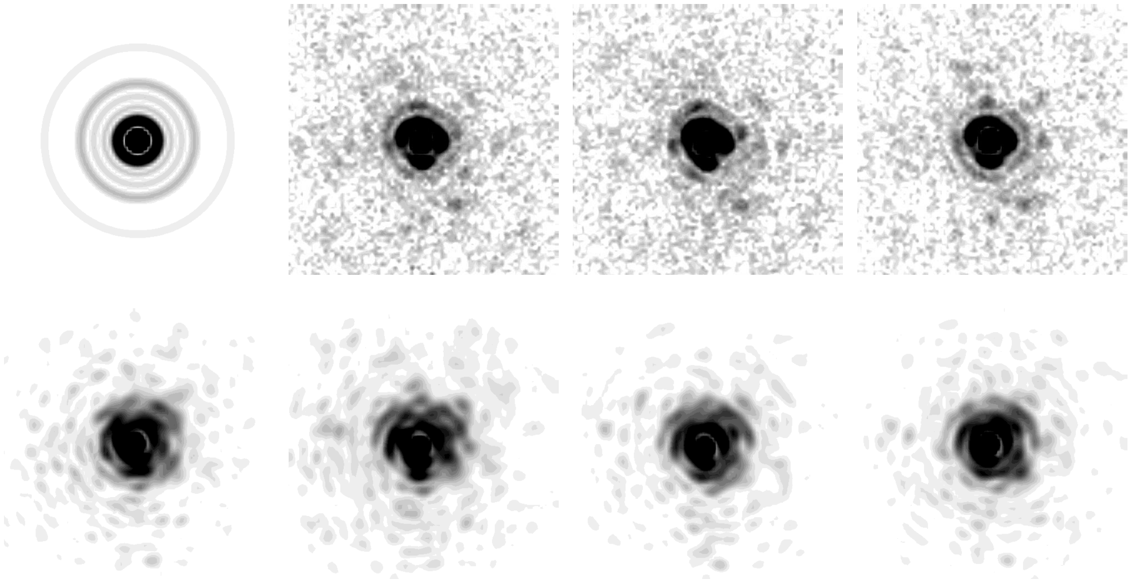

Figure 1. Examples of closed-loop images of a point source taken with the Palomar adaptive optics system. At upper left is a diffraction calculation of the ideal point-spread function from a 5-meter aperture with 36% central obscuration; the effects of spiders (secondary supports) are not included. The next three panels on the top row are successive short exposures at K (2.2 μm) under conditions giving Strehl $\sim$ 30%. The bottom four panels are also successive short exposures at K, but under better atmospheric conditions giving Strehl $\sim$ 60%. Notice, particularly for the high-Strehl data, that speckles tend to lie on circular rings and may show arc-like morphology tracing out the underlying Airy rings.

5. EVIDENCE FOR INTERACTION BETWEEN SPECKLES AND THE DIFFRACTION-LIMITED PSF FROM HIGH-STREHL SIMULATIONS

Even more convincing support for the idea that remnant speckles in a highly corrected adaptive optics imaging system are found only on rings (maxima) of the underlying diffraction-limited point-spread function may be found in simulations of atmospheric phase distortions. These simulations can explore a regime of higher Strehl ratio than is currently feasible with actual ground-based adaptive optics systems.

For this series of simulations, I used a Monte Carlo program to produce successive realizations of random atmospheric phase screens incorporating Zernike components Z_1 through Z_{10} (tip, tilt, and the next eight lowest-order, non-trivial terms). The weights in which these components enter are chosen to correspond to a Kolmogorov power spectrum, with coefficients as given by Beckers[7], although the detailed nature of the remnant phase aberrations is not important so long as they are small. An overall turbulent strength corresponding to a residual post-correction rms phase error of 0.35 radians, or a residual $D/r_0 = 2$, was chosen; this is equivalent to a Strehl ratio of 88%. The results of seven different realizations are shown in Figure 2 (the upper left panel is an ideal, diffraction-limited point-spread function calculated for the same telescope aperture, including effects of blockage by the secondary mirror and by its supports).

