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Calibration of the Barnes-Evans Relation Using Interferometric Observations of Cepheid Variables

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ABSTRACT

Direct diameter measurements of Cepheid variables are used to calibrate the Barnes-Evans Cepheid surface brightness relation. More than 50 separate Cepheid diameter measurements from four different optical interferometers are used to calculate surface brightnesses as a function of magnitude and color. For two Cepheids, η Aquilae and ζ Geminorum, high precision diameter measurements as a function of pulsation phase are available from the Palomar Testbed Interferometer (PTI). Relations using only these diameters are found for each individual Cepheid in order to search for differences between Cepheids of different pulsation period. In all cases the best-fit relations are simple linear relations between surface brightness and color with the constraint that for a spectral type A0 star (where all colors equal zero) all relations must yield the same surface brightness (i.e., there must be a common zero-point). The derived relations found using interferometric Cepheid diameters are consistent with functions in the literature found using interferometric observations of non-variable giant and supergiant stars. In addition, while the separate relations for η Aquilae and ζ Geminorum are marginally consistent within the errors they do differ in the direction predicted for Cepheids of differing pulsation period. Using these new surface brightness relations the distance is calculated to the nearby Cepheid δ Cephei for which a new distance has been found using trigonometric parallax with the Hubble Space Telescope. These distances are well within the errors of the distance derived from trigonometric parallax.

Keywords: Cepheids — stars: fundamental parameters — stars: individual (delta Cephei, eta Aquilae, zeta Geminorum

1. INTRODUCTION

The Period-Luminosity relation for Cepheids is an important means for setting the entire extragallactic distance scale. In order to properly callibrate the P-L relation for Cepheids it is necessary to have accurate and precise distances to a large sample of Cepheids. The pulsation parallax method, by which angular diameter variations are coupled with linear diameter displacements, is one means by which Cepheid distances are determined.

While linear diameter changes are derived in a relatively straight forward manner from radial velocity measurements, angular diameter changes have until recently only been estimated from photometric surface brightness relations.[1] In the last five years, there have been numerous direct measurements of Cepheid angular diameters by long-baseline optical interferometry. Angular diameter variations have been measured at very high signal to noise ratios for the nearby Cepheids ζ Geminorum[2,3] and η Aquilae,[3] while mean angular diameters have been measured for δ Cephei[4,5] and Polaris.[5] Thus, while direct interferometric measurements of pulsation are currently confined to the few Cepheids within half a kiloparsec of the Sun, these observations can be used to calibrate the surface brightness relations necessary for estimating the angular diameters of much more distant Cepheids in this galaxy and beyond. The purpose of this paper is to: (1) present several of these calibrations of the Cepheid surface brightness relation which have been recently published in the literature[3,6]; (2) calculate the distance to the nearby Cepheid δ Cephei using each; and (3) compare the results with the most recent distance as determined from trigonometric parallax.[7]

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2. SURFACE BRIGHTNESS RELATIONS

The Baade-Wesselink method[1] for estimating the angular diameter of Cepheids from their surface brightness has produced several versions which have undergone a variety of calibrations in the last several decades.[8–10] One of these, the Barnes-Evans[8] surface brightness relation has both optical and infrared versions.[11,12] These relations have been used to calculate the angular diameters of, and hence distance to, over a hundred Cepheids in the Milky Way and the Magellanic Clouds.[13,14]

For this method, the surface brightness F in a given band i is related to the apparent magnitude in that band m_i and angular diameter θ by

$$F_i = 4.2207 - 0.1m_i - 0.5\log(\theta). \tag{1}$$

With the above relation and a good estimate of F_i one can determine the angular diameter based on photometry alone. Conversely, given measured angular diameters and multi-band photometry it is possible to calibrate F_i by finding a simple (e.g., linear) relation between F_i and a variety of color indices (e.g., $V-K$). We define the following relations

$$F_V = a + b(V-K) \tag{2}$$

$$F_V = a + c(V-R) \tag{3}$$

Consistency requires a common zero-point (cf. an A0V star where $(V-R) = (V-K) = 0$). Note, all magnitudes and colors in this paper may be assumed to be de-reddened. Detailed analyses of the de-reddening can be found in Refs. 3, 6. Equating these two groups of equations yields the angular diameter of a Cepheid at a given pulsation phase purely as a function of its magnitude and color at that phase:

$$\log\theta = -0.5(a - 4.2207 + 0.1V + b(V-K)) \tag{4}$$

$$\log\theta = -0.5(a - 4.2207 + 0.1V + c(V-R)) \tag{5}$$

where we have chosen $m_i = V$ band.

