distribution of particles due to interactions was made by Peskin,[13] who accounted for fluid forces heuristically.

[1] F. Marble, Phys. Fluids **7**, 1270 (1964).
[2] H. Lamb, *Hydrodynamics* (Dover Publications, Inc., New York, 1932), p. 133.
[3] I. Langmuir and K. B. Blodgett, General Electric Report RL-225 (1945); J. Meteor. **5**, 175 (1948).
[4] W. Ranz and J. Wang, Ind. Eng. Chem. **44**, 1371 (1952).
[5] N. A. Fuchs, *The Mechanics of Aerosols* (The Macmillan Company, New York, 1964), pp. 159, 181.
[6] N. R. Lindblad and R. G. Semonin, J. Geophys. Res. **68**, 1051 (1963).
[7] H. Goldstein, *Classical Mechanics* (Addison-Wesley Publishing Company, Reading, Massachusetts, 1950), p. 58.
[8] J. R. Kliegel and G. R. Nickerson, AIAA Paper 1713–61 (1961).
[9] S. L. Soo, in *Proceedings of Symposium on Interaction between Fluids and Particles* (Institute of Chemcial Engineers, London, 1962), p. 50; J. O. Hinze, Appl. Sci. Res. **A11**, 33 (1962).
[10] S. L. Soo, Project SQUID Report ILL-18-P (1964).
[11] S. Timoshenko, *Theory of Elasticity* (McGraw-Hill Book Company, Inc., New York, 1934), p. 339.
[12] F. E. C. Culick, Phys. Fluids **7**, 1898 (1964).
[13] R. L. Peskin, in *Proceedings of the Heat Transfer and Fluid Mechanics Institute* (Stanford University Press Stanford, California, 1960).

Reply to Comments by S. L. Soo

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If one takes due regard for the condition under which my collision model is valid, explicitly stated by Eq. (15), Ref. 1, the difficulties experienced by Soo[2] will not arise.

Briefly, the condition of validity for use of the geometric cross section (Eq. 15, Ref. 1) states that the velocity equilibration time of a particle must be long in comparison with the time of a particle–particle encounter. If Re is the particle Reynolds number based upon the relative speed between particles before collision, ρ_s the density of the solid material, and ρ the density of the gas, this condition may be expressed as $\frac{2}{9}$ Re $\rho_s/\rho \gg 1$. Referring to the data of Ranz and Wong,[3] one finds that the example cited by Soo corresponds to $\frac{2}{9}$ Re $\rho_s/\rho \approx 0.15$, thus not satisfying the condition of validity. It is clear, therefore, that the dominant mechanism operative in this example has little in common with that described in my paper and, in fact, corresponds more nearly to the "noninertial" encounters discussed by Mason.[4]

On the other hand, if the results of Ranz and Wong (Fig. 11, Ref. 3) are interpreted in terms of $\frac{2}{9}$ Re ρ_s/ρ for particles of nearly equal radii, one finds that the observed number of particle–particle impacts exceeds 80% of the number expected from the geometric cross section when $\frac{2}{9}$ Re ρ_s/ρ is greater than 2. These data suggest that my collision model is nearly correct for $\frac{2}{9}$ Re $\rho_s/\rho \sim 5$. One must avoid, however, the temptation to interpret these measured impacts as momentum exchanging collisions. Hence, until detailed data are available that permit weakening the condition, the appropriate criterion for utilizing the geometric cross section is $\frac{2}{9}$ Re $\rho_s/\rho \gg 1$, as given in Ref. 1.

The direct (solid to solid) heat transfer between colliding particles was justifiably neglected in my paper,[1] although the detailed reasons were not elaborated. The appropriate estimate for the importance of this phenomenon is obtained from the ratio of heat exchanged by direct collision to that transferred between gas and particles. The order of magnitude of this ratio is

$$\kappa\left(\frac{k_s}{k}\right)\left(\frac{\rho}{\rho_s}\right)\left(\frac{v}{\alpha/\sigma}\right)^{1/2}\left(\frac{v}{a_s}\right)^{4/5},$$

where κ is the ratio of particle mass per unit volume to gas mass per unit volume, k_s/k is the ratio of thermal conductivity of solid to that of gas, ρ/ρ_s is the ratio of gas density to density of the solid material, v is the relative particle velocity before impact, α is the thermal diffusivity of the solid, σ is the radius of particle, and a_s is the dilatational wave velocity in solid.

Consider a mixture for which the mass is evenly divided between atmospheric air and a solid, the latter consisting of spherical particles having radii of 1 and 2 μ, the relative velocity between these two particle classes being 250 ft/sec. If the particles are of low carbon steel, the above ratio is 0.02; if the particles are of lead (and collisions are considered elastic), the ratio is 0.019. In both cases, the heat transferred by direct collision is clearly negligible in comparison with that transferred by conduction to the gas. In addition, the collision condition $\frac{2}{9}$ Re $\rho_s/\rho \gg 1$ is satisfied.