It is apparent that remnant speckles have a very strong tendency to appear on Airy rings. In some cases, the Airy rings are themselves slightly distorted by speckles; this does not occur when the correction is extremely high, as expected in the limit of very small phase perturbations.

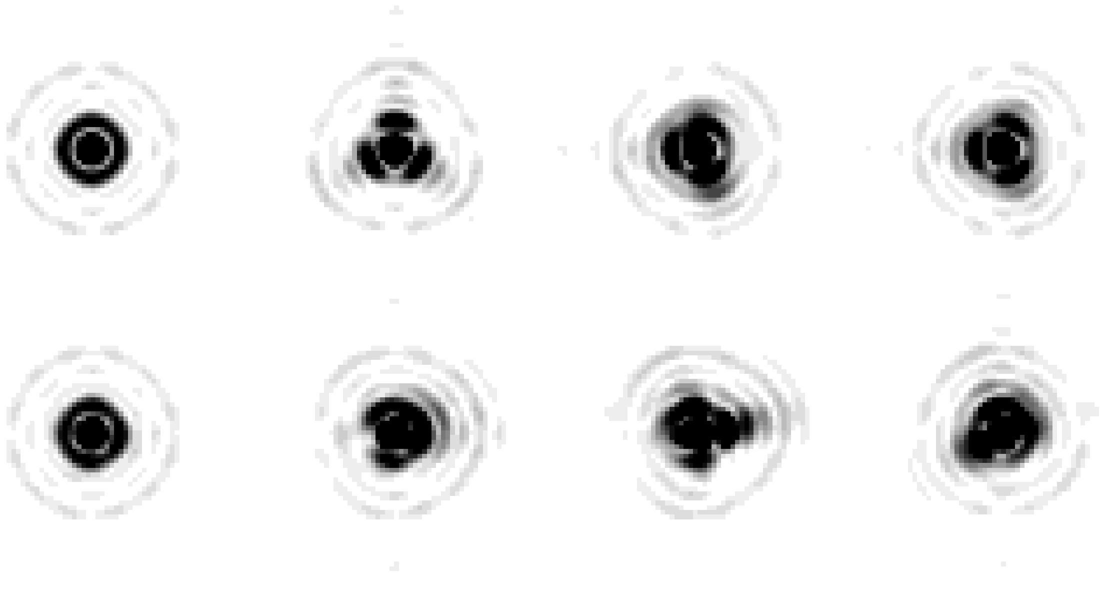

Figure 2. Numerical simulations of an adaptively corrected point-spread function degraded by atmospheric turbulence. The turbulence is modelled by random phase screens incorporating Zernike components Z_0 through Z_{10} in ratios of strength corresponding to a Kolmogorov power spectrum. The upper left panel is a diffraction calculation of the point-spread function of an ideal telescope aperture, including 36% secondary obscuration and including the effects of the spiders (secondary supports). The Strehl assumed for these simulations is 88%, corresponding to a residual post-correction rms phase error of 0.35 radians, or a residual $D/r_0 = 2$. It is apparent that remnant speckles have a very strong tendency to appear on Airy rings. In some cases, the Airy rings are in turn slightly distorted by speckles (this does not occur when the correction is extremely high).

6. ALGEBRAIC DEMONSTRATION OF THE CONNECTION BETWEEN SPECKLES AND THE DIFFRACTION-LIMITED PSF

To sees how the connection between remnant speckles and the diffraction-limited point-spread function arises, it is instructive to examine the Fourier-optical connection between the focal plane, where speckles appear, and the pupil plane of the telescope, where the phase aberrations from atmospheric turbulence that cause speckles are found. If the telescope is described by a real aperture function $A(\xi, \eta)$ in pupil coordinates ξ and η, and phase aberrations over the pupil are described by a phase function $\Phi(\xi, \eta)$, then the focal-plane image will be[8]

$$Image(x, y) = |F.T.\{A(\xi, \eta)e^{i\Phi(\xi,\eta)}\}|^2 \tag{1}$$

Scintillation and any other effects that would change the amplitude of the wavefront are neglected here.