Prior to 1997, no Cepheid interferometric angular diameters were available. These versions of the Barnes-Evans relation were therefore calibrated using interferometric angular diameter observations of non-variable late-type giant and supergiants stars.[12] In that work Fouqué and Gieren[12] used the non-variable stellar observations to set the zero-point of the relation between surface brightness (F_V, and the colors: $(V-R)$, $(V-K)$.

3. CEPHEID CALIBRATION

Over 50 separate interferometric observations of Cepheid angular diameters can now be found in the literature.[2–5,15–17] These observations were made using four separate optical and infrared interferometers: Grand Interféromètre à 2 Télescopes (GI2T), Navy Prototype Optical Interferometer (NPOI), Palomar Testbed Interferometer (PTI), and Infrared-Optical Telescope Array (IOTA). The quite good consistency in the Cepheid observations using these different instruments has been shown elsewhere.[2]

With these recent interferometric observations the Barnes-Evans surface brightness relation for Cepheids has been fully calibrated using nothing but observations of Cepheids.[3,6] Nordgren et al.[6] used the set of all 59 Cepheid observations available in the literature as of 2001 to calibrate relations in $(V-R)$, $(V-K)$. Equation 1 is used to convert the 59 angular diameters to surface brightnesses. Color curves are generated with 0.01 intervals in pulsation phase from published photometry (a detailed analysis is given in Ref. 6). Surface brightness and colors as a function of pulsation phase are then plotted to calculate the coefficients in Equations 2 and 3. Figure 1 is taken from Ref 6 and displays F_V versus $(V-K)$ for the 59 Cepheid observations, where the solid line is the weighted linear least-squares fit to the Cepheid observations which is given in Equation (15) of Ref. 6. The dashed line is the corresponding relation from Fouqué and Gieren[12] given in their Equation (27).

Table 1 lists the values derived for the coefficients in Equations 2 and 3 based upon the 59 Cepheid observations.

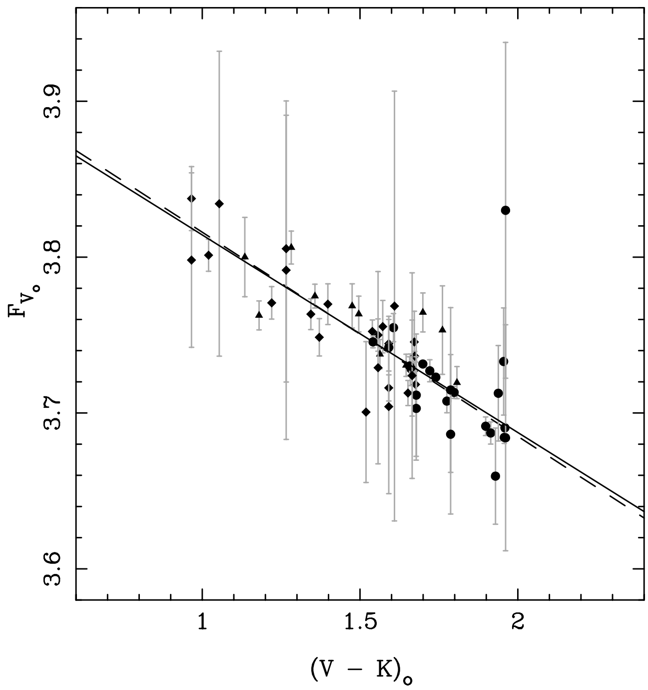

Figure 1. De-reddened F_V versus (V-K) for 59 Cepheid observations. Filled circles are ζ Gem, diamonds are δ Cep, and triangles are η Aql. The solid line is the weighted linear least squares fit. Dashed line is the linear least squares fit of Fouqué and Gieren[12] from their Equation (27). Reprinted from Nordgren et al..[6]

Source	a	b	c
F&G Non-variables	3.947 ± 0.003	-0.131 ± 0.003	-0.380 ± 0.003
59 Cepheids	3.941 ± 0.004	-0.125 ± 0.003	-0.368 ± 0.007
η Aql PTI	3.941 ± 0.005	-0.125 ± 0.004	-0.375 ± 0.002
ζ Gem PTI	3.946 ± 0.011	-0.130 ± 0.002	-0.378 ± 0.003

Table 1: A comparison between the various surface brightness relations (see text for definitions).