In summary: (1) the geometric collision cross section is fully justified under the conditions stated in Ref. 1; (2) the heat transferred through direct particle collision is negligible for ordinary solids under the conditions stated in Ref. 1; (3) the mechanism described and the subsequent results of Ref. 1 are correct as given.

[1] F. E. Marble, Phys. Fluids **7**, 1270 (1964).
[2] S. L. Soo, Phys. Fluids **8**, 1751 (1965).
[3] W. E. Ranz and J. B. Wong, Ind. Eng. Chem. **44**, 1371 (1952).
[4] R. St. J. Manley and S. L. Mason, J. Coll. Sci. **7**, 354 (1952); W. Bartok and S. L. Mason, J. Coll. Sci. **12**, 243 (1957); R. S. Allan and S. L. Mason, J. Coll. Sci. **17**, 383 (1962).

Comments on "General Analysis of the Stability of Superposed Fluids"

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THE stability, with respect to two-dimensional disturbances, of two superposed, perfect, compressible fluids separated by a horizontal vortex sheet (i.e., a discontinuity in tangential velocity) in a gravitational field has been considered by Plesset and Hsieh.[1] They found an infinite, discrete set of modes that contains infinite subsets of stable and of unstable modes. They contrasted their results with those of Landau,[2] who neglected the gravitational field ($g = 0$) and found only a finite number of modes, each of which is stable for sufficiently *large* values of the velocity discontinuity. Landau observed that the high-speed stability predicted by his analysis was not found experimentally and conjectured that the discrepancy might be associated with finite-amplitude effects. (Landau and Lifschitz[3] are quite unequivocal about the instability of vortex sheets emanating from triple intersections of shock waves and, at least in this contex, do not mention Landau's 1944 paper.) Plesset and Hsieh suggested that the unstable gravity waves associated with density stratification, and therefore not comprised by Landau's model, may be responsible for the observed instability. They also stated that their limiting results for $g \to 0$ do not agree with those obtained by Landau in the "supersonic case."

Let us refer to the modes determined by Landau as *inertial waves* and to the additional modes determined by Plesset and Hsieh as *gravity waves*. (These definitions may not be sufficiently distinct in the most general context, but they suffice for the present discussion.) We then assert that: (a) inertial waves travelling at sufficiently oblique angles with respect to the interfacial discontinuity in velocity are unstable for any value of this discontinuity; and (b) the growth rates of the unstable gravity waves are $O(g)$ as $g \to 0$. We also suggest, on the basis of (a) and (b), that instability of obliquely directed inertial waves is a more plausible explanation of the typically observed instabilities than is the instability of two-dimensional gravity waves.

Assertion (a) has been demonstrated for Landau's model by Fejer and Miles,[4] who remarked that obliquely directed small disturbances, characterized by the x, y, t-dependence

$$\exp \{i[nt - \sigma(x \cos \theta + y \sin \theta)]\}, \tag{1}$$

of a vortex sheet of strength $|U_2 - U_1|$ are equivalent to two-dimensional ($\theta = 0$) disturbances of a vortex sheet of strength $|U_2 - U_1| \cos \theta$. This simple extension of Squire's theorem[5] applies equally to the more general Plesset–Hsieh model. Assertion (a) then follows from the fact that an unstable inertial wave exists for $|U_1 - U_2| \to 0$.

To prove (b), we require an appropriate, asymptotic approximation to the dispersion equation. [The approximation (46) of Ref. 1 is, in our opinion, incorrect.] We begin by choosing a reference frame moving with the mean velocity of the two fluids, such that

$$U_1 = -U_2 = U, \tag{2}$$

and introducing the similarity parameters ($M_{1,2}$, Mach numbers; $R_{1,2}$, Richardson numbers; and Z, an acoustic-impedance ratio)

$$M_j = U/a_j, \qquad R_j = (\gamma_j - 1)g/\sigma a_j^2,$$
$$Z = \rho_0^{(1)} a_1^2 / \rho_0^{(2)} a_2^2 \qquad (j = 1, 2), \tag{3}$$

and the frequency-dependent variables

$$\sec \alpha_j = M_j[1 + (-)^j \nu], \qquad \nu = n/\sigma U. \tag{4}$$

We then can rewrite the solutions of (29′) and (31) of Ref. 1 in the form

$$\varphi_j = A_j s_j^{m_j - \frac{1}{2}} W^{(j)}_{-k_j, -m_j}(-s_j), \qquad s_j = 2(\sigma y - R_j^{-1}), \tag{5a, b}$$

$$k_j = -\tfrac{1}{2} R_j^{-1} \sec^2 \alpha_j, \qquad m_j = \tfrac{1}{2}(\gamma_j - 2)/(\gamma_j - 1), \tag{6a, b}$$