The above equation can be used to describe the residual images after correction by an adaptive optics system, if the phase function Φ is simply interpreted as the residual (uncorrected) phase. The situation being considered is an extremely high-quality correction, for which Φ is small (much less than a radian). One may then expand the exponential in equation (1), and continue to evaluate the algebra while keeping only terms up to terms linear in Φ:

$$Image(x, y) = |F.T.\{A(\xi, \eta)e^{i\Phi(\xi,\eta)}\}|^2 \approx |F.T.\{A(\xi, \eta)[1 + i\Phi(\xi, \eta)]\}|^2 \qquad (\Phi \ll 1) \tag{2}$$

$$= |F.T.\{A(\xi,\eta)\} + F.T.\{iA(\xi,\eta)\Phi(\xi,\eta)\}|^2 \tag{3}$$

$$= (F.T.\{A\} + iF.T.\{A\Phi\})((F.T.\{A\})^* - i(F.T.\{A\Phi\})^*) \tag{4}$$

$$\approx |F.T.\{A\}|^2 - iF.T.\{A\}(F.T.\{A\Phi\})^* + i(F.T.\{A\})^* F.T.\{A\Phi\} \tag{5}$$

$$= |F.T.\{A\}|^2 + 2Re[i(F.T.\{A\})^* F.T.\{A\Phi\}] \tag{6}$$

$$= |F.T.\{A\}|^2 + 2Re[i(F.T.\{A\})^*(F.T.\{A\} \star F.T.\{\Phi\})] \tag{7}$$

The first term in the last equation is the diffraction-limited point-spread function, which may be denoted PSF_0, so

$$Image = PSF_0 + 2Re[i(F.T.\{A\})^*(F.T.\{A\} \star F.T.\{\Phi\})] \tag{8}$$

The second term involves a convolution between the Fourier transforms of the aperture function and the phase function; this convolution will be at least as broad as some spatial scales characteristic of each. In the case of the aperture function, the width is related to the diffraction limit of spatial resolution, while phase aberrations will cause additional focal-plane smearing on a scale given by their Fourier transforms.

The multiplication of the second term (the small correction term) in equation (8) by the amplitude of the Fourier transform of the aperture function impresses upon it the structure of the diffraction-limited point-spread function, to which it is related by a squared modulus; its zeroes and maxima will be at the same positions as those of the diffraction-limited point-spread function. So the speckles that correspond to the small perturbations to the Airy pattern will occur only on the rings of the Airy pattern.

7. CONCLUSION

The connection between remnant speckles in a highly-corrected adaptive optics system and the underlying diffraction-limited point-spread function has been demonstrated with experimental data, simulated data, and Fourier-optical imaging theory. Speckles in the highly-corrected case do not appear anywhere within a smooth Gaussian (or similar) envelope, but lie only on the rings of the underlying diffraction-limited point-spread function (which is here loosely called the "Airy" pattern, though that term is strictly appropriate only when the telescope aperture is an unobstructed circle).

This behavior of speckles is currently not widely recognized, but may have important applications to the problem of suppressing speckle noise, which is thought to be the dominant noise source that will hamper direct imaging of stellar companions and exosolar planets by future "extreme" adaptive optics systems. One may engineer the diffraction-limited point-spread function of an adaptive optics system by imposing arbitrary static aberrations on the deformable mirror, and it is possible in this way to create search regions that are free of Airy rings. According to the results presented in this paper, such regions will then be free of speckles and hence of speckle noise. More detailed discussion of these issues may be found in Bloemhof et al. (2001)[9].

Further work remains to be done to determine the practicality of this approach, and the degree of speckle-noise suppression that can be attained. Notwithstanding these questions, a more accurate model of the behavior of speckles in a highly corrected imaging system has been presented.

Further work must also be done to determine the combination of aberrations that are optimal for speckle noise suppression. It will be desirable to have search regions at arbitrary azimuth and radial separation from the peak of the point-spread function, to permit thorough searches for companions at unknown positions. Also, intentionally imposing static aberrations will in general reduce the height of the main lobe of the point-spread function, and thus reduce system sensitivity; selection of the best strategy must incorporate this effect (reduction of signal) in order to maximize signal-to-noise ratio for companion detection.

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