3.1. η Aql and ζ Gem

The majority of the 59 angular diameter measurements making up the previous sample are not of sufficient precision to directly detect the Cepheid pulsation, hence the large scatter exhibited around the least-squares fit line in Figure 1. However, recent observations by PTI of the Cepheids η Aql and ζ Gem have clearly detected their angular diameter change (see Fig 2). In doing so, the distance to these Cepheids has been calculated.[3] Here we use the measured angular diameters to calibrate the same surface brightness relations as in the previous section. Figure 3 shows the surface brightnesses calculated by the angular diameters in Figure 2 versus $(V-K)$. The linear least-squares fit line is shown along with those found from Fouqué and Gieren[12] and Nordgren et al.[6]

The coefficients in Equations 2 and 3 resulting from the PTI data are listted in Table 1. None of the PTI η Aql data in Figure 2 appear in Figure 1, as a result the entry for η Aql in Table 1 is completely independent of the entry for "59 Cepheids." There is, however, some overlap in between the "59 Cepheid" sample and the data for ζ Gem.

The entries in Table 1 together with Equations for $\log\theta$ in Equations 4 and 5 result in eight separate equations whereby a Cepheid's angular diameter as a function of pulsation phase may be determined.

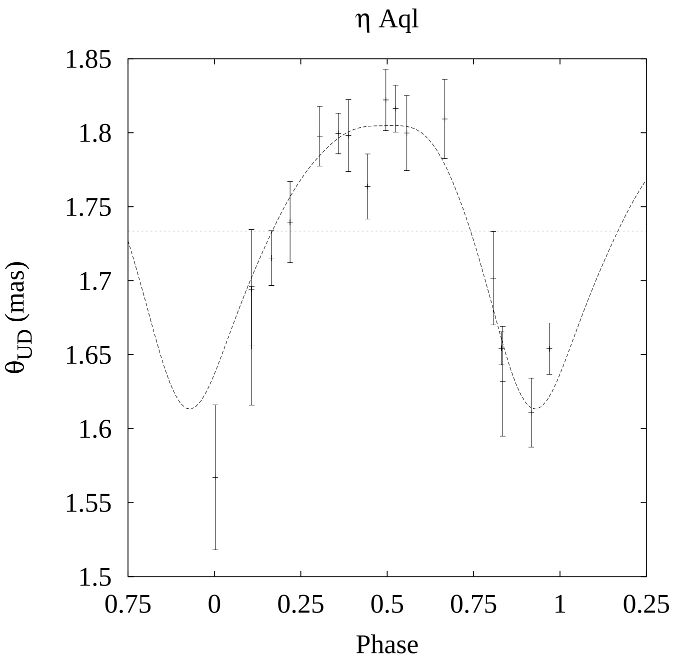

Figure 2. The angular diameter of η Aql as a function of pulsational phase, together with a model based on radial velocity data, but fitting for distance, mean radius and phase shift. Also shown is the result of fitting a straight line to all the data. The fits are extended past phase 0 for clarity. Reprinted from Lane et al.[3]

In the following section, we will use the $V-K$ relations to generate angular diameters for the nearby Cepheid, δ Cephei. These "infrared" surface brigtness relations have a higher precision than the $V-R$ relations and are less suseptible to the reddening effects of dust.[12] With these angular diameters and linear displacements calculated from radial velocities in the literature, we will calculate the distance to δ Cephei using each of the $V-K$ relations in Table 1 and compare them with the distance derived from trigonometric parallax.

4. DISTANCE TO δ CEPHEI

The distance and mean radius of a pulsating star can be found from the equation:

$$D_o + \Delta D(t) = 10^{-3} d \times \theta(t) \tag{6}$$

where D_o is the mean linear diameter in AU, $\Delta D(t)$ is the linear displacement in AU at a time t, d is the distance in parsecs and $\theta(t)$ is the angular diameter in milliarcseconds at time t.

We have chosen to calculate the distance to the nearby Cepheid, δ Cephei, as there is a wealth of photometry and radial velocity measurements from which to derive surface brightnesses and linear displacements. In addition, a distance to this Cepheid has been recently measured based on trigonometric parallax using high precision astrometry with the Hubble Space Telescope's Fine Guidance Sensor.[7] The distance found from the parallax in Ref 7 is 278 $\pm$ 15 pc. An average parallax of the HST, Hipparcos and Allegheny parallaxes[7] results in a distance of 284 $\pm$ 14 pc (see Table 2).

F_V is calculated using 50 V magnitudes in the literature.[18] These magnitudes plus the interpolated $V-K$ color curve from Nordgren et al.[6] and the values in Table 1 yield 50 angular diameters for δ Cephei as a function of pulsation phase (Figure 4).

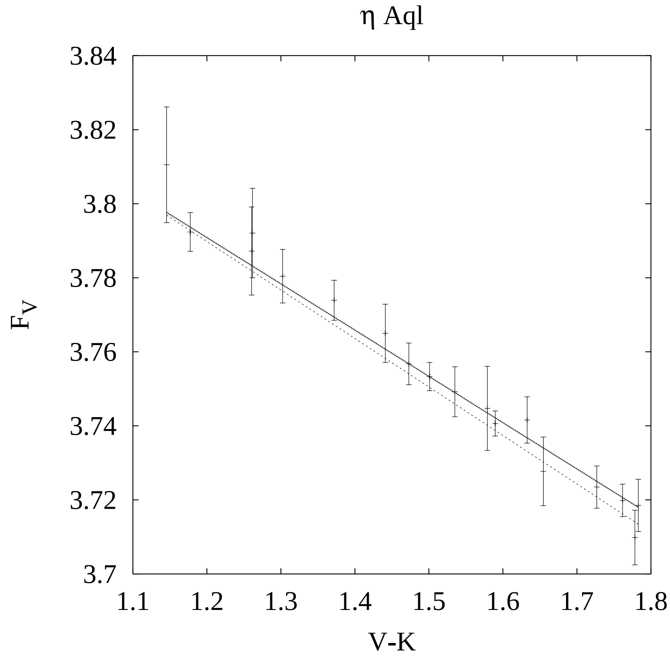

Figure 3. De-reddened F_V vs. $V-K$ for η Aql. The solid line is the weighted linear least-squares fit to the data. The dashed line represents the relation from Fouqué and Gieren,[12] and the dotted line represents the Nordgren et al.[6] result. The solid line dotted lines are nearly coincident. Reprinted from Lane et al.[3]

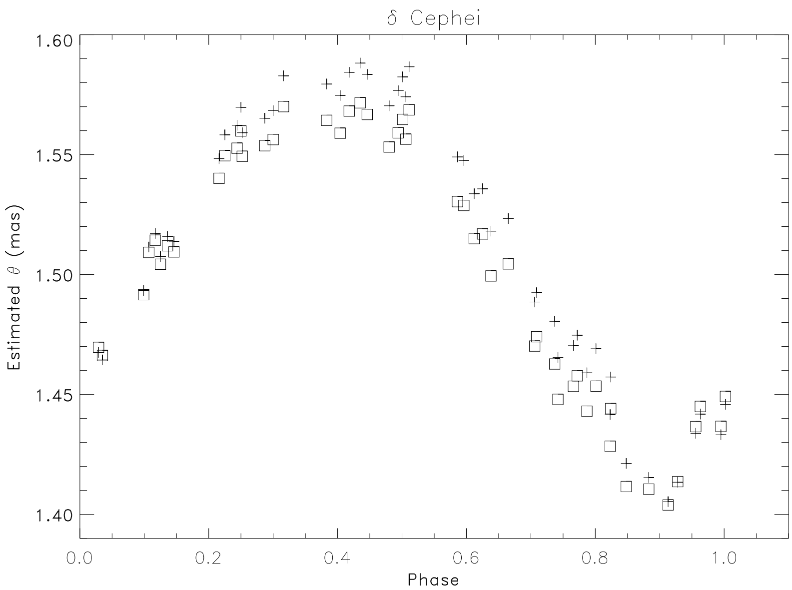

Figure 4. The estimated angular diameter of δ Cep as a function of pulsation phase. Squares are angular diameters using the calibration of all 59 Cepheid observations. Crosses use the calibration of the PTI ζ Gem observations (the η Aql PTI calibration being nearly identical to the 59 Cepheid calibration).

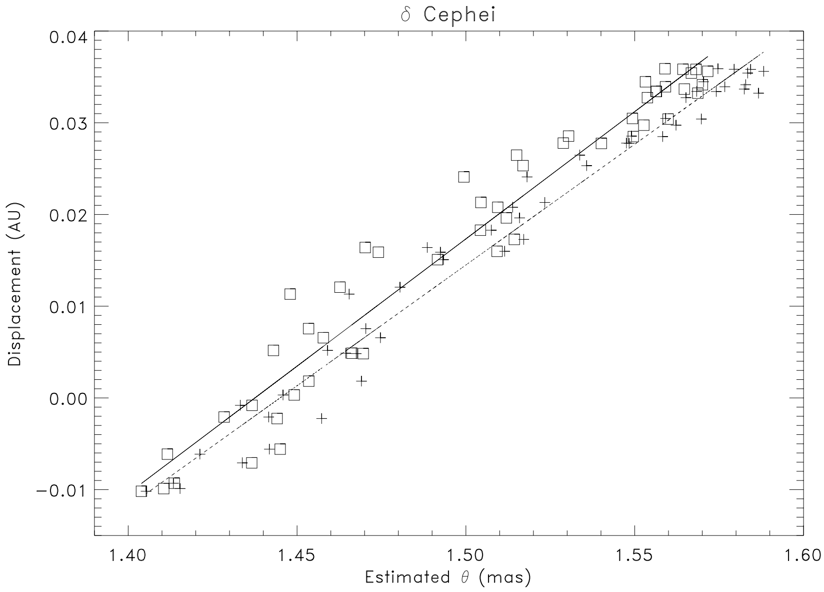

Figure 5. The displacement as a function of estimated angular diameter for δ Cep. Squares are using the calibration of all 59 Cepheid observations with the linear least-squares fit shown by the solid line. Crosses use the ζ Gem PTI calibration with the linear least-squares fit denoted by the dotted line.

Source	Dist. (pc)
HST parallax	278 ± 15
HST + H&A	284 ± 14
F&G Non-variables	259 ± 6
59 Cepheids	277 ± 9
η Aql PTI	279 ± 10
ζ Gem PTI	263 ± 6

Table 2. Comparison between distances calculated from the various surface brightness relations (see text for definitions).

As described in Ref 6 smooth curves were drawn through published radial velocities[19,20] and fitted with cubic splines in order to interpolate in phase. These interpolated radial velocities are converted to pulsation velocities using the projection factor[21] $p = 1.368 \pm 0.03$. It is important to note that the projection factor has a direct effect on the final distance determination. Nordgren et al.[6] used $p = 1.31$. Since that paper was primarily interested in comparing competing calibrations, the exact value of the projection factor was not important as long as it was consistent between calibration models. In this paper we wish to compare the derived distances with the distance found from the HST parallax and therefore we use the more current value from Ref 21.

Linear radii displacements are found by integrating the pulsation velocities. These displacements are then matched to the angular diameters found using the photometry. Level effects in stellar atmospheres would admit a slight phase lag in the linear radii displacements, relative to the angular diameters. In fact the best fit was found with zero phase shift. Using these angular diameters and linear displacements the distance and mean radius for δ Cephei are found. Figure 5 shows the plot of displacement versus angular diameter using two different Cepheid calibrations. Table 2 lists the newely calculated distances to δ Cephei using angular diameters from Table 1, and Equations 5 and 6.

5. RESULTS

The coefficients in Table 1 are all marginally consistent. The two calibrations based on Cepheid observations which are in the best agreement are also the ones which are completely independent of one another. While the two calibrations based on individual Cepheids do differ slightly, it is important to note that this is what is expected. For F_V vs. $V-R$ the slope of the $V-R$ surface brightness relation (c) is weakly dependent on pulsational period[22] (P) according to

$$c = -0.359 - 0.020 \log P, \tag{7}$$

which for η Aql predicts $c = -0.376$ and for ζ Gem $c = -0.379$. These comparisons reveal generally good agreement between the various relations in Table 1.

Table 2 shows that the surface brightness distances fall into two groups marginally consistent with one another. The two calibrations in the best aggreement with the HST result are the ones for η Aql and all 59 Cepheids combined. Again, it should be noted that the data from which each is calculated are completely independent as the η Aql PTI data were not available for the Nordgren et al. sample. Meanwhile the HST parallax incorporating Hipparcos and Allegheny observations differ in the opposite direction from the ζ Gem and Fouqué and Gieren non-variable star calibrations. Given the size of the error bars on the HST parallax observations though, no calibration scheme can be ruled out.

6. CONCLUSION

Direct diameter Measurements of Cepheid variables are used to calibrate the Barnes-Evans Cepheid surface brightness relation. For two Cepheids, η Aquilae and ζ Geminorum, high precision diameter measurements as a function of pulsation phase are available. Relations using only these diameters are found for each individual Cepheid in order to search for differences between Cepheids of different pulsation period. In all cases the best fit relations are simple linear relations between surface brightness and color. The derived relations found using interferometric Cepheid diameters are consistent with functions in the literature found using interferometric observations of non-variable giant and supergiant stars. In addition, while the separate relations for η Aquilae and ζ Geminorum are marginally consistent within the errors they do differ in the direction predicted for Cepheids of differing pulsation period. Using these new surface brightness relations the distance is calculated to the nearby Cepheid δ Cephei for which an HST trigonometric parallax distance is known. These distances are well within the errors of the distance derived from trigonometric parallax.

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